

# ELEMENTS OF MATHEMATICS FOR MICROECONOMICS – Part 2

University of Tor Vergata

Bachelor Degree in Global Governance

MICROECONOMICS

a.a. 2023/2024

Prof. Gustavo Piga

TA Andrea Fazio

[andrea.fazio@uniroma2.it](mailto:andrea.fazio@uniroma2.it)

# Today topics

- First derivative
- Second derivative
- Partial derivatives
- Exercises

# First derivative 1/2

The first derivative of a function  $f(x)$  in the point  $x_0$  is defined as the limit of the difference quotient:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

It represents the slope of the tangent line in the point  $x_0$

The first derivative is given by the formula:

$$f' = \frac{\partial f(x)}{\partial x}$$

## First derivative 2/2

It indicates the impact on the variable  $y$  of an additional unit of the variable  $x$

In Micro it is used to find optimum, to solve maximization (minimization) problems

# Derivation rules

## Function

$$f(x) = k$$

$$f(x) = x$$

$$f(x) = kh(x)$$

$$f(x) = kx$$

$$f(x) = x^{(n)}$$

$$f(x) = g(h(x))$$

$$f(x) = \ln(x)$$

$$f(x) = \log(x)$$

$$f(x) = a^{(x)}$$

$$f(x) = h(x) + g(x)$$

$$f(x) = h(x) * g(x)$$

## First derivative

$$\frac{\partial f(x)}{\partial x} = 0$$

$$\frac{\partial f(x)}{\partial x} = 1$$

$$\frac{\partial f(x)}{\partial x} = k \frac{\partial h(x)}{\partial x}$$

$$f(x) = nx^{(n-1)}$$

$$\frac{\partial f(x)}{\partial x} = \frac{\partial g(x)}{\partial x} \frac{\partial g(h(x))}{\partial x}$$

$$\frac{\partial f(x)}{\partial x} = \frac{1}{x}$$

$$\frac{\partial f(x)}{\partial x} = \frac{1}{x \ln(a)}$$

$$\frac{\partial f(x)}{\partial x} = a^{(x)} \ln(a)$$

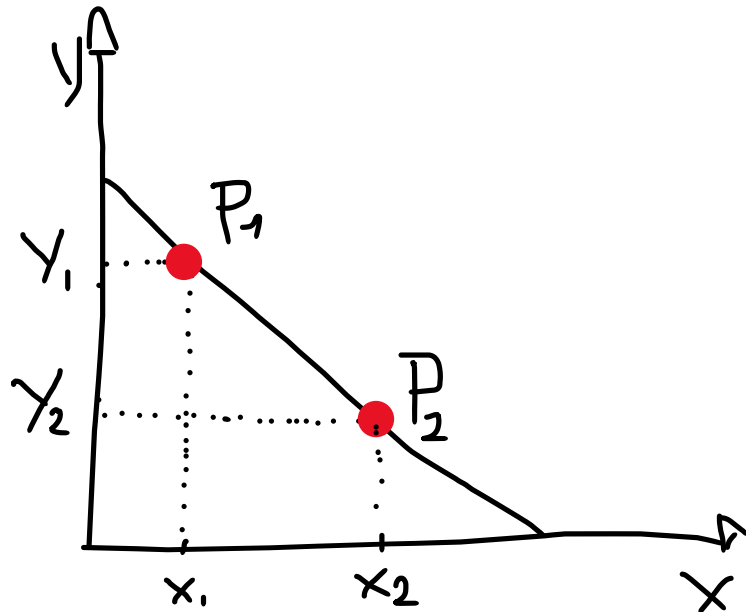
$$\frac{\partial f(x)}{\partial x} = \frac{\partial h(x)}{\partial x} + \frac{\partial g(x)}{\partial x}$$

$$\frac{\partial f(x)}{\partial x} = \frac{\partial h(x)}{\partial x} g(x) + \frac{\partial g(x)}{\partial x} h(x)$$

# First derivative – First difference

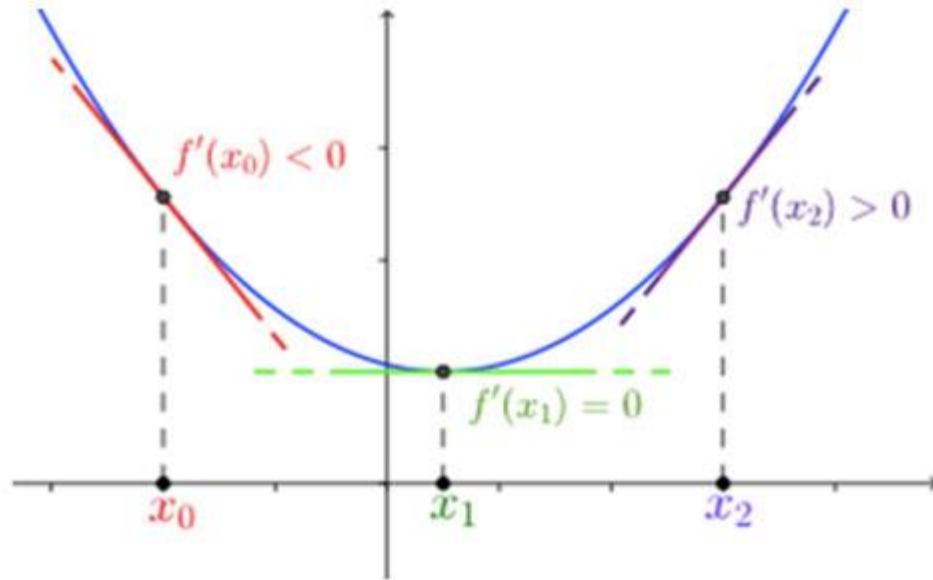
For linear functions (of grade 0 or 1) the first derivative is a constant, equal to the first difference;

For small value of  $h$  the first difference is a good approximation of the derivative



$$\frac{\partial f(x)}{\partial x} = \frac{y_2 - y_1}{x_2 - x_1}$$

# First derivative - graphical representation



The first derivative in  $x_0$  is negative, that is  $f'(x_0) < 0$

In  $x_1 = 0$ ,  $f'(x_1) = 0$

In  $x_2$  positive,  $f'(x_2) = 0$

# Second derivative

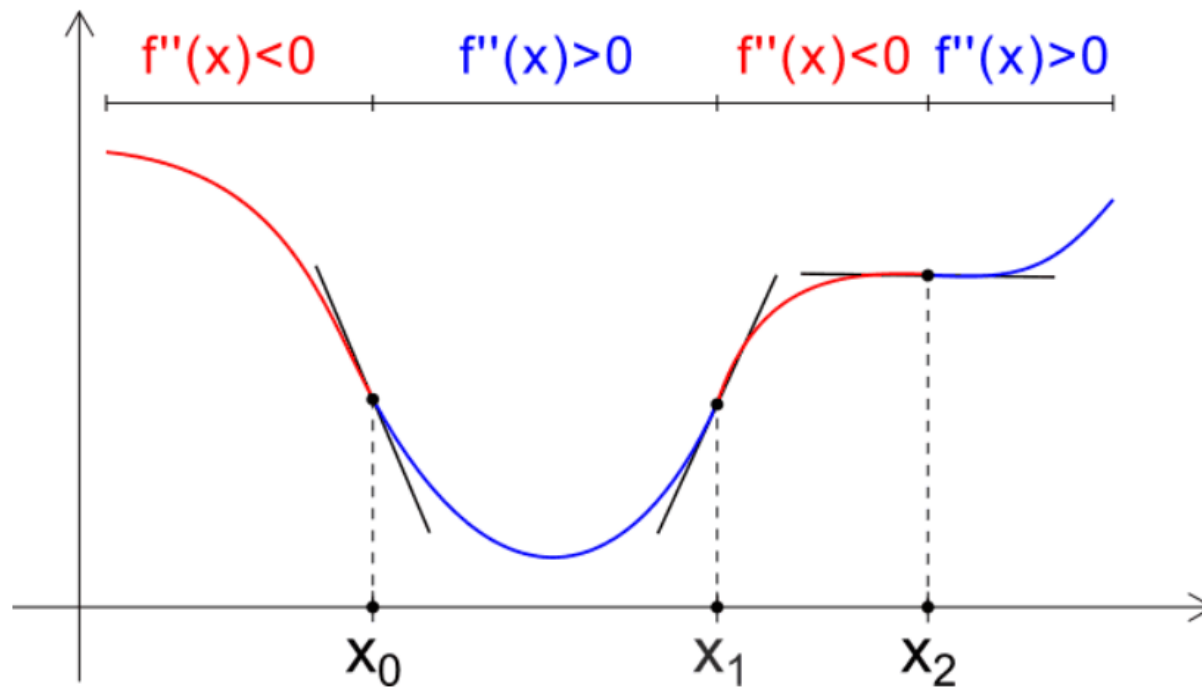
The second derivative of a function  $f(x)$  is the derivative of its first derivative:

$$f'' = \frac{\partial^2 f(x)}{\partial^2 x}$$

It represents the change in the slope of the function



# Second derivative – graphical representation



The second derivative in  $x_0$  is negative, that is  $f''(x_0) < 0$

In  $x_1 > 0$ ,  $f''(x_1) > 0$

In  $x_2$ ? left  $< 0$ , right  $> 0$ , in  $x_2$   $f''(x_2) = 0$

$x_2$  is an inflection point

# Partial derivative 1/3

Given a function  $Z = f(x, y)$  a level curve on the plane  $x, y$  is the set of points representing the  $x, y$  combinations associated to  $z$

Ex:  $Z = x^2y^3$  and the level curve  $8 = x^2y^3$

It represents the combinations giving 8 as a result, like the point (1,2)

To represent it, use the formula

$$Y = \frac{2}{X^{\frac{2}{3}}}$$

# Partial derivative 2/3

The functions in 2 variables are expressed by the formula  $Z = f(x, y)$

They depend on the variables  $x$  and  $y$  and are represented in the tridimensional space given by the axis  $x$ ,  $y$  and  $z$

# Partial derivative 3/3

Give a function  $Z = f(X, Y)$ , it is possible to compute 2 partial derivatives:

- i. the derivative of  $Z$  in  $X$  indicates the variation of  $Z$  after an infinitesimal variation of  $X$ , keeping the  $Y$  constant

$$\frac{\partial f(x,y)}{\partial x}$$

- ii. the derivative of  $Z$  in  $Y$  indicates the variation of  $Z$  after an infinitesimal variation of  $Y$ , keeping the  $X$  constant

$$\frac{\partial f(x,y)}{\partial y}$$

# Exercises

## 1.1 Derive the following functions

1.  $f(x) = 10x + 100$
2.  $f(x) = x^2 + 4x + 10$
3.  $f(x) = 200x - x^2 - 20x - 200$
4.  $f(x) = x + 10 + \frac{(100)}{x}$
5.  $f(x) = x^3 + x^{\frac{1}{2}} + 3281$

## 1.2 Maximise the following functions

1.  $f(x) = 50x - x^2 + 28$
2.  $f(x) = 3x^2 + x + 3$
3.  $f(x) = 4x^2 + 5x - 45$

## 1.3 Minimize the following functions

1.  $f(x) = x + 10 + \frac{10000}{x}$
2.  $f(x) = 4x^2 - 8x - 21$
3.  $f(x) = 2x + \frac{56}{x}$

## 1.4 Find the level curve

1.  $Z = Y^{0.5}X^{0.5}, \text{ where } Z_0 = 3$
2.  $Z = X^2Y, \text{ where } Z_0 = 5$
3.  $Z = Y^{0.25}X^{0.25}, \text{ where } Z_0 = 8$
4.  $Z = X^2Y^3, \text{ where } Z_0 = 27$

## 1.5 Find the partial derivatives of the previous exercise

# Solutions 1/2

## 1.1 Derive the following functions

1.  $\frac{\partial f(x)}{\partial x} = 10$

2.  $\frac{\partial f(x)}{\partial x} = 2x + 4$

3.  $\frac{\partial f(x)}{\partial x} = 180 - 2x$

4.  $\frac{\partial f(x)}{\partial x} = 1 + \frac{100}{x^2}$

5.  $\frac{\partial f(x)}{\partial x} = 3x^2 + \frac{1}{2}x^{-\frac{1}{2}}$

## 1.2 Maximise the following functions

1.  $x = 25$

2.  $x = \frac{1}{6}$

3.  $x = \frac{5}{8}$

## 1.3 Minimize the following functions

1.  $x = \pm 100$

2.  $x = 1$

3.  $x = \sqrt{28}$

## 1.4 Find the level curve

1.  $Y = \frac{9}{X}$

2.  $Y = \frac{5}{X^2}$

3.  $Y = \frac{8^4}{X}$

4.  $Y = \frac{3}{X^{\frac{2}{3}}}$

# Solutions 2/2

1.5 Find the partial derivatives of the previous exercise

$$1. \quad \frac{\partial f(X,Y)}{\partial X} = 0.5X^{-0.5}Y^{0.5}, \quad \frac{\partial f(X,Y)}{\partial Y} = 0.5X^{0.5}Y^{-0.5}$$

$$2. \quad \frac{\partial f(X,Y)}{\partial X} = 2XY; \quad \frac{\partial f(X,Y)}{\partial Y} = X^2$$

$$3. \quad \frac{\partial f(X,Y)}{\partial X} = 0.25X^{-0.75}Y^{0.25}, \quad \frac{\partial f(X,Y)}{\partial Y} = 0.25X^{0.25}Y^{-0.75}$$

$$4. \quad \frac{\partial f(X,Y)}{\partial X} = 2XY^3; \quad \frac{\partial f(X,Y)}{\partial Y} = 3X^2Y^2$$

.