

# Understanding Linear Systems in Microeconomics

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## 1 Introduction

Linear systems are foundational in microeconomics, offering a framework for analyzing multiple variables and their relationships. They are essential in understanding market equilibrium, production functions, and consumer choice, among other applications.

## 2 Basics of Linear Systems

A linear system consists of two or more linear equations involving the same set of variables. The general form of a linear equation in two variables,  $x$  and  $y$ , is  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are constants.

## 3 Solving Linear Systems

Linear systems can be solved using various methods, including substitution, elimination, and matrix operations. The solution to a linear system represents the point(s) at which the equations intersect, corresponding to the values of the variables that satisfy all equations simultaneously.

### 3.1 Substitution Method

Solve one equation for one variable and substitute this expression into the other equation.

### 3.2 Elimination Method

Add or subtract equations to eliminate one variable, solving for the other.

### 3.3 Matrix Method

Express the system in matrix form  $AX = B$  and use matrix operations to solve for  $X$ .

## 4 Application in Microeconomics

Linear systems are used in microeconomics to:

- Determine market equilibrium by setting demand equal to supply.
- Analyze input-output models, determining how changes in one sector affect others.
- Solve constrained optimization problems, such as maximizing utility subject to a budget constraint.

## 5 Exercises

### Exercise 1

Solve the system of linear equations representing market equilibrium:

$$Q_d : 20 - 2P = Q$$

$$Q_s : 5 + 3P = Q$$

### Exercise 2

Given an input-output model with two sectors, where  $x_1 = 0.5x_1 + 0.3x_2 + d_1$  and  $x_2 = 0.2x_1 + 0.4x_2 + d_2$ , solve for  $x_1$  and  $x_2$  when  $d_1 = 100$  and  $d_2 = 200$ .

### Exercise 3

Find the optimal consumption bundle for a consumer with the budget constraint  $8x + 12y = 96$  and utility function  $U(x, y) = x + 2y$ .