

# First exercise lecture

University of Tor Vergata

Bachelor Degree in Global Governance

MICROECONOMICS

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# Today topics

- Choice, preferences, utility
- Preferences approach
- Choice approach
- Utility function and Indifference Curve
- Exercises

Choice, preferences, utility 1/4

Microeconomics is the branch of economics that studies decision-maker choices

Decision makers are economic agents, as consumers, firms or workers

Neoclassical approach: rational economic agents maximizing their objective

# Choice, preferences, utility 2/4

Consumers maximize their utility

Utility is the level of satisfaction consumers derive from consuming goods

Utility is the result of allocation choices, or consumption decision

Consumers choices *reveal* their preferences, given some constraints

Choice, preferences, utility 3/4

Individual choices can be observed, preferences are not

Choices are taken to reveal individual's preferences

Utility is a mathematical construction to model choices and preferences

# Choice, preferences, utility 4/4

Individuals take decision over the set of possible alternatives  $X$

Two different approaches to model individual choices:

1. Preference-based approach: the *preference relation* is the primitive characteristic of the individual, decision maker preferences must be rational
2. Choice based approach: choice behaviour is the primitive characteristic of the individual, behaviour must be rational

# Preference approach

Given observed choice derive the rational preferences over  $X$

A preference  $\succsim$  is a binary relation on  $X$

$x \succsim y \rightarrow x$  *is weakly preferred to*  $y$

Given preference relation  $\succsim$ , we have:

1. strict preference ( $\succ$ ):  $x \succ y$  iff  $x \succsim y$  but  $y \not\succsim x$
2. indifference ( $\sim$ ):  $x \sim y$  iff  $x \sim y$  and  $y \sim x$

# Rational preferences (**strong**) assumption

A preference  $\succsim$  is rational if it is:

1. Complete: for all  $x, y$ , or  $x \succsim y$  or  $y \succsim x$
2. Transitive: for all  $x, y, z$ , if  $x \succsim y$  and  $y \succsim x$ , then it must be that  $x \succsim z$

If  $\succsim$  is rational, then  $\succ$  and  $\sim$  are transitive too



# Theorem of revealed preferences

When does choice data reveal that individual is choosing according to rational preferences?

Given a choice structure  $R, C()$  the revealed preference relation  $\succsim$  is defined as:

$$x \succsim y \iff \exists B \in R \text{ s. t. } x, y \in B \text{ and } x \in C(B)$$

We read  $x \succsim y$  as:  *$x$  is revealed at least as good as  $y$*

To say that  $x$  is revealed preferred to  $y$  we need that  $\exists B \in R \text{ s. t. } x, y \in B, y \notin C(B)$

## Choice approach 1/2

Choice behaviour is represented by a *choice structure*  $(R, C(\cdot))$  where:

$R$  is a family of nonempty subsets of  $X$ , i.e. every element of  $R$  is a set  $B \subseteq X$ ,  $B$  is also called budget set

$C(\cdot)$  is a choice rule that assigns a nonempty set of chosen elements  $C(B) \subset B$  for every budget set  $B \subseteq X$

Note,  $C(B)$  could contain more than one elements

## Choice approach 2/2

Consider  $X = \{x, y, z\}$ ,  $R = \{\{x, y\}, \{x, y, z\}\}$ , and the choice rule  $C_1(\cdot)$  s.t.  $C_1(\{x, y\}) = \{x\}$  and  $C_1(\{x, y, z\}) = \{x\}$

Consider then a choice rule  $C_2(\cdot)$  s.t  $C_2(\{x, y\}) = \{x\}$  and  $C_2(\{x, y, z\}) = \{x, y\}$

The choice structure  $(R, C(\cdot))$  satisfies the Weak Axiom of Revealed Preferences (WARP) if for some  $B \in R$  with  $\{x, y\} \in B$  we have  $x \in C(B)$ , then for any  $B' \in R$  with  $\{x, y\} \in B'$  and  $y \in C(B')$ , we also have  $x \in C(B')$

## Weak Axiom of Revealed Preferences 1/2

Given a choice structure  $R, C(\cdot)$  the *revealed preference relation*  $\succsim^*$  is defined as:

$$x \succsim^* y \iff \exists B \in R \text{ s.t. } x, y \in B \text{ and } x \in C(B)$$

We read  $x \succsim^* y$  *x is revealed at least as good as y*

To say that  $x$  *is revealed preferred to*  $y$  we need that  $\exists B \in R$  s.t.  $x, y \in B$  and  $y \notin C(B)$

## Weak Axiom of Revealed Preferences 2/2

If an individual chooses (only)  $x$  when she faces a budget set  $\{x, y\}$ , she will not choose  $y$  when she faces  $\{x, y, z\}$

If  $x$  is chosen when  $y$  is available then there is no budget set containing  $x$  and  $y$  for which  $y$  is chosen and  $x$  is not

If  $C(\{x, y\}) = x$  we cannot have that  $C(\{x, y, z\}) = \{y\}$

## Utility function 1/4

In economics preference relation can be represented by means of a *utility function*

A utility function  $u(x)$  assigns a numerical value to each element in  $X$ , that is  $u: X \rightarrow R$  is a function representing preference relation  $\succsim$  if for all  $x, y \in X$ ,  
$$x \succsim y \iff u(x) \geq u(y)$$

# Utility function 2/4

Proposition: a preference relation  $\succsim$  can be represented by a utility function only if it is rational

Proof:

Completeness:

- $u(\cdot)$  is a real valued function defined on  $X$
- Then for any  $x, y \in X$  either  $u(x) \geq u(y)$  or  $u(y) \geq u(x)$
- Then it implies that either  $x \succsim y$  or  $y \succsim x$
- Hence  $\succsim$  must be complete

Transitivity:

- Suppose  $x \succsim y$  or  $y \succsim z$
- Then must be  $u(x) \geq u(y)$
- and  $u(y) \geq u(z)$
- Therefore  $u(x) \geq u(z)$ .
- This implies  $x \succsim z$

## Utility function 3/4

A preference relation  $\succsim$  can be represented by a utility function only if it is rational, but not all rational preferences can be represented by an utility function  
Example: lexicographic preferences

Proposition (Representation theorem , Debreu)

A preference relation  $\succsim$  can be represented by a utility function if it is satisfying rationality, continuity and strict monotonicity



# Utility function 4/4

Continuity:

$\forall y \in X, \{x \in X | x \succcurlyeq y\}$  and  $\{x \in X | y \succcurlyeq x\}$  are closed sets

Strict monotonicity:

$$\forall \mathbf{x} \neq \mathbf{y}, x_l \geq y_l \ \forall l \rightarrow \mathbf{x} \succ \mathbf{y}$$

lexicographic preferences do not satisfy continuity

$$x_n = \{1 + \frac{1}{n}, 1\} \quad x_0 = \{1, 3\}$$

For  $n > 0$   $x_n \succ x_0$  but for  $n \rightarrow \infty$   $x_0 \succ x_1$

# Indifference Curve

Given a utility function, the indifference curve(IC) plots all the combinations of two goods that provide the same level of satisfaction, or utility

An individual is indifferent across all the combinations of the two goods along the IC

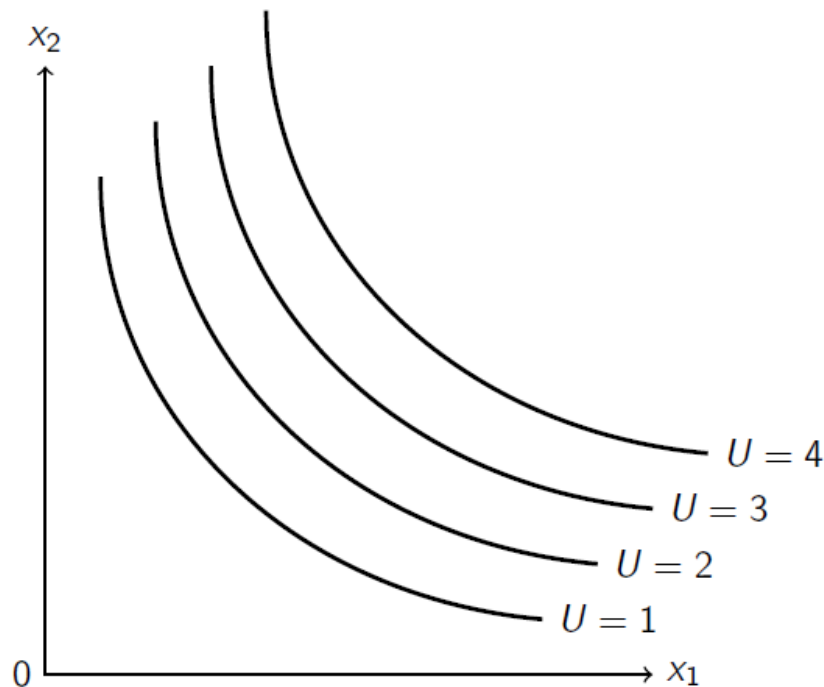
# Exercise 1

A consumer has a utility function  $u(x_1, x_2) = x_1 x_2$ , an income  $I = 100$ . The price of good  $x_1$  is  $p_1 = 10$  and the price of good  $x_2$  is  $p_2 = 20$

1. Derive the indifference curve
2. Compute the optimal consumption bundle

# Derive the indifference curve 1/3

The IC is the curve plotting all the combinations of goods  $x_1$  and  $x_2$  that gives the same level of utility



Derive the indifference curve 2/3

1. Set the level of utility to any positive value, say 100 for example

$$u(x_1, x_2) = x_1 x_2 = 100$$

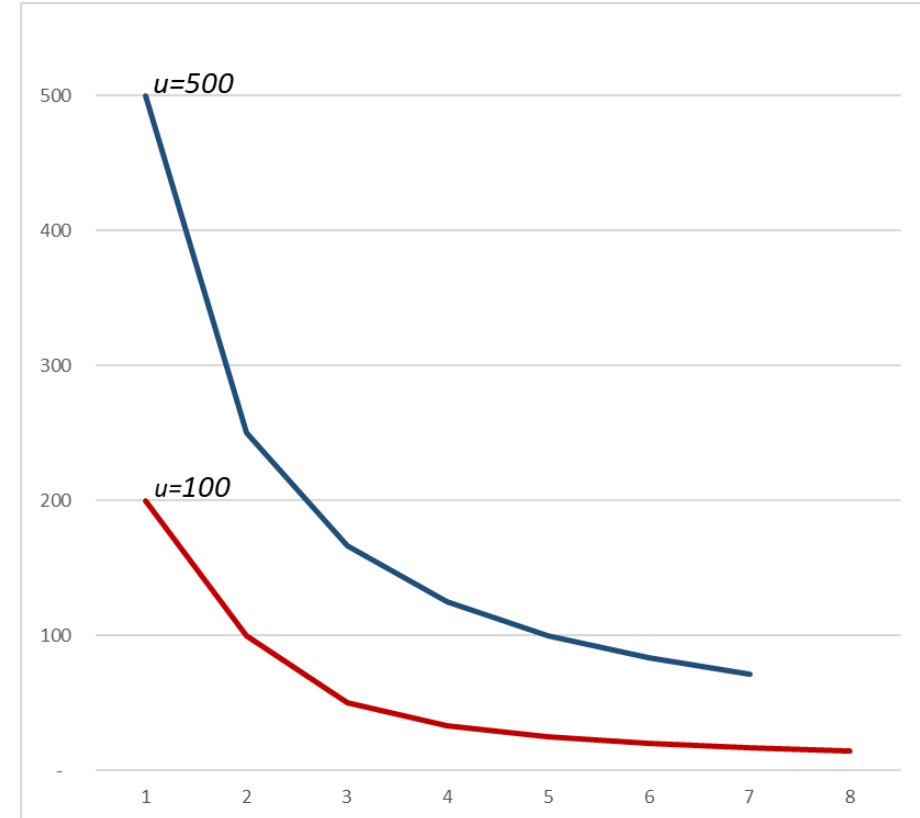
2. Express the utility function in terms of  $x_2$

$$x_2 = \frac{100}{x_1}$$

# Derive the indifference curve 3/3

3. Find the combinations of  $x_1$  and  $x_2$  that provides a utility of 100

<b>x1</b>	<i>u=100</i> <b>x2</b>	<i>u=500</i> <b>x2'</b>
1	100	500
2	50	250
3	33	167
4	25	125
5	20	100
6	17	83



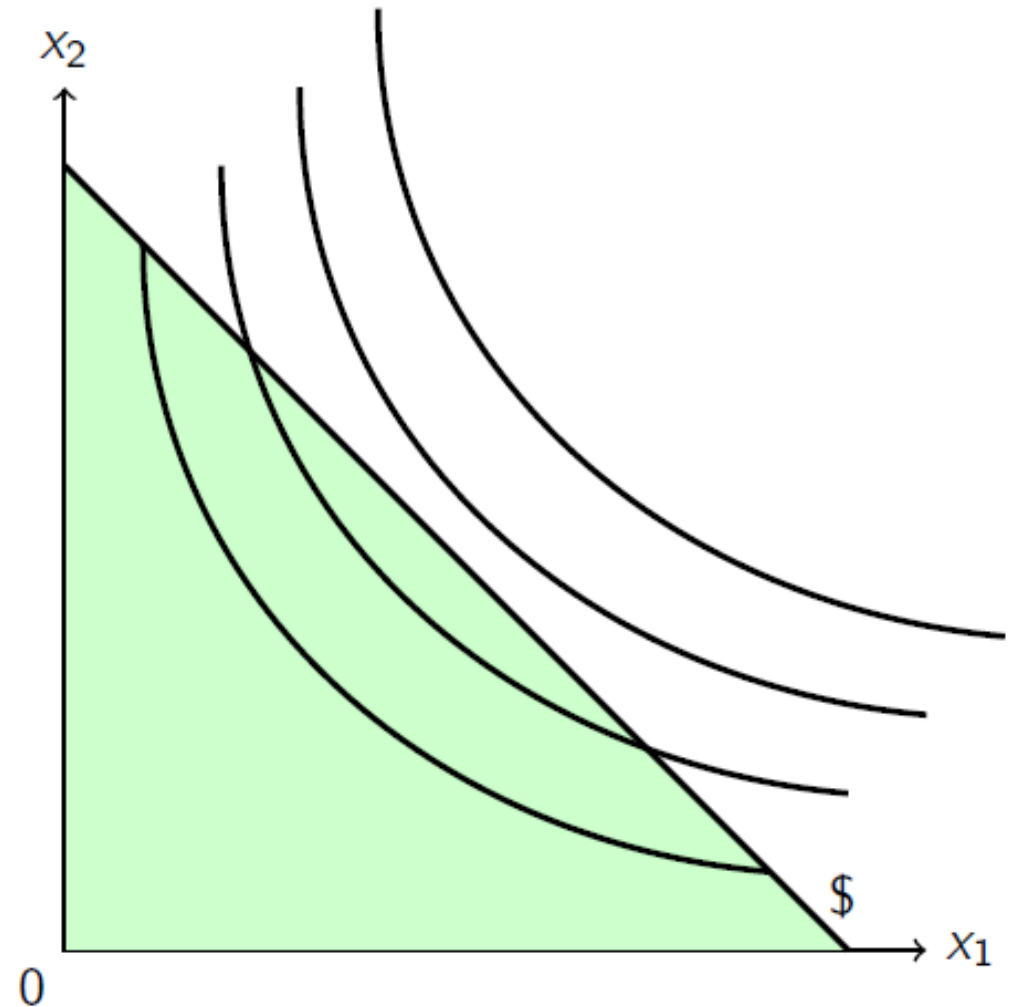
Find the optimal consumption bundle 1/2

The optimal consumption bundle is the bundle that gives consumer the highest utility, given her budget constraint

The optimal bundle is the allocation of  $x_1$  and  $x_2$  that on the budget line identifies the tangent point with the utility function

# Find the optimal consumption bundle 2/2

Indifference curves are parallel, and each one of them represents a specific level of satisfaction, utility





# Constrained maximization problem 1/2

$$\max_{\{x_1, x_2\}} u$$

Given that  $I = p_1x_1 + p_2x_2 = 100$

The tangency point between the budget line and the indifference curve is the point in which the MRS equals the slope of the budget line

# Constrained maximization problem 1/2

$$\max_{\{x_1, x_2\}} u$$

Condition

$$MRS = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \text{slope } B.C. = \frac{p_1}{p_2}$$

# Solution

$$\text{Condition: } \frac{x_2}{x_1} = \frac{p_1}{p_2} \rightarrow \frac{x_2}{x_1} = \frac{10}{20} = \frac{1}{2}$$

$$\begin{cases} x_1 = 2x_2 \\ 10 * 2x_2 + 20x_2 = 100 \end{cases}$$

$$x_1^* = 5 ; x_2^* = 2,5$$

## Exercise 2

A consumer has a utility function  $u(x, y) = x^{1/2}y^{1/4}$ , and income  $I = 150$

The price of good  $x$  is  $p_x=2$  and the price of good  $y$  is  $p_y=4$

## Solutions ex.2 1/4

$$\begin{cases} |MRS| = \frac{MU_x}{MU_y} \\ I = p_x x + p_y y \end{cases}$$

$$\blacktriangleright MU_x = \frac{\partial u(x,y)}{\partial x} \Rightarrow \frac{\partial x^{\frac{1}{2}} \cdot y^{\frac{1}{4}}}{\partial x} = \frac{1}{2} \cdot x^{\frac{1}{2}-1} \cdot y^{\frac{1}{4}} \Rightarrow \frac{1}{2} x^{-\frac{1}{2}} \cdot y^{\frac{1}{4}}$$

$$\blacktriangleright MU_y = \frac{\partial u(x,y)}{\partial y} \Rightarrow \frac{\partial x^{\frac{1}{2}} \cdot y^{\frac{1}{4}}}{\partial y} = \frac{1}{4} \cdot x^{\frac{1}{2}} \cdot y^{\frac{1}{4}-1} \Rightarrow \frac{1}{4} x^{\frac{1}{2}} \cdot y^{-\frac{3}{4}}$$

## Solutions ex.2 2/4

$$|MRS| = \frac{MU_x}{MU_y} \Rightarrow |MRS| = \frac{\frac{1}{2}x^{-\frac{1}{2}} \cdot y^{\frac{1}{4}}}{\frac{1}{4}x^{\frac{1}{2}} \cdot y^{-\frac{3}{4}}}$$

$$|MRS| = \frac{1}{2}x^{-\frac{1}{2}} \cdot y^{\frac{1}{4}} \cdot \left(\frac{1}{4}x^{\frac{1}{2}} \cdot y^{-\frac{3}{4}}\right)^{-1}$$

$$|MRS| = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{4}} \cdot \left(4^{-1}x^{\frac{1}{2}}y^{-\frac{3}{4}}\right)^{-1}$$

$$|MRS| = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{4}} \cdot 4x^{-\frac{1}{2}}y^{\frac{3}{4}}$$

$$|MRS| = \frac{4}{2} \cdot x^{-\frac{1}{2}} \cdot x^{-\frac{1}{2}} \cdot y^{\frac{1}{4}} \cdot y^{\frac{3}{4}}$$

$$|MRS| = 2 \cdot x^{\frac{-1-1}{2}} \cdot y^{\frac{1+3}{4}} \Rightarrow |MRS| = 2 \cdot x^{-\frac{2}{2}} \cdot y^{\frac{4}{4}}$$

$$|MRS| = 2 \cdot x^{-1} \cdot y^1 \Rightarrow |MRS| = 2 \cdot \frac{y}{x}$$

## Solutions ex.2 3/4

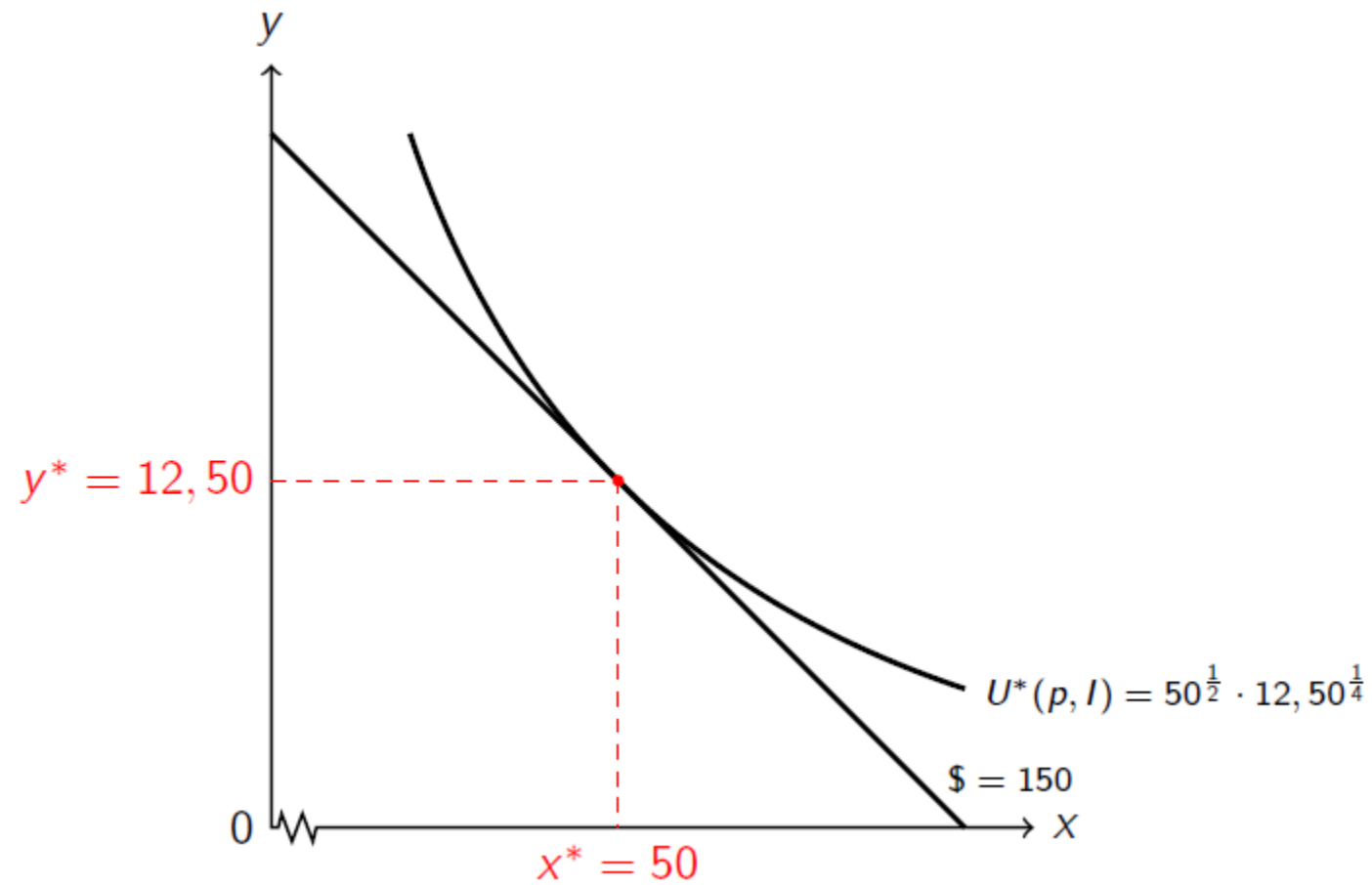
$$\begin{cases} 2 \cdot \frac{y}{x} = \underbrace{\frac{2}{4}}_{p_x/p_y} \Rightarrow \frac{y}{x} = \frac{1}{4} \Rightarrow y = \frac{x}{4} \\ 2x + 4y = 150 \end{cases}$$

$$2x + 4 \cdot \left(\frac{x}{4}\right) = 150 \rightarrow 3x = 150 \rightarrow x^* = 50$$

$$y = \frac{x}{4} \rightarrow y^* = 12.50$$

**Optimal Consumption Bundle  $(x^*, y^*) = (50, 12.50)$**

# Solutions ex.2 4/4





# Take home exercises

1.  $u(x, y) = x^2y$

$$I = 240$$

$$p_x = 8$$

$$p_y = 2$$

2.  $u(x, y) = x^{\frac{1}{2}}y^{\frac{1}{2}}$

$$I = 200$$

$$p_x = 10$$

$$p_y = 10$$

3.  $u(x, y) = \ln(x) + 2\ln(y)$

$$I = 60$$

$$p_x = 3$$

$$p_y = 1$$

4.  $u(x, y) = x^{\frac{2}{5}}y^{\frac{3}{5}}$

$$I = 200$$

$$p_x = 20$$

$$p_y = 30$$