Understanding Powers and Radicals in Microeconomics

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1 Introduction

Powers and radicals are fundamental in mathematics, with significant applications in microeconomics. They are used to represent exponential growth or decay, model non-linear production functions, and analyze diminishing returns to scale, among other applications.

2 Definition and Properties

2.1 Powers

A power is an expression that represents repeated multiplication of the same factor. The general form is a^n, where a is the base and n is the exponent. If n is positive, a^n represents a multiplied by itself $n-1$ times. If n is negative, $a^n = \frac{1}{a^{-n}}$. If $n = 0$, $a^0 = 1$ (assuming $a \neq 0$).

2.2 Radicals

A radical expression involves roots, such as square roots, cube roots, etc. The general form is $\sqrt[n]{a}$, which is the nth root of a. Specifically, $\sqrt{a}$ is the square root of a, and $\sqrt[3]{a}$ is the cube root of a.

2.3 Properties

Powers and radicals have several important properties:

- $a^m \cdot a^n = a^{m+n}$

- $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$
- $(a^m)^n = a^{mn}$
- $a^{-n} = \frac{1}{a^n}$, for $a \neq 0$
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

3 Application in Microeconomics

Powers and radicals are used in microeconomics to model various phenomena:

- Exponential growth or decay in populations, prices, or technology.
- Non-linear production functions, such as Cobb-Douglas or CES functions, often involve powers to represent returns to scale.
- Utility functions may use radicals to model diminishing marginal utility.

4 Exercises

Exercise 1

Given a production function $Q = L^{0.7}K^{0.3}$, find the returns to scale.

Exercise 2

If the utility function is $U(X, Y) = \sqrt{X} + \sqrt{Y}$, find the marginal utility with respect to X and Y.

Exercise 3

A population grows following the function $P(t) = P_0 e^{rt}$, where P_0 is the initial population, r is the growth rate, and t is time. Calculate the population after 5 years if $P_0 = 1000$ and $r = 0.03$.