Differentiation of $\frac{PQ}{R}$ with Respect to R (page 95 of the book)

April 6, 2024

Given a function $f(R) = \frac{PQ(R)}{R}$, where P is a constant and $Q(R)$ is a function of R, we aim to differentiate this function with respect to R. The differentiation involves the application of the quotient rule.

1 Quotient Rule

The quotient rule for differentiation is given by:

$$\frac{d}{dR}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dR} - u\frac{dv}{dR}}{v^2} \tag{1}$$

where $u = PQ(R)$ and $v = R$.

2 Differentiation Steps

1. Differentiate $u = PQ(R)$ with respect to R: Since P is a constant and $Q(R)$ is a function of R, the derivative of u with respect to R is $P\frac{dQ}{dR}$.
2. Differentiate $v = R$ with respect to R: The derivative of v with respect to R is simply 1.
3. Substitute u, v, $\frac{du}{dR}$, and $\frac{dv}{dR}$ into the quotient rule formula:

$$\frac{d}{dR}\left(\frac{PQ(R)}{R}\right) = \frac{R\left(P\frac{dQ}{dR}\right) - PQ(R)(1)}{R^2} \tag{2}$$

   Simplifying, we obtain:

$$\frac{P\left(R\frac{dQ}{dR} - Q(R)\right)}{R^2} \tag{3}$$

Please note that in the book Q is simply Q, here we are underlying that Q is a function of R.

3 Conclusion

Through the application of the quotient rule and careful differentiation of u and v, we successfully derive the differential equation of $\frac{PQ}{R}$ with respect to R, taking into account that P is a constant and $Q(R)$ is a function dependent on R.