

PRACTICE I - MICROECONOMICS

Bachelor Degree in Global Governance

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FUNCTIONS

- A function is a law that associates each element of a set called the domain with one and only one element of another set (codomain): $Y=f(x)$.
- Functions with a single variable are represented in the Cartesian plane, constructed with a horizontal axis (x -axis) and a vertical axis (y -axis).
- A function is drawn as the set of points in the Cartesian plane that represent combinations of x and y , identified according to the relationship expressed by the function itself.

LINEAR FUNCTIONS

- The generic equation of a straight line is: $y = mx + q$
- The slope coefficient is m , which indicates the slope of the line. 4 cases are important to remember:
 1. $m > 0$ indicates that the line is positively sloped (goes upward)
 2. $m < 0$ indicates that the line is negatively sloped (goes downward)
 3. $m = 0$ indicates that the straight line is horizontal, and its equation is: $y=q$
 4. does not exist (or, more precisely, is equal to ∞) in the case of a vertical line, of the type $x = k$
- The intercept q , which indicates the value assumed by the variable y when x is zero and thus the intersection of the line and the vertical axis; obviously, in the case of a vertical line, this value does not exist, since the line does not intersect the vertical axis but is parallel to it.

LINEAR SYSTEM OF EQUATIONS

- To carry out such an exercise, we need to place within a linear system the equations of the two lines and find the values of x and y (or, more generally, of the two variables in the equations) that solve the system itself.
- To solve a system, we use the method by substitution.

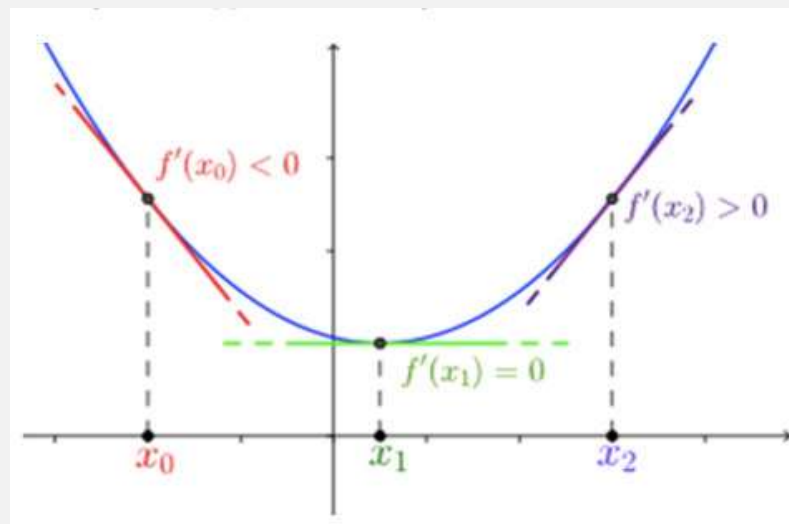
LOGARITHMIC AND EXPONENTIAL FUNCTIONS

- Exponential function: $a^x = b$
- Logarithm: $\log_a(b) = x$. The logarithm is a function whose value is the index of the power (x) to which we must elevate the base (a) to obtain the argument (b).
- Special case: natural logarithm (has Nepero's number as its base), that is, $\ln(b)=x$

DERIVATIVES

- The first derivative of a function $f(x)$ at a point x_0 , $f'(x) = \frac{\partial f(x)}{\partial x}$, indicates the slope of the tangent line to the function $f(x)$ at x_0 . The first derivative of a function indicates the impact on the output variable y of an additional unit of the input variable x .
- Computing the derivative of a function is of fundamental importance since it allows us to understand what the trend of the function is:
 1. If the derivative is less than zero, it means that the function is decreasing (i.e., it goes downward);
 2. If the derivative is greater than zero, it means that the function is increasing (i.e., it goes upward);
 3. If the derivative is zero, it means that the function is neither increasing nor decreasing: it is characterized by a stationary point and the tangent line to the function is horizontal.

DERIVATIVES

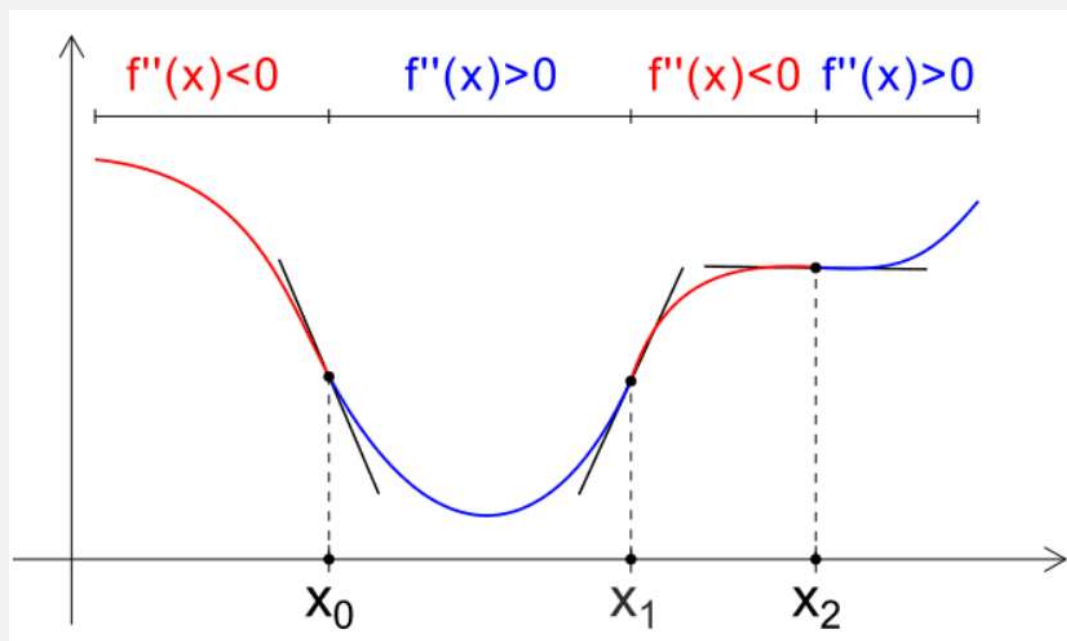


Function	First Derivative
$f(x) = k$	$\frac{\partial f(x)}{\partial x} = 0$
$f(x) = x$	$\frac{\partial f(x)}{\partial x} = 1$
$f(x) = kh(x)$	$\frac{\partial f(x)}{\partial x} = k \frac{\partial h(x)}{\partial x}$
$f(x) = kx$	$\frac{\partial f(x)}{\partial x} = k$
$f(x) = x^n$	$\frac{\partial f(x)}{\partial x} = nx^{n-1}$
$f(x) = g(h(x))$	$\frac{\partial f(x)}{\partial x} = \frac{\partial g(x)}{\partial x} \frac{\partial g(h(x))}{\partial x}$
$f(x) = \ln(x)$	$\frac{\partial f(x)}{\partial x} = \frac{1}{x}$
$f(x) = \log(x)$	$\frac{\partial f(x)}{\partial x} = \frac{1}{x \ln(a)}$
$f(x) = e^x$	$\frac{\partial f(x)}{\partial x} = e^x$
$f(x) = a^x$	$\frac{\partial f(x)}{\partial x} = a^x \ln(a)$
$f(x) = h(x) + g(x)$	$\frac{\partial f(x)}{\partial x} = \frac{\partial h(x)}{\partial x} + \frac{\partial g(x)}{\partial x}$
$f(x) = h(x)g(x)$	$\frac{\partial f(x)}{\partial x} = \frac{\partial h(x)}{\partial x} g(x) + \frac{\partial g(x)}{\partial x} h(x)$
$f(x) = \frac{h(x)}{g(x)}$	$\frac{\partial f(x)}{\partial x} = \frac{\frac{\partial h(x)}{\partial x} g(x) + \frac{\partial g(x)}{\partial x} h(x)}{[g(x)]^2}$

SECOND DERIVATIVE

- The second derivative of a function $f(x)$, $f''(x) = \frac{\partial^2 f(x)}{\partial^2 x}$, is calculated as the derivative of the first derivative $\frac{\partial f(x)}{\partial x}$.
- The second derivative is useful since it indicates whether the function is concave or convex: a concave function is characterized by a negative second derivative, while a convex function has positive second derivative.
- Studying the second derivative of a function is useful for minimizing or maximizing the function. Specifically, a function has a point of minimum (maximum), that is, a point such that there are no lower (higher) values of y when:
 - The first derivative is zero (the point is stationary)
 - The second derivative is positive (negative), that is, the function is convex (concave)

SECOND DERIVATIVE



The second derivative in x_0 is negative, that is $f''(x_0) < 0$

In $x_1 > 0$, $f''(x_1) > 0$

In x_2 ? left < 0 , right > 0 , in x_2 $f''(x_2) = 0$

x_2 is an inflection point

FUNCTIONS OF TWO VARIABLES

- Functions with two variables are expressed in the form $Z = f(X, Y)$, depend on two variables (X and Y) and are represented in the space identified by three axes (X, Y, Z).
- Given a function $Z = f(X, Y)$, a level curve, constructed on the X - Y plane, is the set of points representing combinations of X and Y associated by the function with the same level of Z .

PARTIAL DERIVATIVES

Given the function $Z = f(X, Y)$, two partial derivatives can be calculated:

- The partial derivative of Z with respect to X , $\frac{\partial f(X,Y)}{\partial X}$, indicates the change undergone by Z with respect to an infinitesimal change in the variable X , holding Y constant.
- The partial derivative of Z with respect to Y , $\frac{\partial f(X,Y)}{\partial Y}$, indicates the change undergone by Z in correspondence with an infinitesimal change in the variable Y , holding X constant.

PARTIAL DERIVATIVES

To compute the partial derivative of a function $Z = f(X, Y)$ with respect to X , one derives the function Z by considering X as a variable and Y as a constant. To calculate the partial derivative with respect to Y , we consider Y as a variable and X as a constant.