

MICROECONOMICS - PERFECT COMPETITION

1 EXERCISE 1 - PROFIT MAXIMIZATION IN THE SHORT RUN

1.1 Theoretical premises

Profit maximization: the producer as a rational economic agent acts in order to maximize his net benefit, which is referred to as profit. Mathematically speaking, the objective of a firm is:

$$\max \pi = TR - TC$$

Which means that the firm wants to maximize the profit, that is defined as the difference between the total revenue and the total cost.

Calculus tells us that such objective is reached providing that the following condition is fulfilled:

$$MR = MC$$

Which means that the marginal revenue equals the marginal cost.

Profit maximization problem in the short run: the profit maximization problem in the short run is a constrained maximization problem; namely:

$$\begin{aligned}\max \pi &= TR - TC = P(Q)Q - TC \\ s.t. \quad Q &= f(L; K) \\ s.t. \quad P(Q) &= P_0 \\ s.t. \quad TC &= wL + rK \\ s.t. \quad K &= K_0\end{aligned}$$

The first constraint indicates that the firm has to convert inputs into outputs according to a given production function; the second constraint indicates that the firm considers the market price as a given: this assumption is valid with reference to a perfectly competitive market; the third constraint refers to the structure of the cost function; the last constraint is a peculiarity of the short run: assuming the capital cost to be a sunk cost, the firm is obliged to use the capital at the maximum level, in an effort to minimize the total cost and therefore to maximize the profit.

The system which represents the solution to this constrained maximization problem is:

$$\begin{cases} K = K_0 \\ MP_L = \frac{w}{P_0} \end{cases}$$

1.2 An example

Given that $Q = L^{0.5}K$, $w = 30$, $r = 20$, $K_0 = 16$, $P_0 = 30$, perform the following steps assuming a short run time horizon and sunk costs with respect to capital.

1. Set the profit maximization problem;
2. Set the system which represents the solution to this problem;
3. Determine the input bundle and the quantity which maximize the profit;
4. Calculate the maximum profit.

1.3 How to solve the example

1. In accordance with Paragraph 1.1, the constrained maximization problem is:

$$\begin{aligned}\max \pi &= P(Q)Q - TC \\ s.t. \quad Q &= L^{0.5}K \\ s.t. \quad P(Q) &= 30 \\ s.t. \quad TC &= 30L + 20K \\ s.t. \quad K &= 16\end{aligned}$$

2. As it can be argued from Paragraph 1.1, the system which represents the solution to the previous problem is defined as:

$$\begin{cases} K = 16 \\ MP_L = \frac{30}{30} \end{cases}$$

As a consequence, one is supposed to determine the marginal product function with respect to labor. Given that the production function is a Cobb Douglas function, one knows:

$$MP_L = \alpha L^{\alpha-1} K^{\beta} = 0.5 \frac{K}{L^{0.5}}$$

Knowing that $\frac{30}{30} = 1$, one gets:

$$\begin{cases} K = 16 \\ 0.5 \frac{K}{L^{0.5}} = 1 \end{cases}$$

3. In order to determine the optimal input bundle, one needs to solve the above determined system. This is a very easy business, since one variable (capital) is a given. Consequently, one is supposed to perform a substitution within the second equation of the system. Namely:

$$\begin{cases} K = 16 \\ 0.5 \frac{16}{L^{0.5}} = 1 \end{cases}$$

Performing the required calculations:

$$\begin{cases} K = 16 \\ \frac{8}{L^{0.5}} = 1 \end{cases} \leftrightarrow \begin{cases} K = 16 \\ \frac{L^{0.5}}{8} = 1 \end{cases} \leftrightarrow \begin{cases} K = 16 \\ L^{0.5} = 8 \end{cases} \leftrightarrow \begin{cases} K = 16 \\ L = 64 \end{cases}$$

In order to calculate the quantity of product which is associated with this optimal input bundle, one is supposed to perform a substitution within the production function; namely, 64 and 16 substitute for labor and capital.

$$Q^{max\pi} = 64^{0.5} 16 = 128$$

4. To start with, one knows that profit is generally defined as:

$$\pi = TR - TC$$

Consequently, with the aim of calculating the maximum profit, one needs to calculate the total revenue and the total cost associated with the quantity and the input bundle which maximize the profit. As regards the total revenue, it is easy to catch that:

$$TR^{max\pi} = P_0 Q^{max\pi} = 30 \cdot 128 = 3840$$

As regard the total cost, it important to remember that:

$$TC = wL + rK$$

Consequently:

$$TC^{max\pi} = wL^{max\pi} + rK^{max\pi} = 30 \cdot 64 + 20 \cdot 16 = 2240$$

As a result:

$$\pi^{max} = 3840 - 2240 = 1600$$

2 EXERCISE 2 - PROFIT MAXIMIZATION IN THE LONG RUN

2.1 Theoretical premises

Profit maximization problem in the long run: the profit maximization problem in the long run is the following constrained maximization problem:

$$\begin{aligned}\max \pi &= P(Q)Q - TC \\ s. t. Q &= f(L; K) \\ s. t. P(Q) &= P_0 \\ s. t. TC &= wL + rK\end{aligned}$$

As it is intuitive to get, the constraints are the same as those associated with the short run, except for the constraint related to the use of capital, which is a peculiar element of the short run problem.

The system which represents the solution to this problem is:

$$\begin{cases} MP_L = \frac{w}{P_0} \\ MP_K = \frac{r}{P_0} \end{cases}$$

This system implies the fulfillment of the following condition:

$$MSRP = \frac{w}{r}$$

Which is the cost minimization condition in the long run. This makes a lot of sense, since, in order for profit to be maximized, costs are to be minimized.

2.2 An example

Given that $Q = L^{0.25}K^{0.25}$, $w = 5$, $r = 5$, $P_0 = 40$, perform the following steps with reference to a long run time horizon.

1. Set the profit maximization problem;
2. Set the system which represents the solution to this problem;
3. Determine the optimal input bundle and the quantity of product which maximizes the profit;
4. Determine the maximum profit.

2.3 How to solve the example

1. As it can be argued from Paragraph 2.1, the profit maximization problem is:

$$\begin{aligned}\max \pi &= P(Q)Q - TC \\ s. t. Q &= L^{0.25}K^{0.25} \\ s. t. P(Q) &= 40 \\ s. t. TC &= 5L + 5K\end{aligned}$$

2. The system which represents the solution to the previously determined problem is:

$$\begin{cases} MP_L = \frac{5}{40} \\ MP_K = \frac{5}{40} \end{cases}$$

Since the production function is a Cobb Douglas function, one obtains:

$$\begin{aligned}MP_L &= \alpha L^{\alpha-1} K^{\beta} = 0.25 \frac{K^{0.25}}{L^{0.75}} \\ MP_K &= \beta L^{\alpha} K^{\beta-1} = 0.25 \frac{L^{0.25}}{K^{0.75}}\end{aligned}$$

Therefore, taking into consideration that $\frac{5}{40} = 0.125$, one obtains:

$$\begin{cases} 0.25 \frac{K^{0.25}}{L^{0.75}} = 0.125 \\ 0.25 \frac{L^{0.25}}{K^{0.75}} = 0.125 \end{cases}$$

3. In order to determine the optimal input bundle, one is required to solve the above determined system. It is not an easy business as the system is not linear and the exponents are quite demanding to handle. There is no one best way. Who is writing recommend you to follow this way. One can argue that the fulfillment of the equations of the system implies the fulfillment of this equation:

$$\frac{\alpha K}{\beta L} = \frac{w}{r}$$

Which, in this situation, is:

$$\frac{0.25 K}{0.25 L} = \frac{5}{5} \leftrightarrow \frac{K}{L} = 1$$

Given this, one is supposed to carry out two steps:

- Express K as a function of L ;
- Select the easiest equation of the system and perform a substitution.

As regards step a), one gets:

$$\frac{K}{L} = 1 \leftrightarrow K = L$$

As regards step b), in this case the equation of the system are equally easy to handle; one can choose for instance the first equation; by performing the substitution (L substitutes for K), one gets:

$$0.25 \frac{K^{0.25}}{L^{0.75}} = 0.125 \leftrightarrow 0.25 \frac{L^{0.25}}{L^{0.75}} = 0.125$$

Now, one is supposed to perform the required calculations in order to come up with the value of L . Namely:

$$0.25 \frac{L^{0.25}}{L^{0.75}} = 0.125 \leftrightarrow \frac{0.25}{L^{0.5}} = 0.125 \leftrightarrow \frac{L^{0.5}}{0.25} = \frac{1}{0.125} \leftrightarrow L^{0.5} = \frac{0.25}{0.125} \leftrightarrow L = 4$$

Since we have previously obtained that $K = L$, we can infer that the optimal input bundle is:

$$\begin{cases} L = 4 \\ K = 4 \end{cases}$$

In order to calculate the quantity which maximize the profit, one is required to perform the proper substitution within the production function; namely:

$$Q^{max\pi} = 4^{0.25} 4^{0.25} = 2$$

4. By applying the same rationale as that of Exercise 1, one obtains:

$$TR^{max\pi} = 40 \cdot 2 = 80$$

$$TC^{max\pi} = 5 \cdot 4 + 5 \cdot 4 = 40$$

As a result:

$$\pi^{max} = 80 - 40 = 40$$

3 EXERCISE 3 - SUPPLY CURVE UNDER COMPETITIVE CONDITIONS

3.1 Theoretical premises:

Perfect competition: a market is defined as perfectly competitive providing that four requirements are fulfilled:

1. **Homogeneity of products:** all the products that are marketed by the various firms are exactly identical or perfect substitutes; this means that the only lever that can be exploited by a single firm to compete is price.
2. **High number of economic agents:** there is a significant number of consumers and producers that are not able to individually influence the level of the market demand and the market supply.
3. **Perfect information:** any producer is perfectly informed about the preferences of all consumers and any consumer is perfectly informed about the characteristics of the products marketed by all firms.
4. **Absence of barriers to entry and exit:** firms can easily enter or exit the market.

The combination of these requirements causes each firm to consider the market price as a given.

Market supply curve in perfect competition: a market supply curve is generally built as the horizontal summation of the individual supply curves. In a competitive market, as firms are identical, the market supply curve is obtained as the individual supply curve multiplied by the number of firms which are present in the marketplace.

Each firm supplies a quantity which allows it to maximize its profit and therefore to fulfill this condition:

$$MR(Q) = MC(Q)$$

Since in perfect competition the market price is a given, the marginal revenue equals P ; as a result, the individual supply curve boils down to the marginal cost curve, in accordance with the following relationship:

$$P = MC(Q)$$

Market supply curve in perfect competition in the long run: in order to build the supply curve in the long run under competitive conditions one has to pay attention to the fourth requirement, according to which there is no barrier to entry and exit. This means that in the long run there is a market equilibrium providing that no potential entrant is interested in entering the market and no firm is interested in exiting the market. Such a situation is achieved if the individual surplus of each firm equals zero. This implies that each firm, in equilibrium, supplies the quantity that minimizes the average cost and therefore fulfills the following condition:

$$P = AC^{min}$$

As consequence, the supply curve in the long run is a horizontal straight line, at the level of the minimum average cost.

3.2 An example

In the short run 20 identical firms are present in a perfectly competitive market. Each of them is characterized by a total cost function $TC = Q^2 + 3Q + 16$. Perform the following steps:

1. Build the individual supply curve;
2. Build the market supply curve in the short run;
3. Build the market supply curve in the long run.

3.3 How to solve the example

1. In order to perform this step, one is supposed to determine the marginal cost function, which is defined as the derivative of the total cost function. Referring to the data:

$$MC(Q) = TC'(Q) = 2Q + 3$$

As it can be argued from Paragraph 3.1, the individual supply curve is:

$$P_i = 2Q_i + 3$$

2. The market supply curve is built by performing three steps:

- Use the individual supply curve to obtain the quantity as a function of the price;
- Multiply by the number of firms;
- Invert the equation to obtain the price as a function of the quantity.

As regards step a), one is supposed to invert the individual supply curve. Namely:

$$P_i = 2Q_i + 3 \leftrightarrow Q_i = \frac{P_i - 3}{2}$$

As regards step b), one obtains:

$$Q = 20 \left(\frac{P - 3}{2} \right) = 10P - 30$$

As the supply curve usually expresses the price as an output variable, one is supposed to invert the previously determined equation by performing step c). Namely:

$$P = \frac{Q}{10} + 3$$

3. In order to carry out this step, it is important to start with the determination of the average cost function, which is defined as the total cost divided by the quantity; namely:

$$AC(Q) = \frac{TC(Q)}{Q} = \frac{Q^2 + 3Q + 16}{Q} = Q + 3 + \frac{16}{Q}$$

As it can be argued from Paragraph 3.1, one is required to minimize this function. In order to do that, it is needed to determine the derivative and to make it equal zero. Namely:

$$AC'(Q) = 1 - \frac{16}{Q^2}$$

In an effort to make this derivative equal zero, one obtains:

$$\frac{16}{Q^2} = 1 \leftrightarrow Q = 4$$

As a consequence, the minimum average cost is obtained when 4 substitutes for Q within the average cost function. Namely:

$$AC^{min} = 4 + 3 + \frac{16}{4} = 11$$

4 EXERCISE 4 - EQUILIBRIUM UNDER COMPETITIVE CONDITIONS

4.1 Theoretical premises

Market equilibrium: the market equilibrium is a situation in which the market demand equals the market supply. Geometrically speaking, it is represented by the point on the $(Q; P)$ in which the supply curve and the demand curve intersect.

4.2 An example

In a perfectly competitive market there are 10 firms. Each of them is characterized by a total cost function $TC = Q^2 + 4$. The demand is $P = 10 - \frac{4}{5}Q$. Perform the following steps:

1. Determine the market supply curve;
2. Determine the market equilibrium in the short run;
3. Determine the consumer surplus and the producer surplus.

4.3 How to solve the example

1. In order to perform this step, one is supposed to come up with an individual supply curve, which can be derived from the cost function, applying the same rationale as that of Exercise 3. Namely, the individual supply curve is obtained by calculating the derivative of the total cost function:

$$P_i = 2Q_i$$

By inverting it one obtains:

$$Q_i = \frac{P_i}{2}$$

From this individual supply curve, it is possible to come up with a market supply curve. Multiplying by the number of firms, one gets:

$$Q = 5P$$

Inverting this function, one gets:

$$P = \frac{Q}{5}$$

This is the market supply curve.

2. In order to calculate the equilibrium quantity, one has to build the equilibrium equation, in which the demand curve (a given piece of information) equals the supply curve. Namely:

$$10 - \frac{4}{5}Q = \frac{Q}{5}$$

By solving this (extremely simple) equation, one gets:

$$Q^{sr} = 10$$

Where sr stands for “short run”.

The equilibrium price is calculated by performing a substitution within either the supply curve or the demand curve. Choosing the supply curve, one gets:

$$P^{sr} = \frac{10}{5} = 2$$

3. The consumer surplus, geometrically speaking, is the region below the demand curve and above the equilibrium price. Mathematically speaking, assuming the demand curve to be linear (and this will always be the case in this course of Microeconomics), it is determined in accordance with this formula:

$$CS = \frac{(\text{demand curve intercept} - \text{equilibrium price}) \cdot \text{equilibrium quantity}}{2}$$

In accordance with our numbers:

$$CS = \frac{(10 - 2) \cdot 10}{2} = 40$$

The producer surplus, geometrically speaking, is the region above the demand curve and below the equilibrium price. Mathematically speaking, assuming the demand curve to be linear (and this

will be always the case in this course of Microeconomics), it is determined in accordance with this formula:

$$PS = \frac{(\text{equilibrium price} - \text{supply curve intercept}) \cdot \text{equilibrium quantity}}{2}$$

In accordance with our numbers:

$$PS = \frac{(2 - 0) \cdot 10}{2} = 10$$

EXERCISES:

1 EXERCISE 1

With reference to the following data, perform these steps taking into consideration a short run time horizon and the hypothesis of sunk costs:

1. Set the profit maximization problem;
2. Set the system which solves this problem;
3. Determine the input bundle and the quantity which maximize the profit;
4. Determine the maximum profit.

1) $Q = L^{0.25}K^2$ $P_0 = 40$ $w = 20$ $r = 2$ $K_0 = 5$

2) $Q = L^{0.75}K$ $P_0 = 50$ $w = 25$ $r = 5$ $K_0 = 10$

3) $Q = L^{0.5}K^{0.5}$ $P_0 = 60$ $w = 20$ $r = 1$ $K_0 = 9$

2 EXERCISE 2

With reference to the following data, perform these steps taking into consideration a long run time horizon:

1. Set the profit maximization problem;
2. Set the system which solves this problem;
3. Determine the input bundle and the quantity which maximize the profit;
4. Determine the maximum profit.

1) $Q = L^{0.2}K^{0.2}$ $w = 1$ $r = 1$ $P_0 = 10$

2) $Q = L^{0.4}K^{0.4}$ $w = 2$ $r = 2$ $P_0 = 20$

3) $Q = L^{0.25}K^{0.25}$ $w = 3$ $r = 3$ $P_0 = 18$

3 EXERCISE 3

Perform the following steps with reference to these data.

1. Determine the individual supply curve;
2. Determine the market supply curve in the short run;
3. Determine the market supply curve in the long run.

1) $TC = \frac{1}{2}Q^2 + 2$ *n. of firms* = 6

2) $TC = 3Q^2 + 6Q + 1$ *n. of firms* = 6

3) $TC = Q^2 + 4$ *n. of firms* = 10

4 EXERCISE 4

Carry out the following steps with reference to these data and taking into consideration a short run time horizon.

1. Determine the market supply curve;
2. Determine the equilibrium quantity and the equilibrium price;
3. Determine the consumer surplus and the producer surplus.

1) $TC = 1.5Q^2 + 2$ $P = 10 - \frac{2}{3}Q$ *n. of firms* = 9

2) $TC = 2Q^2 + 6$ $P = 20 - Q$ *n. of firms* = 4

3) $TC = 5Q^2 + 26$ $P = 28 - 2Q$ *n. of firms* = 5

SOLUTIONS

1 EXERCISE 1

1)

3. $L = 29 \quad K = 5 \quad Q = 58$

4. $\pi = 1730$

2)

3. $L = 50625 \quad K = 10 \quad Q = 33750$

4. $\pi = 421825$

3)

3. $L = 20.25 \quad K = 9 \quad Q = 13.5$

4. $\pi = 396$

2 EXERCISE 2

1)

3. $L = 3.17 \quad K = 3.17 \quad Q = 1.59$

4. $\pi = 9.56$

2)

3. $L = 1024 \quad K = 1024 \quad Q = 256$

4. $\pi = 1024$

3)

3. $L = 2.25 \quad K = 2.25 \quad Q = 1.5$

4. $\pi = 13.5$

3 EXERCISE 3

1)

1. $P_i = Q_i$

2. $P = \frac{Q}{6}$

3. $P = 2$

2)

1. $P_i = 6Q_i + 6$

2. $P = Q + 6$

3. $P = 9.46$

3)

1. $P_i = 2Q_i$

2. $P = \frac{Q}{5}$

3. $P = 4$

4 EXERCISE 4

1)

1. $P = \frac{Q}{3}$

2. $Q = 10 \quad P = 3.33$

3. $CS = 33.35 \quad PS = 16.65$

2)

1. $P = Q$

2. $Q = 10 \quad P = 10$

3. $CS = 50 \quad PS = 50$

3)

1. $P = 2Q$

2. $Q = 7 \quad P = 14$

3. $CS = 49 \quad PS = 49$

If you have any kind of doubts or remarks, write to me (federico.marciano97@gmail.com). Good luck for your Microeconomics exam!