

Understanding Linear Systems in Microeconomics

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1 Introduction

Linear systems are foundational in microeconomics, offering a framework for analyzing multiple variables and their relationships. They are essential in understanding market equilibrium, production functions, and consumer choice, among other applications.

2 Basics of Linear Systems

A linear system consists of two or more linear equations involving the same set of variables. The general form of a linear equation in two variables, x and y , is $ax + by = c$, where a , b , and c are constants.

3 Solving Linear Systems

Linear systems can be solved using various methods, including substitution, elimination, and matrix operations. The solution to a linear system represents the point(s) at which the equations intersect, corresponding to the values of the variables that satisfy all equations simultaneously.

3.1 Substitution Method

Solve one equation for one variable and substitute this expression into the other equation.

3.2 Elimination Method

Add or subtract equations to eliminate one variable, solving for the other.

3.3 Matrix Method

Express the system in matrix form $AX = B$ and use matrix operations to solve for X .

4 Application in Microeconomics

Linear systems are used in microeconomics to:

- Determine market equilibrium by setting demand equal to supply.
- Analyze input-output models, determining how changes in one sector affect others.
- Solve constrained optimization problems, such as maximizing utility subject to a budget constraint.

5 Exercises

Exercise 1

Solve the system of linear equations representing market equilibrium:

$$Q_d : 20 - 2P = Q$$

$$Q_s : 5 + 3P = Q$$

Exercise 2

Given an input-output model with two sectors, where $x_1 = 0.5x_1 + 0.3x_2 + d_1$ and $x_2 = 0.2x_1 + 0.4x_2 + d_2$, solve for x_1 and x_2 when $d_1 = 100$ and $d_2 = 200$.

Exercise 3

Find the optimal consumption bundle for a consumer with the budget constraint $8x + 12y = 96$ and utility function $U(x, y) = x + 2y$.