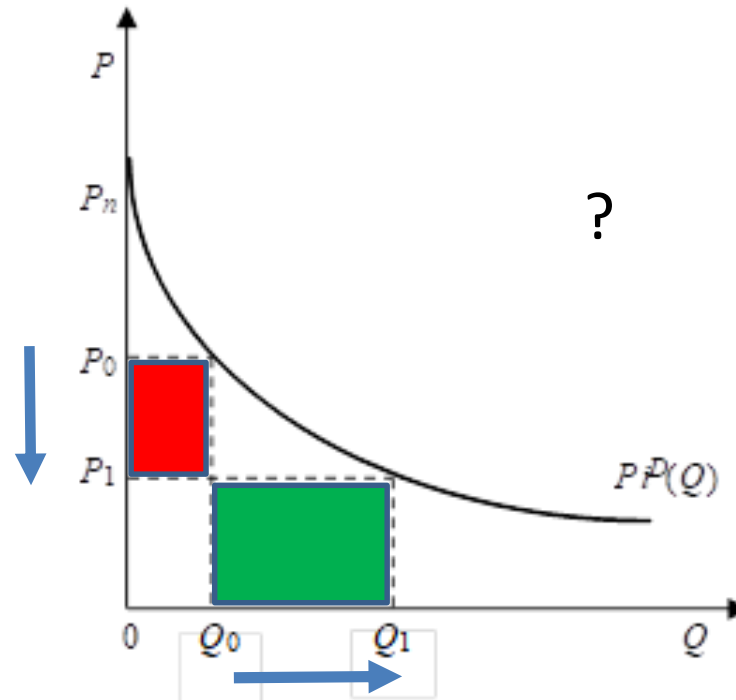




TOR VERGATA
UNIVERSITY OF ROME

The revenue dilemma, from Q_0 to $Q_1=Q_0+1$





An example

$P = 6$ $Q = ?$

$P = 10$ $Q = ?$

$$P_i^d(Q) = 10 - 2Q$$

$$2Q = 10 - P$$

$$(2Q/2) = (10/2) - P/2$$

$$Q_i^d(P) = 5 - \left(\frac{1}{2}\right) P$$

What happens to Total Revenues when quantities sold change?

$$P^d_i(Q) = 10 - 2Q$$

$$Q^d_i(P) = 5 - \left(\frac{1}{2}\right)P$$

$$Q = 1$$

$$Q = 3$$

$$\Delta Q = +2$$

$$P = ?$$

$$P = ?$$

$$\Delta P = -4$$

$$RT(1) = 8$$

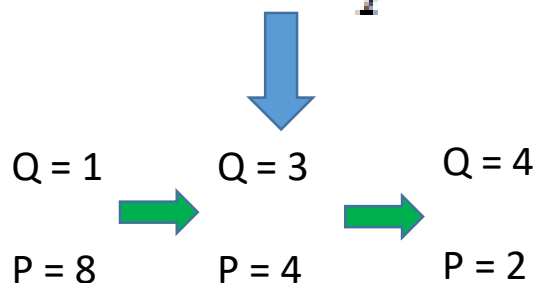
$$RT(3) = 12$$

May we build an index, an indicator, that can help us understand what happens to Total Revenues when quantities sold change?



$$Q^d_i(P) = 5 - \left(\frac{1}{2}\right) P$$

$$\sum_P^D = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} \quad \longrightarrow \quad \sum_P^D = \frac{\frac{\delta Q}{Q}}{\frac{\delta P}{P}} = \frac{\delta Q}{\delta P} \times \frac{P}{Q} \quad \longrightarrow \quad \sum_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$



$$\Sigma = (2/1)/(-4/8) = -4 \quad \Sigma = (1/3)/(-2/4) = -2/3$$

$$P^d_i(Q) = 10 - 2 Q$$



An example



$$\sum_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$

$$P(Q) = a - bQ,$$

$$Q(P) = (a/b) - (1/b)P,$$

The elasticity of the demand curve
is:

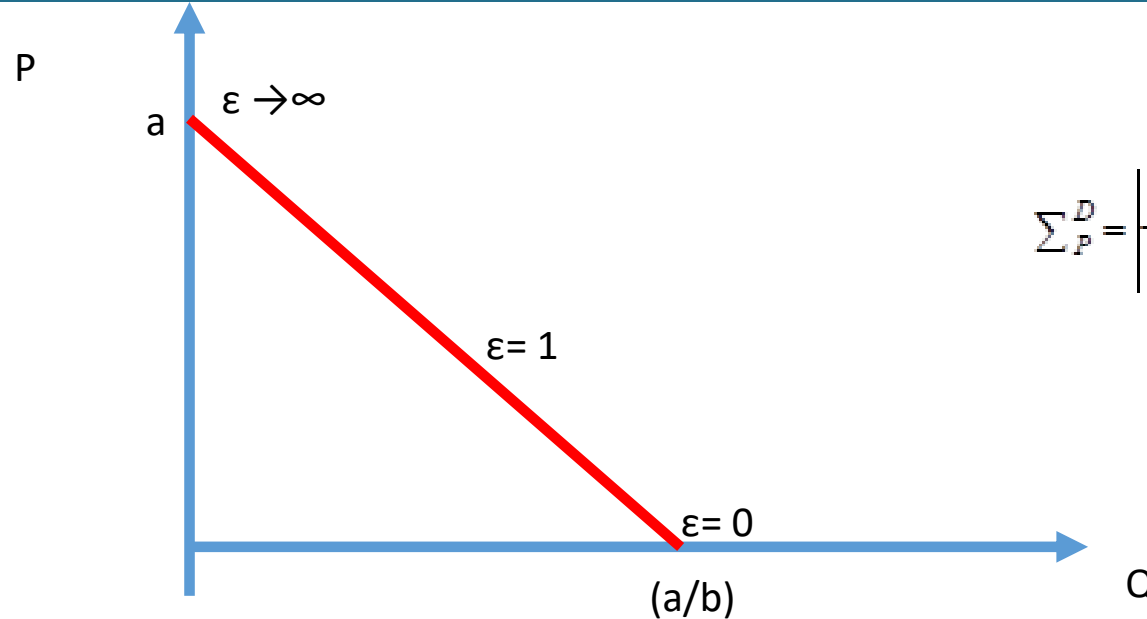
$$\sum_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a - bQ}{Q} \right) \right|$$



Marginal revenue function and elasticity



$$P(Q) = a - bQ,$$
$$Q(P) = (a/b) - (1/b)P$$



$$\sum_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a-bQ}{Q} \right) \right|$$

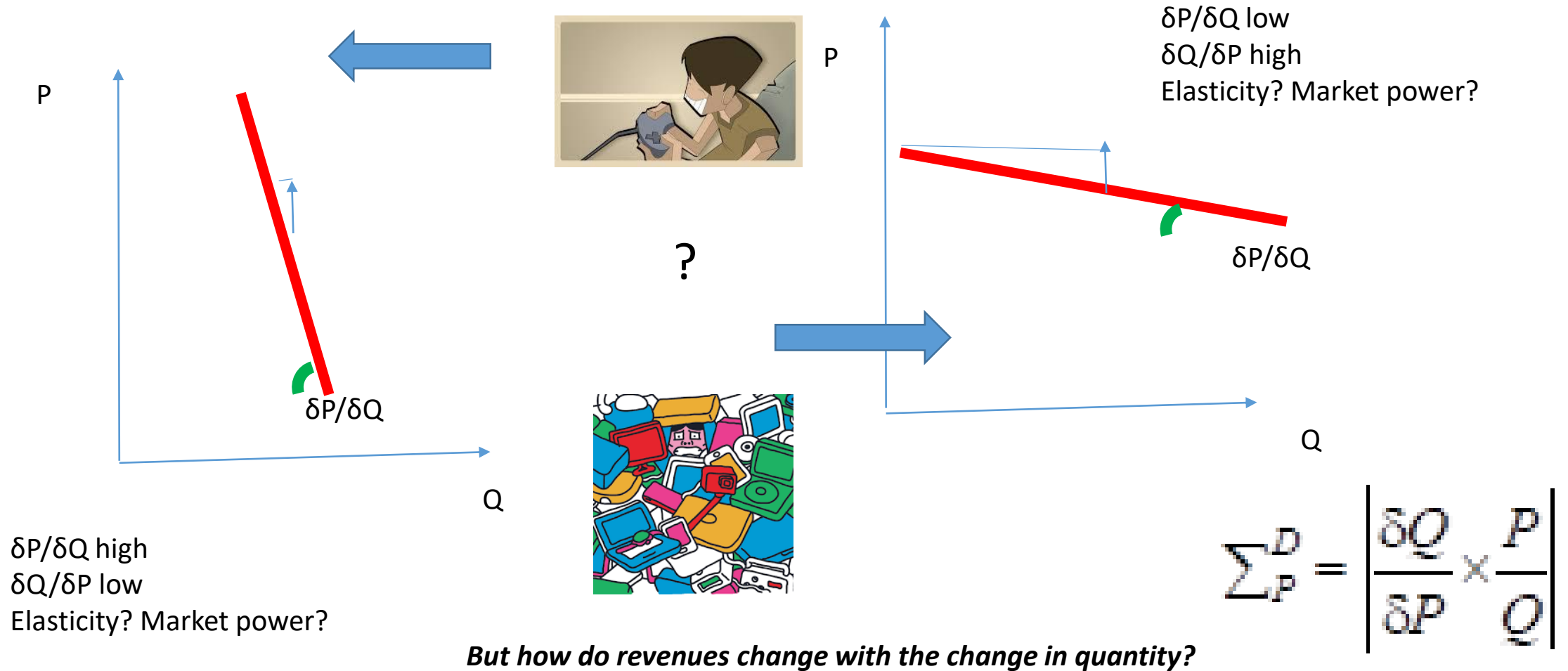
The elasticity (sensitivity) of consumption desires of an individual as prices change **changes** depending on how much we are consuming already!

And what about among individuals, given what they are consuming?

$$\sum_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$



Understanding elasticities



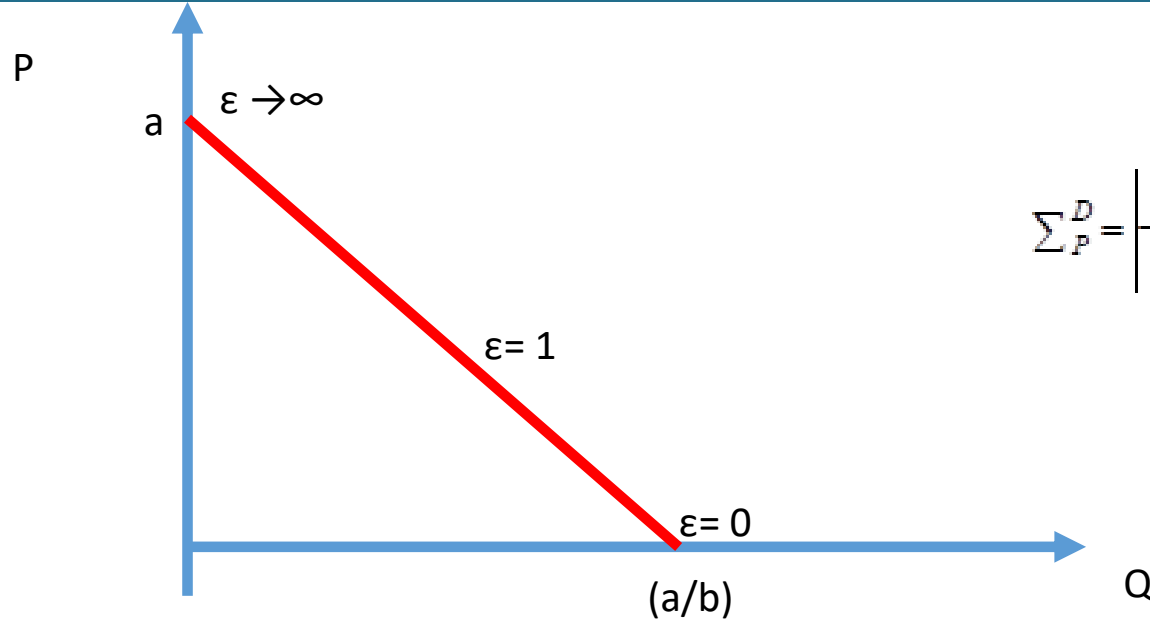


Marginal revenue function and elasticity



$$P(Q) = a - bQ,$$

$$Q(P) = (a/b) - (1/b)P$$



$$\sum_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a-bQ}{Q} \right) \right|$$

$$\frac{\delta TR}{\delta Q} = \frac{\delta [P(Q)Q]}{\delta Q}$$



Derivative of $U \times V = (UV)'$

$$= U'V + UV'$$

$$\sum_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$

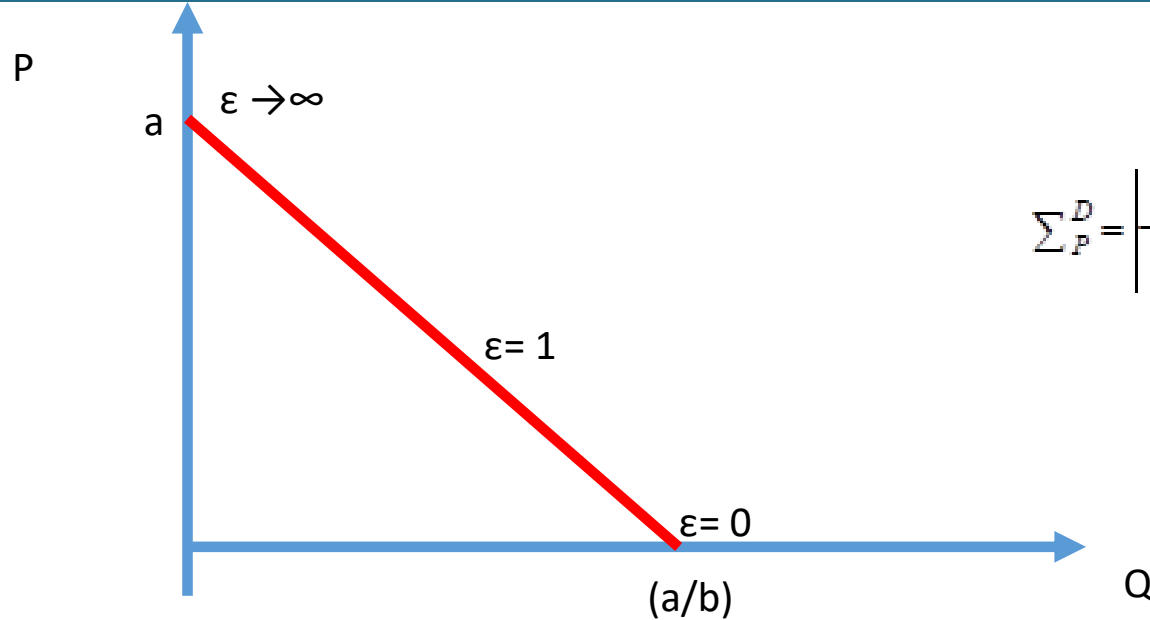


Marginal revenue function and elasticity



$$P(Q) = a - bQ,$$

$$Q(P) = (a/b) - (1/b)P$$



$$\sum_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a-bQ}{Q} \right) \right|$$

$$\frac{\delta TR}{\delta Q} = \frac{\delta [P(Q)Q]}{\delta Q} = \frac{\delta P}{\delta Q} Q + P(Q)$$

Derivative of $U \times V = (UV)'$
 $= U'V + UV'$

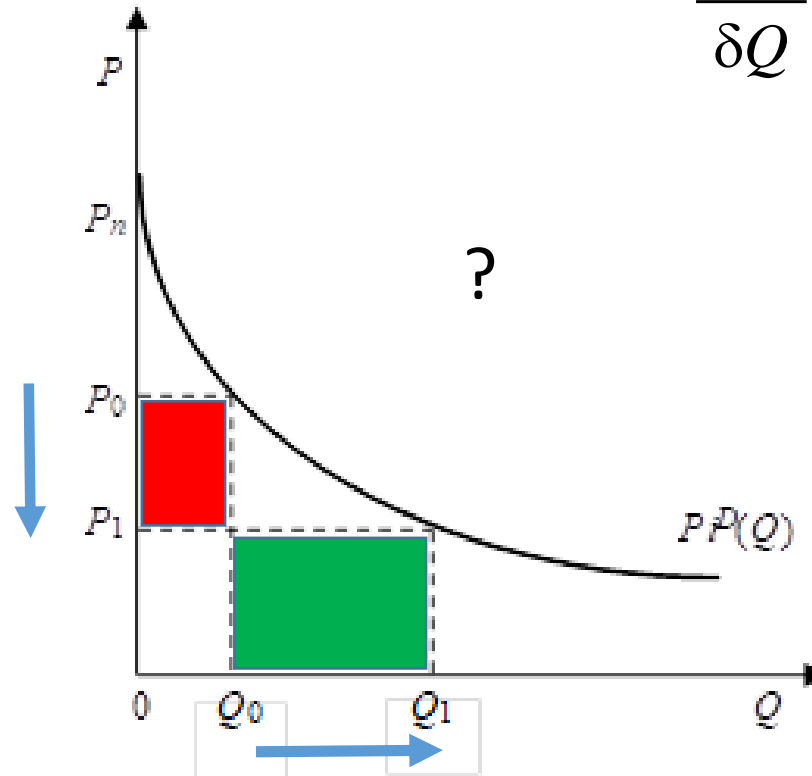
$$\sum_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$



The revenue dilemma, from Q_0 to $Q_1=Q_0+1$



By how much do we have to lower the price (increase the discount) to convince the consumer to buy one more unit in order to sell it?



$$\frac{\delta TR}{\delta Q} = \frac{\delta [P(Q)Q]}{\delta Q} = \frac{\delta P}{\delta Q} Q + P(Q)$$



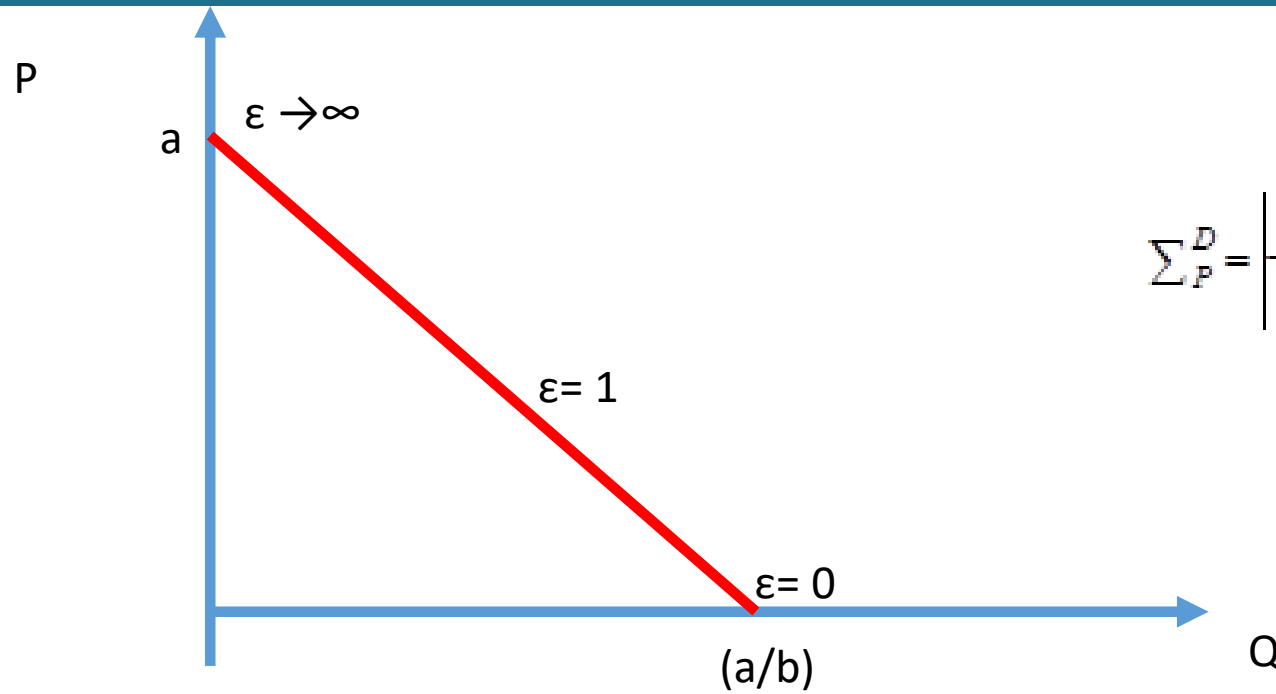


Marginal revenue function and elasticity



$$P(Q) = a - bQ,$$

$$Q(P) = (a/b) - (1/b)P$$



$$\Sigma_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a - bQ}{Q} \right) \right|$$

$$\Sigma_P^D = \left| \frac{\delta Q}{\delta P} \times \frac{P}{Q} \right|$$

$$\frac{\delta TR}{\delta Q} = \frac{\delta [P(Q)Q]}{\delta Q} = \frac{\delta P}{\delta Q} Q + P(Q)$$

$$\frac{\delta TR}{\delta Q} = \frac{\delta [P(Q)Q]}{\delta Q} = \frac{\delta P}{\delta Q} P \frac{Q}{P} + P(Q)$$

$$\frac{\delta TR}{\delta Q}(Q) = P(Q) \left[1 - \left(\frac{1}{\varepsilon(Q)} \right) \right]$$



The revenue dilemma: solved!

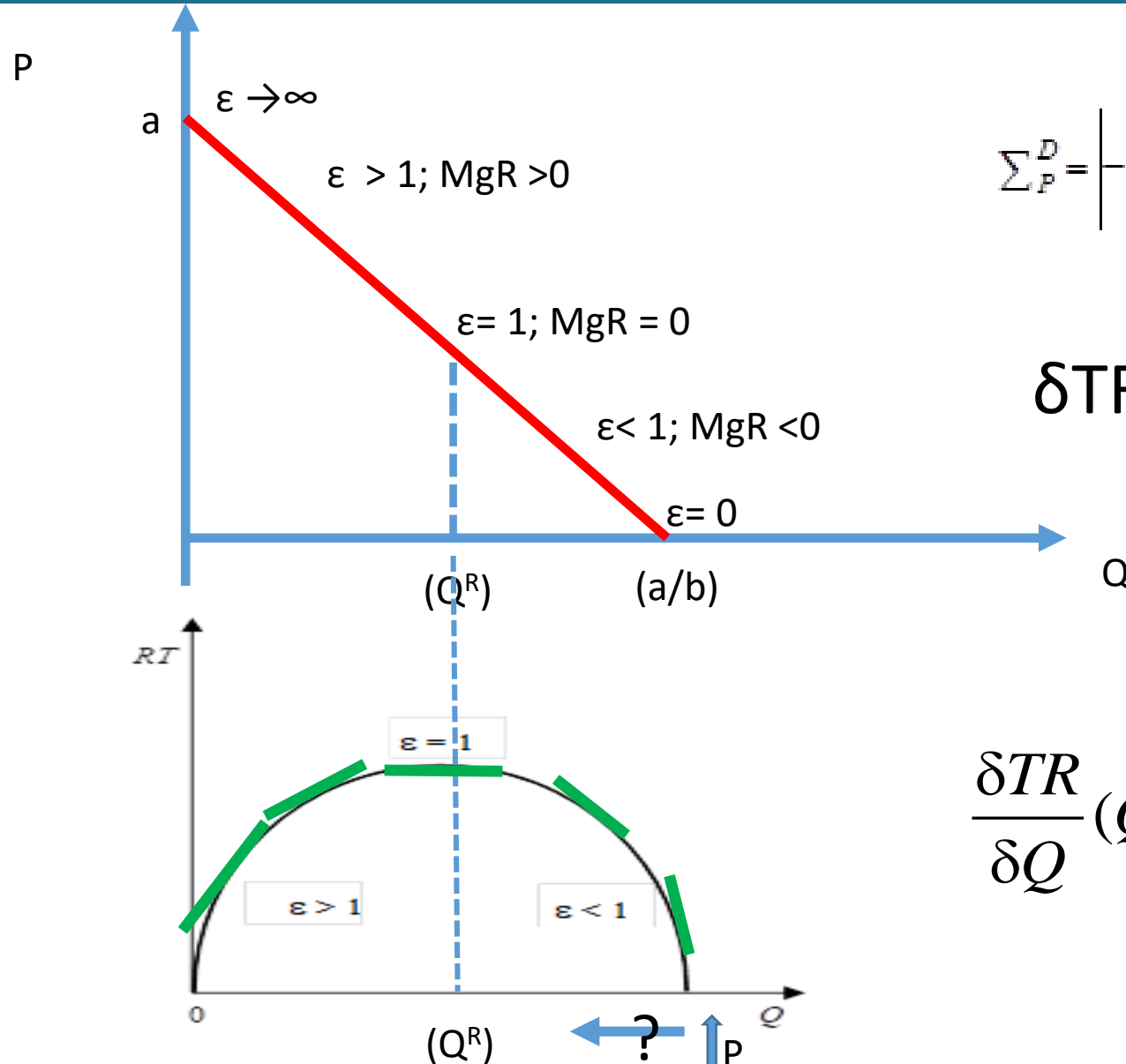


Expenditure function?

TR (150) = 12.043 €
MR (150) = 50 €
TR (151) = ? €
= 12.093 €

TR (23) = 2300 €
TR (24) = 2200 €
MR (23) = ? €
= -100 €

MR (0) = 8€
MR (1) = 6€
TR (2) = ? €
= 14 €



$$\Sigma_P^D = \left| -\frac{1}{b} \times \frac{P}{Q} \right| = \left| \left(-\frac{1}{b} \right) \times \left(\frac{a-bQ}{Q} \right) \right|$$

$$\delta TR / \delta Q \equiv MR(Q) = ?$$

$$\frac{\delta TR}{\delta Q}(Q) = P(Q) \left[1 - \left(\frac{1}{\varepsilon(Q)} \right) \right]$$

Product cannibalization?



LOCK-IN



Quanto mi costa 



An application of elasticity



«What is the impact of prohibitionism on drugs over crime?»

Prohibitionism raises the «price» cost of a unit of drugs.

Demand effect on drug consumption? \searrow

Drug consumption increases crime via: pharmacological and theft effects.

Pharmacologically-induced crime: reduced.

Crime? \searrow

Money-need crime: ? The role of elasticity.

Crime? \nearrow

Final impact: $\searrow + \nearrow = ?$

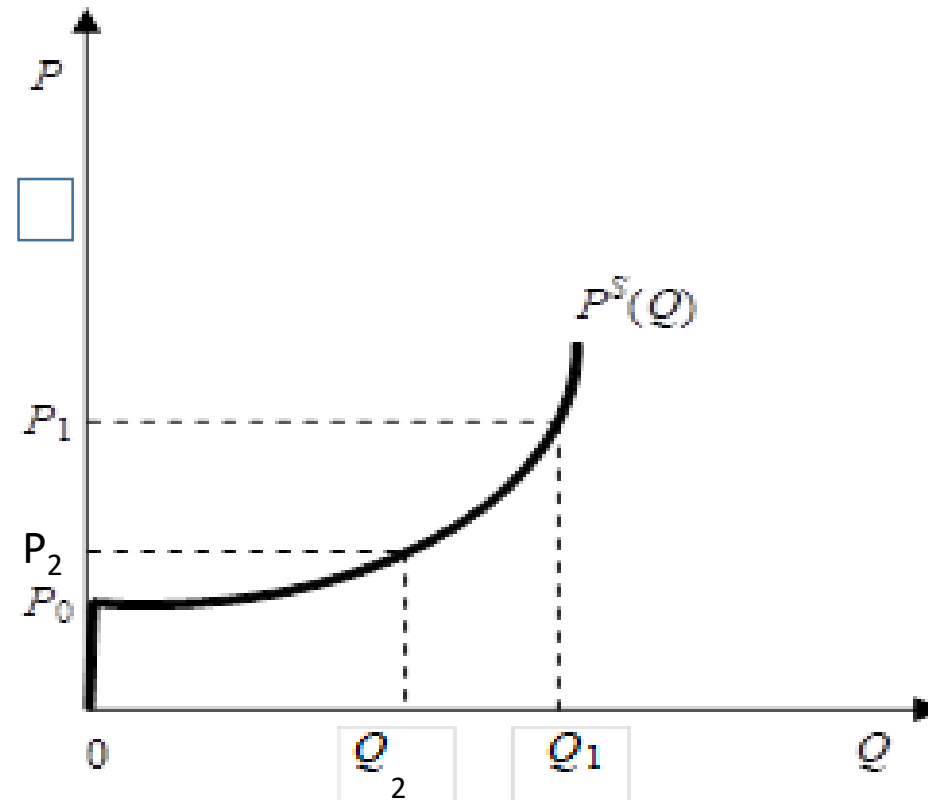


A supply curve



The individual firm's supply curve for good Q tells us **for every possible price** how many units the firm sells of good Q

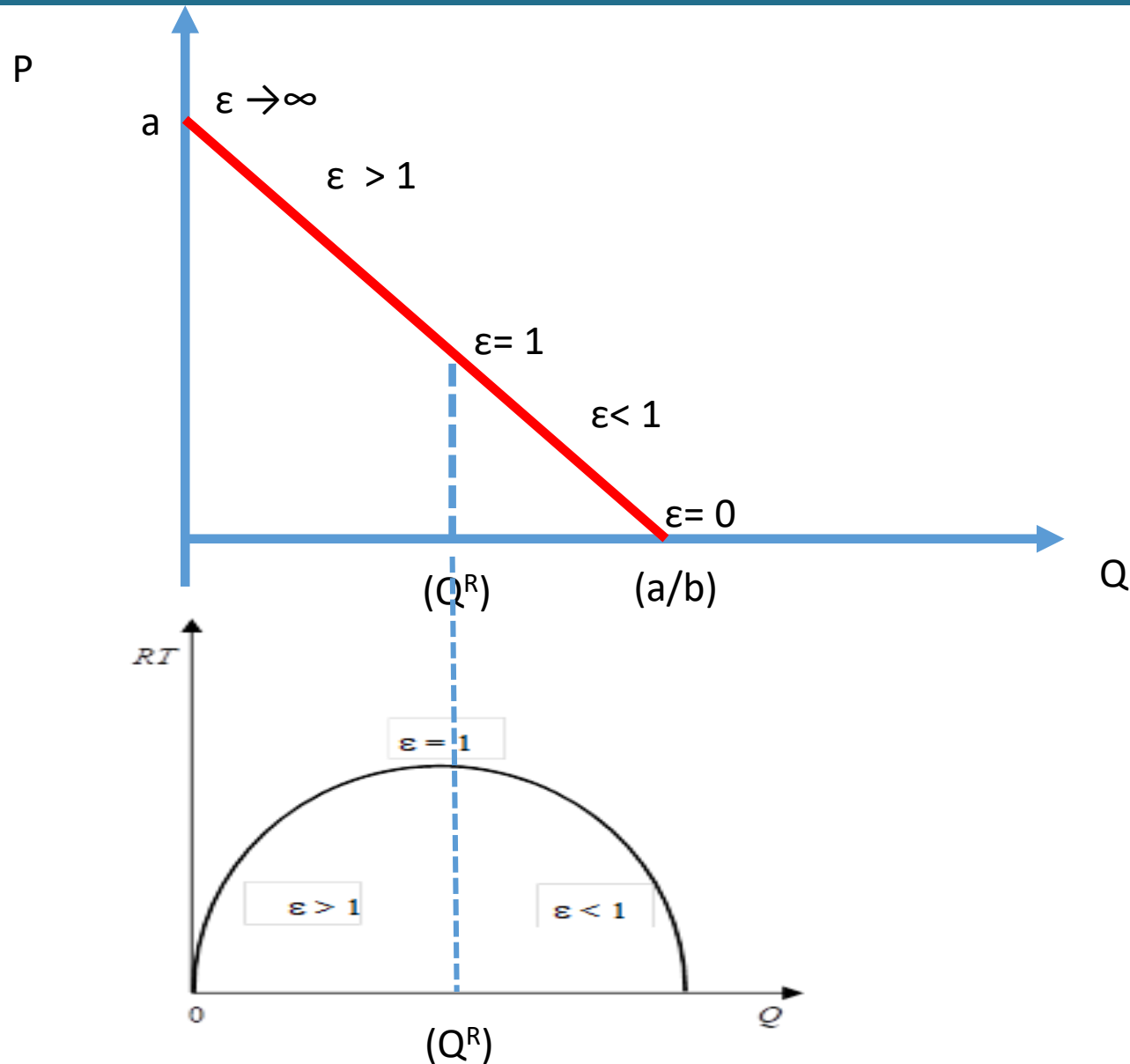
The individual firm's supply curve for good Q tells us **for every possible price** how many units the firm **desires to sell** of good Q **given**....



At price P_1 the firm sells Q_1 units of good Q if there is somebody willing to buy them from him *(if allowed to raise prices at P_1)*

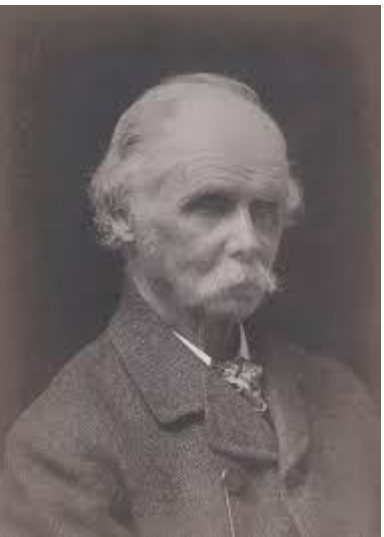


The goal of a firm: maximize revenues?





“The chemist or the physicist may happen to make money by his inventions, but that is seldom the chief motive of his work. . . . business men are very much of the same nature as scientific men; they have the same instincts of the chase, and many of them have the same power of being stimulated to great and even feverish exertions by emulations that are not sordid or ignoble. This part of their nature has however been confused with and thrown into the shade by their desire to make money. . . . And so all the best business men want to get money, but many of them do not care about it much for its own sake; they want it chiefly as the most convincing proof to themselves and others that they have succeeded.”





Q^* such that:

$$\text{Max } \Pi (Q) = P^d(Q) Q - TC$$

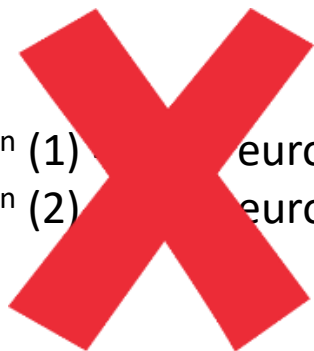
Q^* such that:

$$\text{Max } \Pi (Q) = P^d(Q) Q - TC (Q)$$

Where $TC (Q) \equiv TC^{\min} (Q)$

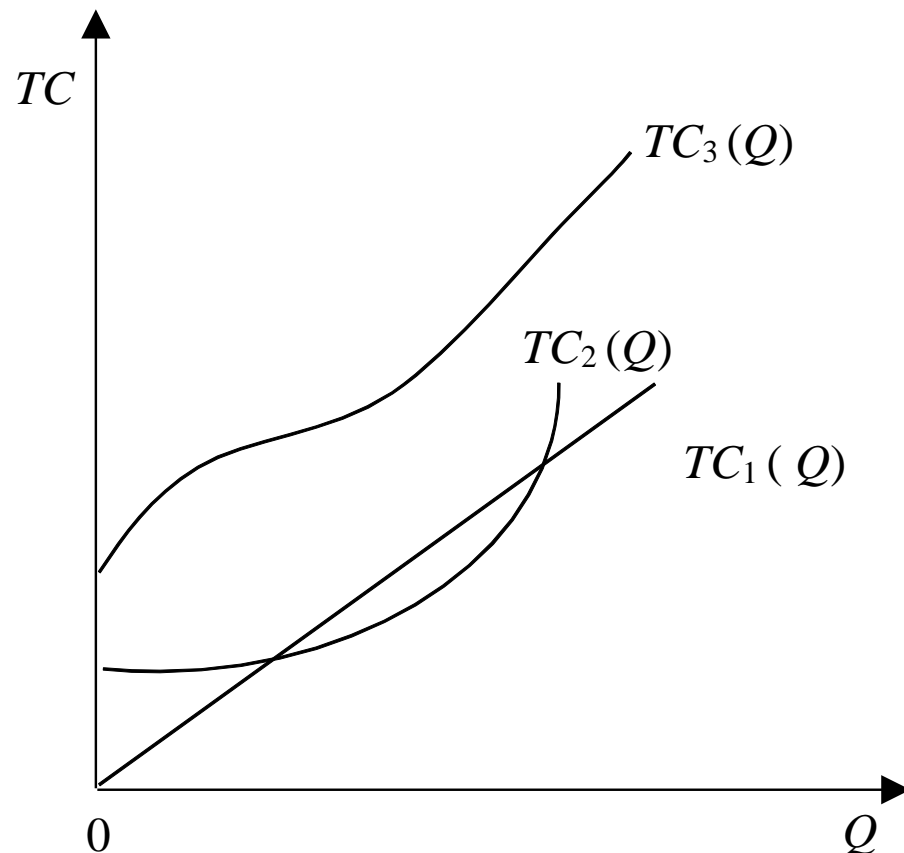
Where $TC (Q) \equiv TC^{\min} (Q, T^{\circ}, w^{\circ}, r^{\circ}, \text{Leg}^{\circ} \dots)$

$TC^{\min} (1)$ euro
 $TC^{\min} (2)$ euro?



TC?





Q^* such that:

$$\text{Max } \Pi(Q) = P(Q) Q - TC(Q)$$

Where $TC(Q) \equiv TC^{\min}(Q, w^{\circ}, r^{\circ}, \text{Leg}^{\circ} \dots)$

Where $TC(Q) \equiv TC^{\min}(Q)$

For each possible Q ,
 $TC(Q)$ tells me the
 cost of producing that
 Q , given all the
 elements that
 influence the
 minimum cost of
 producing Q .

For each possible Q ,
 $CT(Q)$ tells me the
minimum cost of
 producing that Q ,
 given all the elements
 that influence the
 minimum cost of
 producing Q .



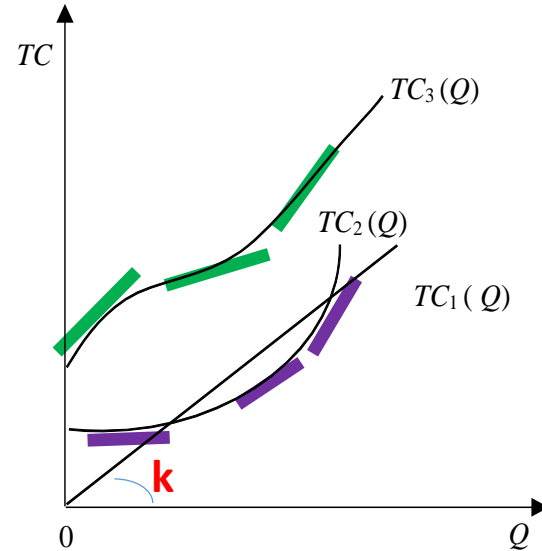
Cost curves



$$\begin{aligned} MC_1(0) &= k \text{ €} \\ MC_1(1) &= k \text{ €} \\ TC_1(2) &= ? \text{ €} \\ &= 2k \text{ €} \end{aligned}$$

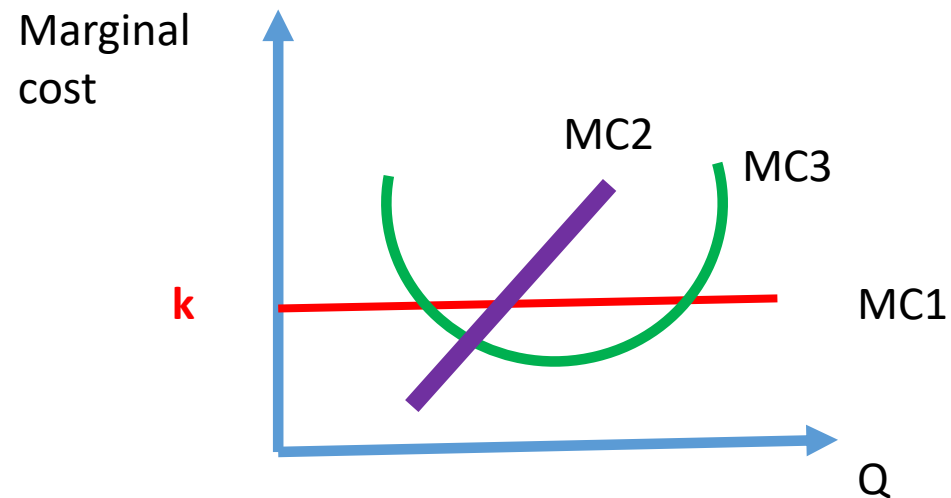
$$\begin{aligned} TC_2(23) &= 230 \text{ €} \\ MC_2(23) &= 15 \text{ €} \\ TC_2(24) &= ? \\ &= 245 \text{ €} \end{aligned}$$

$$\begin{aligned} TC_3(150) &= 80 \text{ €} \\ TC_3(151) &= 86 \text{ €} \\ MC_3(150) &= ? \\ &= 6 \text{ €} \end{aligned}$$



$$\delta TC / \delta Q \equiv MC(Q) = ?$$

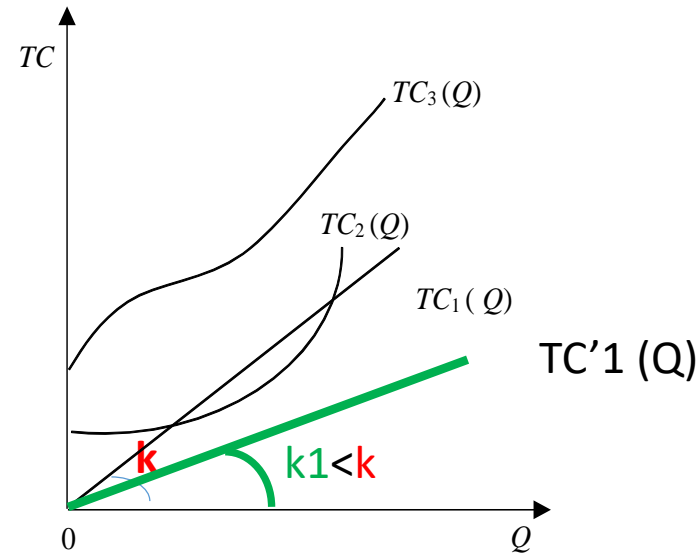
$$MC(Q) > 0$$





Distinguish:

- movements along the cost curve (impact of changes of quantity on minimum costs)
- Movements of the cost curve (impact of changes of all other variables that affect minimum cost):
Bravura, unit cost of factors of production...



$$\delta TC / \delta Q \equiv MC(Q) = ?$$

$$MC(Q) > 0$$

