Understanding Logarithms in Microeconomics

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1 Introduction

Logarithms are a fundamental concept in mathematics, with extensive applications in microeconomics. They help in simplifying complex mathematical expressions and are particularly useful in analyzing growth rates, elasticity, and optimizing production functions.

2 Definition of Logarithms

A logarithm answers the question: to what exponent must we raise a given base number to produce another number? Mathematically, if $a^x = b$, then we can express this relationship as $x = \log_a b$, where:

- a is the base of the logarithm,
- b is the number we are taking the logarithm of, and
- x is the logarithm of b with base a.

3 Properties of Logarithms

Logarithms have several important properties that simplify the manipulation of exponential expressions:

1. $\log_a(xy) = \log_a x + \log_a y \quad \text{(Product Rule)}$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \quad \text{(Quotient Rule)}$
3. $\log_a(x^r) = r \log_a x \quad \text{(Power Rule)}$
4. $\log_a a = 1$ and $\log_a 1 = 0$

4 Application in Microeconomics

One common application of logarithms in microeconomics is in the calculation of elasticity. Elasticity measures how much the quantity demanded of a good responds to a change in price. Mathematically, it is expressed as:

$$E_{d,p} = \frac{\%\Delta Q_d}{\%\Delta P}$$

Using logarithms, we can represent elasticity as:

$$E_{d,p} = \frac{d\log(Q_d)}{d\log(P)}$$

This representation simplifies the calculation of elasticity, making it easier to analyze the responsiveness of demand to price changes.

5 Exercises

Exercise 1

Given that the quantity demanded for a product changes from 100 units to 150 units when the price drops from \$20 to \$15, calculate the price elasticity of demand using logarithms.

Exercise 2

If a country's GDP grows from \$1 trillion to \$1.1 trillion in a year, use logarithms to calculate the annual growth rate of the GDP.

Exercise 3

Consider a production function represented as $Q = L^{0.75}K^{0.25}$, where Q is the output, L is labor, and K is capital. Use logarithms to linearize this production function.