PRACTICE 6 - MICROECONOMICS

Bachelor Degree in Global Governance

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RETURNS TO SCALE

A production function can have:

- **Increasing returns to scale (IRS):** if as inputs increase, output increases more than proportionally.

$$f(\lambda L, \lambda K) > \lambda f(L, K),$$

Therefore, by increasing the inputs of a constant the increase in the quantity produced is greater than the increase in the quantity of the same constant.

- **Constant returns to scale (CRS):** If as the inputs increase, the output increases proportionally.

$$f(\lambda L, \lambda K) = \lambda f(L, K),$$

Thus, by increasing the inputs of a constant the increase in the quantity produced is equal to the increase in the quantity of the same constant.

RETURNS TO SCALE

A production function can have:

- **Decreasing returns to scale (DRS):** If as the inputs increase, the output increases less than proportionally.

$$f(\lambda L, \lambda K) < \lambda f(L, K),$$

Thus, by increasing the inputs of a constant, the increase in the quantity produced is less than the increase in the quantity of the same constant.

In the case of the Cobb-Douglas function,

$$f(K, L) = AK^{\alpha}L^{\beta}$$
$$f(\lambda K, \lambda L) \gtreqless \lambda AK^{\alpha}L^{\beta}$$

$$A\lambda^{\alpha}K^{\alpha}\lambda^{\beta}L^{\beta} \gtreqless \lambda AK^{\alpha}L^{\beta}$$
$$A\lambda^{\alpha+\beta}K^{\alpha}L^{\beta} \gtreqless A\lambda K^{\alpha}L^{\beta}$$

$$\alpha + \beta \gtreqless 1$$

If $\alpha + \beta > 1$ the function has IRS; if $\alpha + \beta = 1$ the function has CRS; if $\alpha + \beta < 1$ the function has DRS.

EXERCISE

Define the returns to scale of the following production functions:

1. $f(L, K) = 2(L + K)$
2. $f(L, K) = L^{\frac{1}{2}}K^{\frac{2}{6}}$
3. $f(L, K) = 2(LK)^{\frac{1}{2}}$
4. $f(L, K) = L + K^2$
5. $f(L, K) = L^3K^5$

RETURNS TO SCALE AND AVERAGE COST

- If we figure out how the average cost of a firm varies, we also understand what happens as its output increases:
- **Constant average cost:** If average cost is constant as output increases the average cost for all units produced remains stable, this means that costs also remain stable because inputs increase proportionally and, therefore, **returns to scale are constant.**
- **Increasing average cost:** If the average cost increases as output increases the average cost for all units produced increases, this means that costs also increase because inputs increase less than proportionally. **The returns to scale are, therefore, decreasing**.
- **Decreasing average cost**: If the average cost decreases as production increases the average cost for all units produced decreases, this means that costs also decrease because inputs increase more than proportionally. **Returns to scale are increasing.**

EXERCISE

Given the following cost functions, calculate the returns to scale via the average cost trend:

1. $TC(Q) = 5Q$
2. $TC(Q) = 3Q^{1.3}$
3. $TC(Q) = 8Q^{0.4}$

PROFIT MAXIMIZATION

Until now, we faced the following optimization problems:

- **Utility maximization**: $max U(x_1, x_2)\ s.t.\ I = p_1 x_1 + p_2 x_2$
- **Cost minimization**: $\min wL + rK\ s.t.\ \bar{q} = f(L, K)$

Now, the optimization problem is about profit maximization. What is profit? We calculate it as all revenues minus all costs incurred:

$$\pi = pY - (wL + rK) = pY - wL - rK$$

From our last class, we know that $Y = f(L, K)$ and, therefore, the producer's problem is the following:

$$\underset{L,K}{max}\ \pi = pf(L, K) - wL - rK$$

PROFIT MAXIMIZATION

In the short run, we hold one of the two inputs constant (capital, K). Therefore, the previous equation becomes:

$$\max_{L,K} \ \pi = pf(L, \overline{K}) - wL - r\overline{K}.$$

To maximize such a function, we calculate the first derivative with respect to L and set it equal to zero.

- **Exercise:** Given the following production function and the following data, calculate the labor demand function that solves the profit maximization problem in the short run. Compute, then, the optimal quantity produced and the corresponding profits.

$$f(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$p = 10, \overline{K} = 100, w = 5, r = 5$$