Perfect Competition in the Long Run and Monopoly

Microeconomics Exercises

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Theoretical background: perfect competition in the long run

In the long run, free entry and exit guarantee that economic profit tends to zero. The long-run equilibrium for a price-taking firm simultaneously satisfies three conditions:

$$\boxed{p^{LR} = \mathrm{MC}(Q^{LR}) = \mathrm{AC}(Q^{LR}) = \mathrm{AC}_{\min}}$$

- $p = \mathrm{MC}$: profit-maximisation condition.
- $p = \mathrm{AC}$: zero-profit condition (no incentive to enter or exit).
- The production level coincides with the *minimum* of average cost.

The number of active firms in the long run is determined by dividing the quantity demanded at price p^{LR} by the optimal output of the individual firm:

$$n^{LR} = \frac{Q^d(p^{LR})}{Q_i^{LR}}$$

Exercise 1 — Long-run equilibrium with fixed costs

Problem. In a perfectly competitive market, 100 identical firms are initially active, each with total cost function:

$$TC(Q_i) = Q_i^2 + 10$$

Market demand is:

$$Q^d = 300 - 20p$$

Determine:

1. The price and the output of the individual firm in the long-run equilibrium.
2. The total quantity supplied by the industry in the long run.
3. The number of active firms in the long run.
4. The profit of each individual firm in the long run, assuming that plant size cannot change.

Solution

Point 1 — Price and optimal output for the individual firm.
Average cost is:

$$\mathrm{AC}(Q) = \frac{Q^2 + 10}{Q} = Q + \frac{10}{Q}$$

We minimise it by setting the first derivative to zero:

$$\frac{\partial\,\mathrm{AC}}{\partial Q} = 1 - \frac{10}{Q^2} = 0 \;\Rightarrow\; Q^2 = 10 \;\Rightarrow\; \boxed{Q_i^{LR} = \sqrt{10}}$$

The long-run price is obtained by evaluating average cost at its minimum:

$$p^{LR} = \mathrm{AC}\Big(\sqrt{10}\Big) = \sqrt{10} + \frac{10}{\sqrt{10}} = \sqrt{10} + \sqrt{10} = \boxed{2\sqrt{10} \approx 6.32}$$

Point 2 — Aggregate industry supply.
With 100 firms each producing $\sqrt{10}$:

$$Q^s_{LR} = 100 \cdot \sqrt{10} \approx 316$$

Point 3 — Number of firms in the long run.
Substituting $p^{LR} = 2\sqrt{10}$ into the demand function:

$$Q^E = Q^d\Big(2\sqrt{10}\Big) = 300 - 20 \cdot 2\sqrt{10} = 300 - 40\sqrt{10} \approx 174$$

The required number of firms:

$$n^{LR} = \frac{Q^E}{Q_i^{LR}} = \frac{300 - 40\sqrt{10}}{\sqrt{10}} \approx \boxed{55}$$

Since the initial supply (≈ 316) exceeds long-run demand (≈ 174), approximately 45 out of 100 firms will exit the market.

Point 4 — Individual profit in the long run.

$$\pi_i^{LR} = p^{LR} \cdot Q_i^{LR} - TC(Q_i^{LR}) = 2\sqrt{10} \cdot \sqrt{10} - \Big((\sqrt{10})^2 + 10\Big) = 20 - 10 - 10 = \boxed{0}$$

Profit is zero, confirming the long-run equilibrium condition: the price exactly covers minimum average cost and there is no incentive to alter the number of firms.

Theoretical background: monopoly

A **monopolistic firm** is the sole seller of the good in the market. Unlike the competitive firm, it is a *price-setter*: it chooses quantity Q taking into account that price is a decreasing function of Q itself, i.e. $p = p(Q)$.
The profit-maximisation problem is:

$$\max_{Q} \ \pi^m = p(Q) \cdot Q - TC(Q)$$

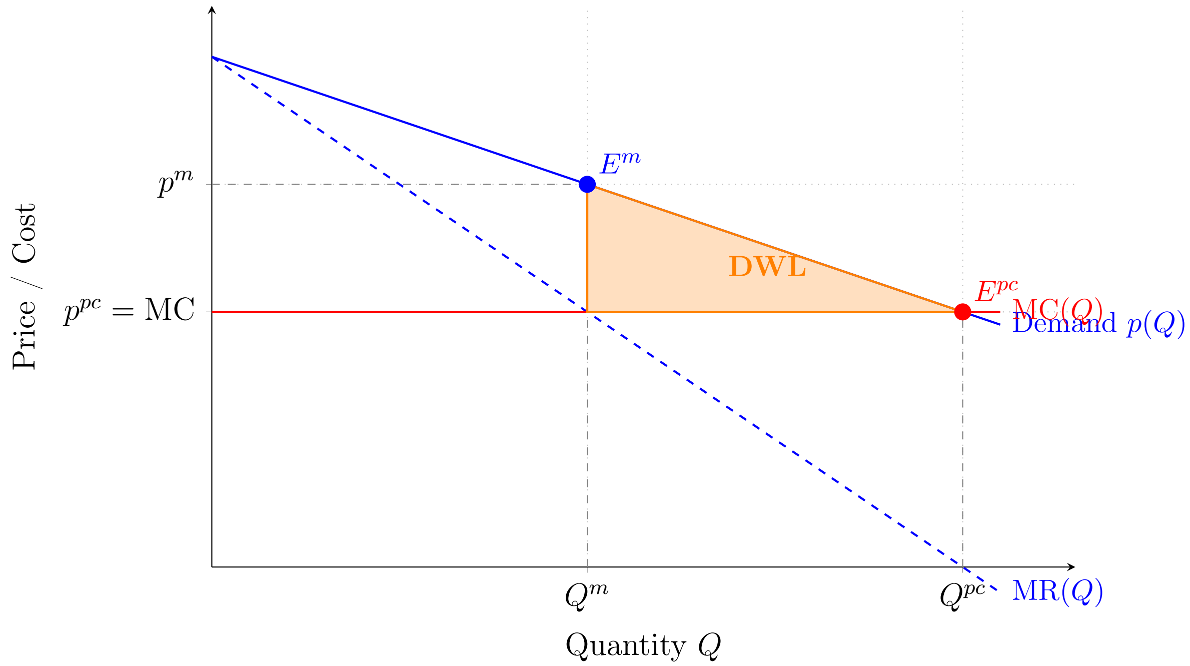
The first-order condition yields the **monopoly rule**:

$$\frac{\partial \pi^m}{\partial Q} = \underbrace{\frac{\partial [p(Q) \cdot Q]}{\partial Q}}_{\mathrm{MR}(Q)} - \underbrace{\frac{\partial TC(Q)}{\partial Q}}_{\mathrm{MC}(Q)} = 0 \qquad \Longrightarrow \qquad \boxed{\mathrm{MR}(Q) = \mathrm{MC}(Q)}$$

The monopolist chooses Q^m such that **marginal revenue** equals **marginal cost**; the price p^m is then read off the demand curve.

Graph of the monopoly equilibrium

The figure below illustrates the monopoly equilibrium relative to the competitive equilibrium, highlighting the deadweight loss.



The **orange triangle** represents the **deadweight loss** (DWL): it is the surplus that neither the monopolist nor consumers can realise due to the restriction of output from Q^{pc} to Q^m. The monopolist maximises profit at the intersection MR = MC (point E^m), whereas the socially efficient allocation would be E^{pc}, where $p = \mathrm{MC}$.

Exercise 2 — Comparison between monopoly and perfect competition

Problem. An industry is served by a single firm with total cost function:

$$TC(Q) = 100Q$$

Market demand is:

$$Q^d = 400 - 2p$$

Determine:

1. The market equilibrium when the firm acts as a monopolist (*price-setter*).
2. The market equilibrium if the same firm behaves as a *price-taker* (as under perfect competition).
3. The change in profit between the two regimes and comment on the result from a social welfare perspective.

Solution

Point 1 — Monopoly equilibrium.
Inverting the demand function to obtain $p(Q)$:

$$Q = 400 - 2p \;\Rightarrow\; p(Q) = 200 - \tfrac{1}{2}Q$$

Total revenue and marginal revenue:

$$TR(Q) = 200Q - \tfrac{1}{2}Q^2 \qquad \Rightarrow \qquad \text{MR}(Q) = 200 - Q$$

Marginal cost (constant):

$$\text{MC}(Q) = 100$$

From the condition MR = MC:

$$200 - Q^m = 100 \;\Rightarrow\; \boxed{Q^m = 100} \qquad p^m = 200 - \tfrac{1}{2}\cdot 100 = \boxed{150}$$

$$\text{Eq}^m = \{Q^m = 100,\; p^m = 150\}$$

Point 2 — Competitive equilibrium.
Under perfect competition $p = \text{MC}$:

$$200 - \tfrac{1}{2}Q^{pc} = 100 \;\Rightarrow\; \boxed{Q^{pc} = 200} \qquad p^{pc} = 200 - \tfrac{1}{2}\cdot 200 = \boxed{100}$$

$$\text{Eq}^{pc} = \{Q^{pc} = 200,\; p^{pc} = 100\}$$

As expected, the competitive regime produces a higher quantity at a lower price compared to monopoly.

Point 3 — Profit comparison and welfare analysis.

$$\pi^m = p^m Q^m - TC(Q^m) = 150\cdot 100 - 100\cdot 100 = 15\,000 - 10\,000 = \boxed{5\,000}$$
$$\pi^{pc} = p^{pc} Q^{pc} - TC(Q^{pc}) = 100\cdot 200 - 100\cdot 200 = 20\,000 - 20\,000 = \boxed{0}$$

The monopolist earns an extra profit of 5 000 relative to the competitive scenario. However, this private gain comes with a **net loss of social welfare**: the units between $Q^m = 100$ and $Q^{pc} = 200$ are not produced, even though consumers value them above the cost of production. The deadweight loss (area of the DWL triangle in the graph above) equals:

$$\text{DWL} = \frac{1}{2}(p^m - p^{pc})(Q^{pc} - Q^m) = \frac{1}{2}(150 - 100)(200 - 100) = \frac{1}{2}\cdot 50\cdot 100 = \boxed{2\,500}$$

Exercise 3 — Monopoly, competition and second-best regulation

Problem. In a market the inverse demand curve is:

$$p = 100 - Q$$

A single firm operates with the following total cost function:

$$TC(Q) = 200 + 10Q$$

Determine the market equilibrium — quantity, price, and profit — under the three following scenarios:

1. The firm behaves as a *price-taker* (competitive conditions).
2. The firm operates as a monopolist (*price-setter*).
3. A public regulator requires the firm to set price equal to its own average cost (*second-best solution*).

Preliminary quantities

From the total cost function we immediately derive:

$$\mathrm{MC}(Q) = \frac{\partial TC}{\partial Q} = 10 \qquad\qquad \mathrm{AC}(Q) = \frac{TC(Q)}{Q} = \frac{200}{Q} + 10$$

Marginal cost is constant and equal to 10; average cost is decreasing (fixed costs equal to 200), so the competitive *first-best* necessarily generates a loss.

Solution

Scenario 1 — Competitive conditions ($p = \mathrm{MC}$).

$$p = \mathrm{MC} \;\Rightarrow\; 100 - Q^{pc} = 10 \;\Rightarrow\; \boxed{Q^{pc} = 90, \quad p^{pc} = 10}$$

$$\pi^{pc} = p^{pc}Q^{pc} - TC(Q^{pc}) = 10 \cdot 90 - (200 + 10 \cdot 90) = 900 - 1100 = \boxed{-200}$$

The firm records a **loss equal to fixed costs**: price exactly covers variable costs but not the fixed cost of 200. This is why a natural monopoly (with high fixed costs and a low, constant marginal cost) cannot survive under a purely competitive regime without subsidies.

Scenario 2 — Monopoly ($\mathrm{MR} = \mathrm{MC}$).
Total revenue and marginal revenue:

$$TR(Q) = p(Q) \cdot Q = (100 - Q)Q = 100Q - Q^2 \;\Rightarrow\; \mathrm{MR}(Q) = 100 - 2Q$$

From the optimality condition:

$$\mathrm{MR} = \mathrm{MC} \;\Rightarrow\; 100 - 2Q^m = 10 \;\Rightarrow\; \boxed{Q^m = 45, \quad p^m = 100 - 45 = 55}$$

$$\pi^m = 55 \cdot 45 - (200 + 10 \cdot 45) = 2\,475 - 650 = \boxed{1\,825}$$

The monopolist earns a profit of 1 825, but produces less than half the competitive quantity, generating a deadweight loss of social welfare.

Scenario 3 — *Second-best* regulation ($p = \mathrm{AC}$).
The *first best* ($p = \mathrm{MC}$) is unfeasible because it generates losses that would require a public subsidy. The *second-best* solution imposes $p = \mathrm{AC}$, guaranteeing zero profit without subsidies.
We impose the condition on the demand curve:

$$p(Q) = \mathrm{AC}(Q) \;\Rightarrow\; 100 - Q = \frac{200}{Q} + 10$$

Multiplying both sides by $Q > 0$:

$$(90 - Q)\,Q = 200 \;\Rightarrow\; Q^2 - 90Q + 200 = 0$$

Applying the quadratic formula:

$$Q = \frac{90 \pm \sqrt{90^2 - 4 \cdot 200}}{2} = \frac{90 \pm \sqrt{8100 - 800}}{2} = \frac{90 \pm \sqrt{7300}}{2} = 45 \pm 5\sqrt{73}$$

The two solutions are $Q_1 = 45 + 5\sqrt{73} \approx 87.7$ and $Q_2 = 45 - 5\sqrt{73} \approx 2.3$.
The economically relevant solution is Q_1: it is the one closest to the competitive optimum, realising the largest quantity compatible with the firm's break-even constraint. Q_2 corresponds to an unstable equilibrium with minimal output and is of no regulatory interest.

$$\boxed{Q^{sb} = 45 + 5\sqrt{73} \approx 87.7} \qquad p^{sb} = 100 - Q^{sb} = 55 - 5\sqrt{73} \approx 12.3$$

$$\pi^{sb} = p^{sb} \cdot Q^{sb} - TC(Q^{sb}) = \mathrm{AC}(Q^{sb}) \cdot Q^{sb} - TC(Q^{sb}) = \boxed{0}$$

Profit is zero by construction: $p = \mathrm{AC}$ implies $pQ = TC(Q)$.

Comparative summary.

Scenario	Q	p	π
Perfect competition	90	10	-200
Monopoly	45	55	1 825
Second best	≈ 87.7	≈ 12.3	0

Second-best regulation recovers nearly all the competitive efficiency ($Q^{sb} \approx 87.7$ versus $Q^{pc} = 90$) while simultaneously eliminating the monopoly profit and ensuring the firm's survival without public subsidies.