Exercise on hashing (Hashing.pdf, slide 44)

Let's consider a hash table of size 7 and open addressing with quadratic probe using the hash function $h(k) = k \bmod 7$. Insert the keys 18, 14, 21, 1 and 35 (in this order) and write down the entries of the hash table after each insertion.

Recall that quadratic probing uses a quadratic function to calculate the next slot to check. Therefore, the probe sequence is

$h_i(k) = (h(k) + i^2) \bmod \text{TableSize}$

where $h(k)$ is the original hash function (in our case $h(k) = k \bmod 7$). The `mod TableSize` ensures that the new index wraps around to stay within the bounds of the table.

That means:

0^{th} probe: $h(k)$

1^{st} probe: $h_1(k) = (h(k) + 1^2) \bmod \text{TableSize} = (h(k) + 1) \bmod \text{TableSize}$

2^{nd} probe: $h_2(k) = (h(k) + 2^2) \bmod \text{TableSize} = (h(k) + 4) \bmod \text{TableSize}$

3^{rd} probe: $h_3(k) = (h(k) + 3^2) \bmod \text{TableSize} = (h(k) + 9) \bmod \text{TableSize}$

4^{th} probe: $h_4(k) = (h(k) + 4^2) \bmod \text{TableSize} = (h(k) + 16) \bmod \text{TableSize}$

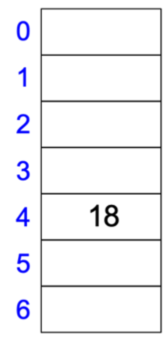
5^{th} probe: $h_5(k) = (h(k) + 5^2) \bmod \text{TableSize} = (h(k) + 25) \bmod \text{TableSize}$

…

We consider a new probe until an empty (i.e., *open*) slot is found.

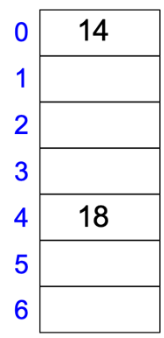
Informally, starting from the initial slot in which we should insert the key (i.e., 0^{th} probe), we move up by 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 … slots until we found an empty slot.

Let's **insert key 18**.
$h(18) = 18 \bmod 7 = 4$
and slot 4 is empty, so we insert key 18 into slot 4.



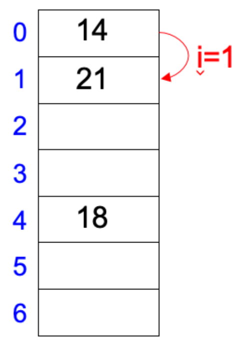
Insert 18

Let's **insert key 14**.
Since $h(14) = 14 \bmod 7 = 0$
and slot 0 is empty, we insert key 14 into slot 0.



Insert 14

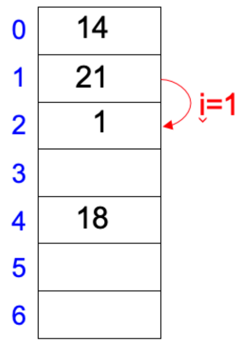
Let's **insert key 21**.
Since $h(21) = 21 \bmod 7 = 0$
and slot 0 is full, there is a collision.
Let's consider the quadratic probe sequence, starting from i=1
$h_1(21) = ((21 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$
Slot 1 is empty, so we insert key 21 into slot 1.



Insert 21

Let's **insert key 1**.
Since $h(1) = 1 \bmod 7 = 1$
and slot 1 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from i=1
$h_1(1) = ((1 \bmod 7) + 1^2) \bmod 7 = (1 + 1) \bmod 7 = 2 \bmod 7 = 2$
Slot 2 is empty, so we insert key 1 into slot 2.

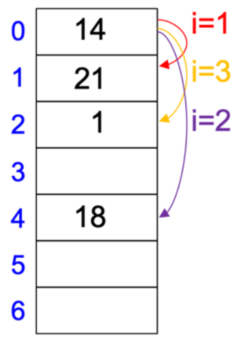


Insert 1

Let's **insert key 35**.
Since $h(35) = 35 \bmod 7 = 0$
and slot 0 is full, there is a collision.

Let's consider the quadratic probe sequence, starting from i=1
$h_1(35) = ((35 \bmod 7) + 1^2) \bmod 7 = (0 + 1) \bmod 7 = 1$
Slot 1 is full, again a collision.

Let's try with the 2nd step in the probe sequence (i=2):
$h_2(35) = ((35 \bmod 7) + 2^2) \bmod 7 = (0 + 4) \bmod 7 = 4$
Slot 4 is full, again a collision.

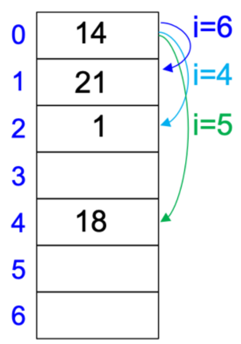


Insert 35

Let's try with the 3rd step in the probe sequence (i=3):
$h_3(35) = ((35 \bmod 7) + 3^2) \bmod 7 = (0 + 9) \bmod 7 = 2$
Slot 2 is full, again a collision.

Let's try with the 4th step in the probe sequence (i=4):
$h_4(35) = ((35 \bmod 7) + 4^2) \bmod 7 = (0 + 16) \bmod 7 = 2$
Slot 2 is full, again a collision.

Let's try with the 5th step in the probe sequence (i=5):
$h_5(35) = ((35 \bmod 7) + 5^2) \bmod 7 = (0 + 25) \bmod 7 = 4$
Slot 4 is full, again a collision.



Insert 35

Let's try with the 6th step in the probe sequence (i=6):
$h_6(35) = ((35 \bmod 7) + 6^2) \bmod 7 = (0 + 36) \bmod 7 = 1$
Slot 1 is full, again a collision.

Let's try with the 7th step in the probe sequence (i=7):
$h_7(35) = ((35 \bmod 7) + 7^2) \bmod 7 = (0 + 49) \bmod 7 = 0$
Slot 0 is full, again a collision.

Let's try with the 8th step in the probe sequence (i=8):

$h_8(35) = ((35 \bmod 7) + 8^2) \bmod 7 = (0 + 64) \bmod 7 = 1$
Slot 1 is full, again a collision.

Let's try with the 9th step in the probe sequence (i=9):
$h_9(35) = ((35 \bmod 7) + 9^2) \bmod 7 = (0 + 81) \bmod 7 = 4$
Slot 4 is full, again a collision.

Let's try with the 10th step in the probe sequence (i=10):
$h_{10}(35) = ((35 \bmod 7) + 10^2) \bmod 7 = (0 + 100) \bmod 7 = 2$
Slot 2 is full, again a collision.

Let's try with the 11th step in the probe sequence (i=11):
$h_{10}(35) = ((35 \bmod 7) + 11^2) \bmod 7 = (0 + 121) \bmod 7 = 2$
Slot 2 is full, again a collision.

Let's try with the 12th step in the probe sequence (i=12):
$h_{10}(35) = ((35 \bmod 7) + 12^2) \bmod 7 = (0 + 144) \bmod 7 = 4$
Slot 4 is full, again a collision.

Let's try with the 13th step in the probe sequence (i=13):
$h_{10}(35) = ((35 \bmod 7) + 13^2) \bmod 7 = (0 + 169) \bmod 7 = 1$
Slot 1 is full, again a collision.

Let's try with the 14th step in the probe sequence (i=14):
$h_{10}(35) = ((35 \bmod 7) + 14^2) \bmod 7 = (0 + 196) \bmod 7 = 0$
Slot 0 is full, again a collision.

This is a very unlucky situation, where we get the sequence 0, 1, 4, 2, 2, 4, 1, 0 which repeats itself endlessly.
Therefore we are not able to find a free slot.
The motivation is that the probing sequence does not cover all slots.