TOR VERGATA
UNIVERSITY OF ROME

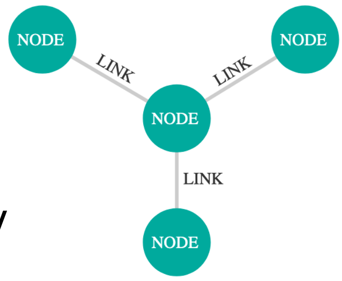
Social Networks

Algorithms, Data and Security
A.Y. 2024/25

Valeria Cardellini

Global Governance, 3rd year
Science and Technology Major

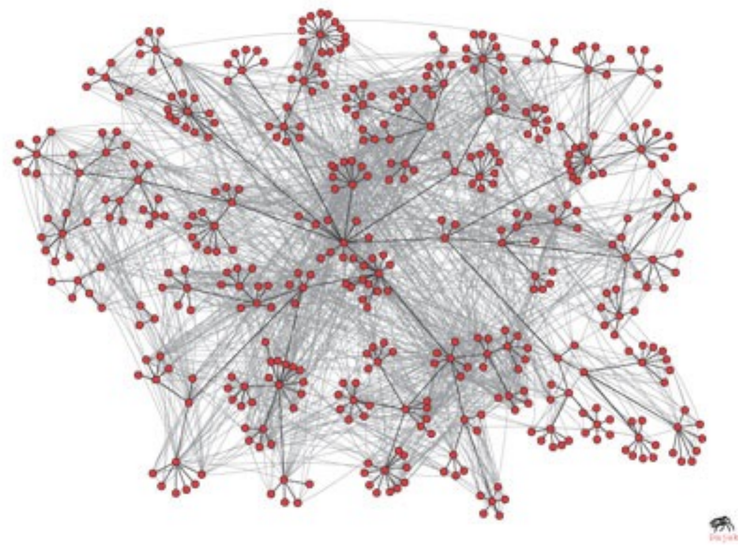
Networks



- Networks are a representation of **interaction** structure among units
  - These units are **nodes**
  - Some pairs of nodes are connected by **links**
  - In case of social and economic networks, nodes are individuals or organizations
- The study of networks can encompass the study of all kinds of interactions, e.g.,
  - Transportation
  - Communication
  - **Social**
  - Spread of epidemics

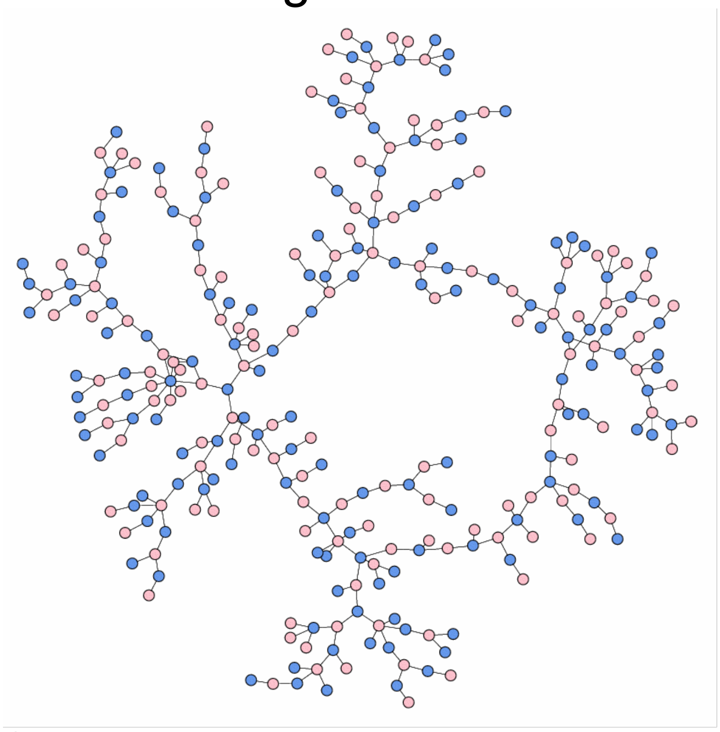
Examples of social networks

- Corporate e-mail communication
  - Links indicate people connected by e-mail exchange



Examples of social networks

- High school dating

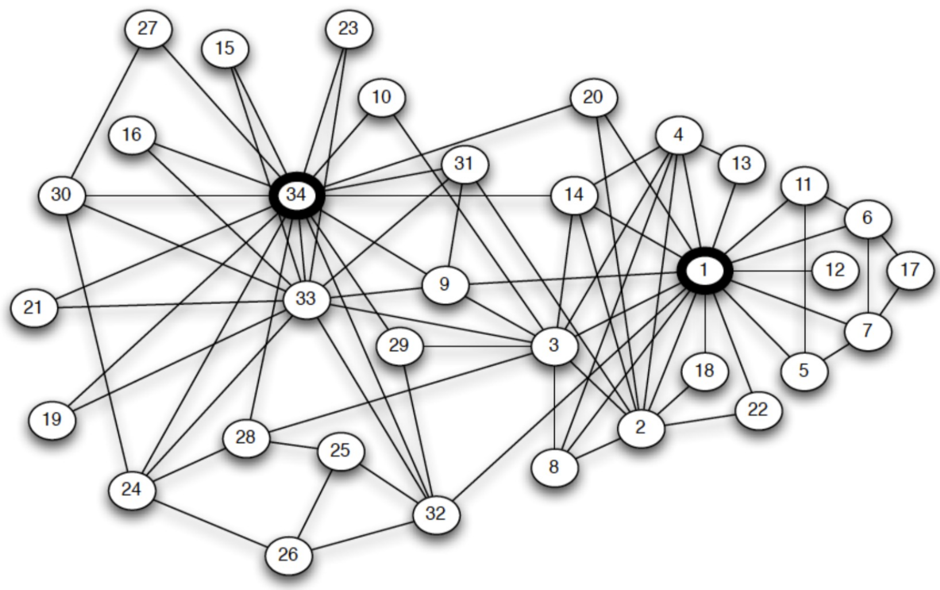


Examples of social networks

- Trails of Flickr users in Manhattan

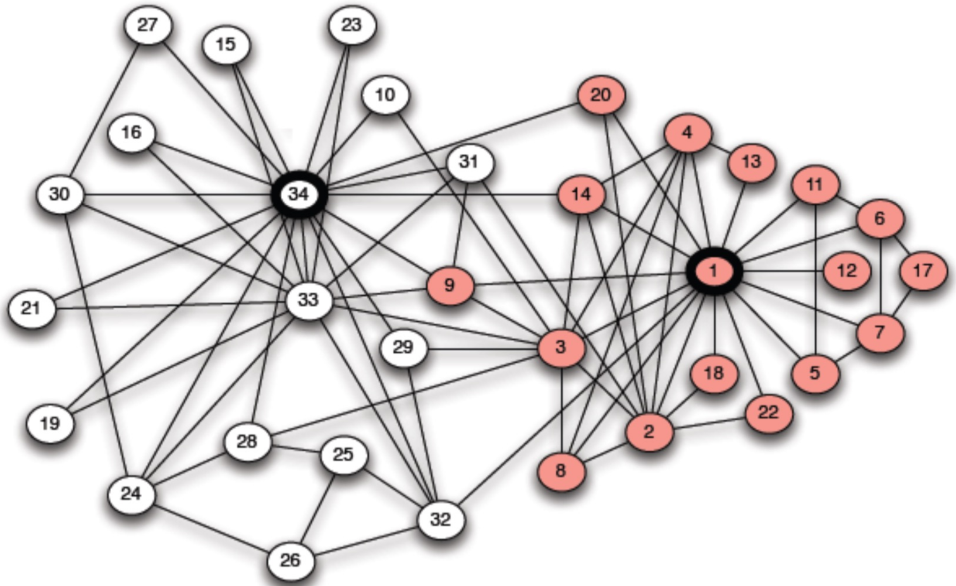
Examples of social networks

- The social network of friendships within a 34-person karate club



Examples of social networks

- The social network of friendships within a 34-person karate club provides clues to the fault lines that eventually split the club apart



Networks: structure and behavior

- Networks are complex and highly connected systems
- Two perspectives to analyze them

1. Connectedness at the level of **structure**: who is linked to whom (**graph theory**)
   - E.g., in a social network like Instagram we can consider structural features such as who is linked to whom, centrality measures, communities, etc.
2. Connectedness at the level of behavior: each individual's actions have implicit consequences for the outcomes of everyone (game theory)
   - E.g., each user chooses what content to share or engage with

Network dynamics: population effects

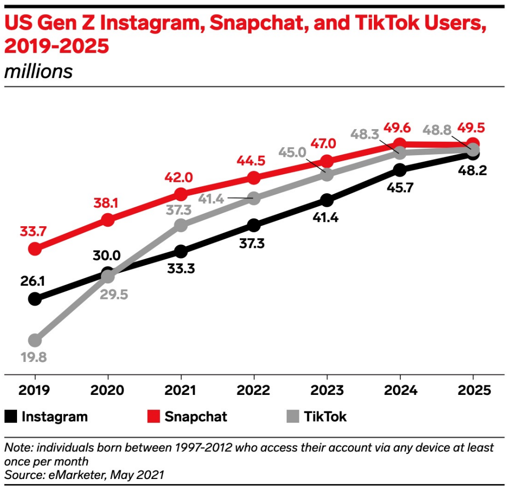
- Observing a large population over time, we see a recurring pattern by which new ideas, beliefs, opinions, innovations, technologies, products, and social conventions emerge and evolve
- Collectively, we can refer to these as social practices (holding opinions, adopting products, behaving according to certain principles) that people can choose to adopt or not
- How new practices spread through a population depends in large part on the fact that people influence each other's behavior

Network dynamics: population effects

- We could hypothesize that people imitate the decisions of others because of the human tendency to conform
- But there are multiple reasons why even purely rational agents - individuals with no a priori desire to conform to what others are doing - will copy the behavior of others
- One class of reasons is based on the fact that the behavior of others conveys information: information cascades

Network dynamics: population effects

- Cascading adoption of a new technology or service can be the result of individual incentives to use the most widespread technology; can happen for:
  - Informational effects of seeing many other people adopt the technology
  - Direct benefits of adopting what many others are already using

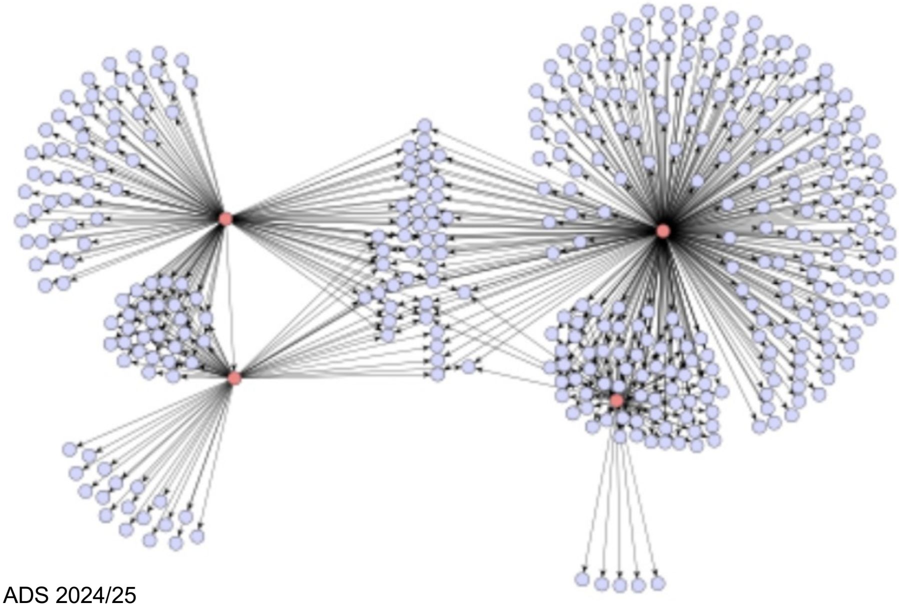


Network dynamics: structural effects

- Taking network structure into account provides important further insights into how such kinds of influence take place
- The underlying mechanisms - based on information and direct benefits - are present both at the global level of whole population, and also at a local level in the network
  - Between an individual and their set of friends or colleagues
- When people are influenced by the behaviors of their neighbors in the network, the adoption of a new product or innovation can *cascade* through the network structure

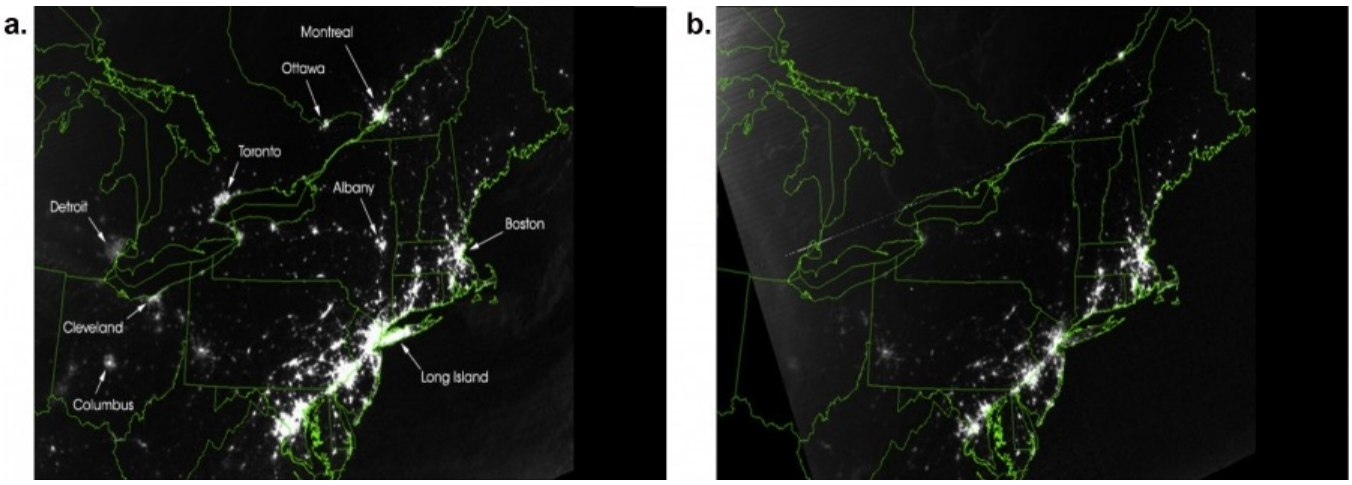
Network dynamics: structural effects

- Example: e-mail recommendations for a Japanese graphic novel spread in a kind of informational or social contagion



Network dynamics: structural effects

- Example: 2003 North American blackout



a) Satellite image on Northeast United States on August 13th, 2003, at 9:29pm (EDT), 20 hours before the 2003 blackout

b) The same as above, but 5 hours after the blackout

Vulnerability due to interconnectivity

- 2003 blackout: typical example of cascading failure, where a local failure may not stay local any longer
  - When a network acts as a transportation system, a local failure shifts loads to other nodes
  - If the extra load is negligible, the system can seamlessly absorb it and the failure goes unnoticed
  - If the extra load is too much for the neighboring nodes, they will redistribute the load to their neighbors
  - In no time, we are faced with a cascading event, whose magnitude depends on the position and the capacity of the nodes that failed initially

Networks: interdisciplinarity

- Theory of network structure and behavior addresses simultaneous challenges deriving from
  - **Economics**: theories for strategic interaction among small numbers of parties, as well as for cumulative behavior of large, homogeneous populations
  - **Sociology**: some of fundamental insights into structure of social networks, but network methodology refined only in domains and scales where data-collection has traditionally been possible (well-defined groups with tens to hundreds of people)
  - **Computer Science**: with rise of Web and social media, we deal with design constraints on large computing systems which are not only technological but also human (complex feedback that human audiences create when humans collectively use Web for communication, self-expression, and creation of knowledge)

Network science

- Academic field which studies complex networks
  - Including telecommunication, computer, biological, cognitive and semantic, social networks
- Considers distinct elements or actors represented by nodes (or vertices) and the connections between elements or actors as links (or edges)
- Theories and methods from multiple disciplines
  - Graph theory from mathematics, statistical mechanics from physics, data mining and information visualization from computer science, inferential modeling from statistics, and social structure from sociology

Elements of any social network

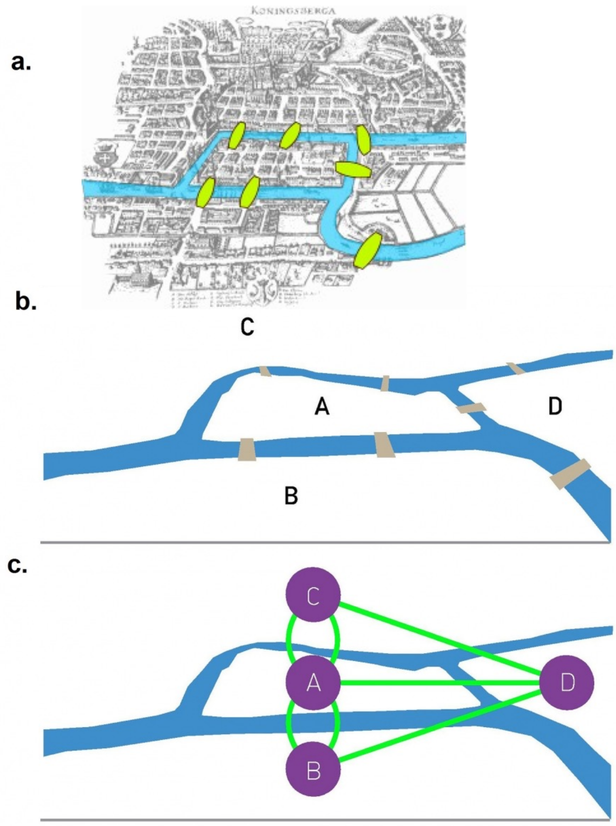
- We focus on social networks
- Components of networks consist of actors and relations
- Actors can be individuals (e.g., students, employees) or organizations
  - Those actors are modeled as nodes in a graph
- The connections between actors make up relations (modeled as links or edges)
  - Those relations can represent friendship, advice, hindrance or communication

A brief history of networks (graphs)

- The earliest known paper in this field is the famous Seven Bridges of Königsberg written by Leonhard Euler in 1735
- Laid the foundations of graph theory
- City of Königsberg with 7 bridges across the river Pregel that surrounded the town
- Can one walk across all 7 bridges and never cross the same one twice?

Seven bridges of Königsberg

- Can one walk across all 7 bridges and never cross the same one twice?
- Euler constructed a graph that has 4 nodes (A, B, C, D), each corresponding to a patch of land, and 7 links, each corresponding to a bridge
- He then showed that there is no continuous path that would cross the 7 bridges while never crossing the same bridge twice

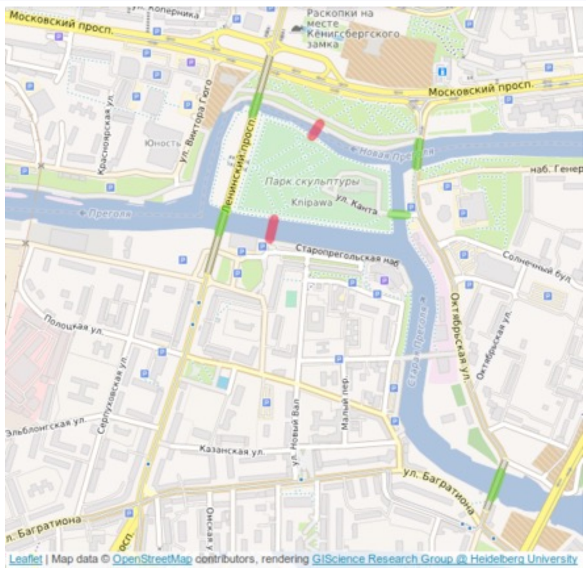


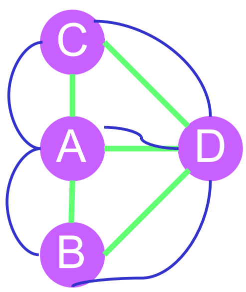
Seven bridges of Königsberg

- Euler showed that the possibility of a walk through a graph, traversing each link exactly once, depends on the degrees of the nodes
  - Node degree: number of links touching it
- Necessary condition: the graph is connected and has exactly 0 or 2 nodes of odd degree
  - Not satisfied by Königsberg graph: 4 nodes with odd number of links (A, B, C, and D)
- Theorem (Euler): A graph is Eulerian if and only if:
  - All nodes have even degree
  - Or only 2 nodes have odd degree

Seven Bridges of Königsberg: today

- 2 bridges were destroyed in World War II, while 2 others were later demolished and replaced, the 3 other bridges remain
- Now 2 nodes (B and C) have degree 2, and the other 2 have degree 3
- An Eulerian path is now possible, but it must begin on one island (A or D) and end on the other (D or A)
  - E.g., A -> B -> D -> C -> A ->D





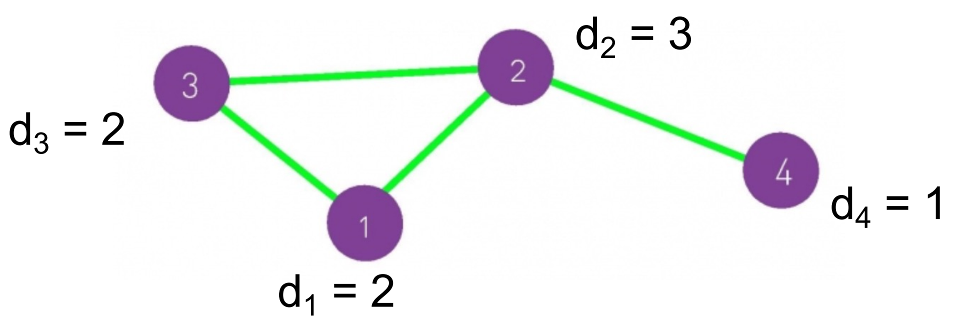
Networks or graphs?

- In scientific literature the terms network and graph are used interchangeably:

Network science	Graph theory
Network	Graph
Node	Vertex
Link	Edge

Graph theory: degree

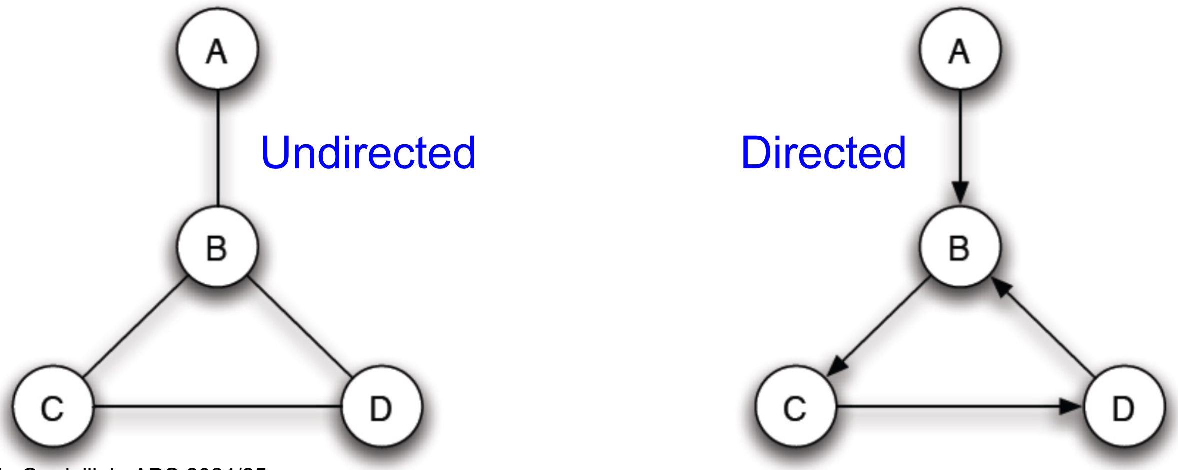
- Node degree: number of links the node has to other nodes



- Average degree: average node degree of all the nodes in the graph
  - Example: (2+3+2+1)/4 = 2

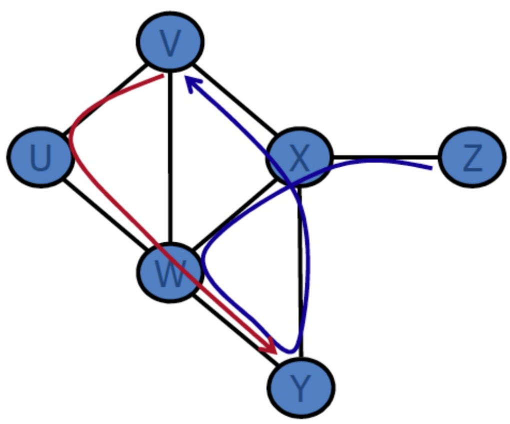
Graph theory: undirected or directed

- Undirected or directed graph
  - Links can be directed or undirected
  - A graph is called directed if all of its links are directed; it is called undirected if all of its links are undirected



Graph theory: path

- Path: a sequence of nodes with each consecutive pair in the sequence connected by a link
  - Red path: V -> U -> W -> Y
- Length of the path: number of links the path contains
  - Red path has length 3
  - Blue path has length 5

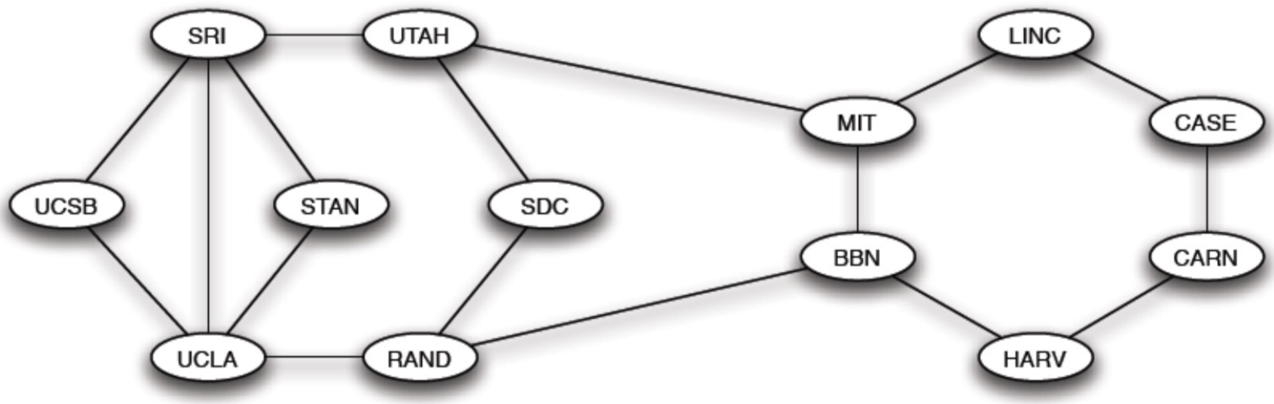


Graph theory: shortest path

- Several algorithms exist for finding shortest paths in a graph (i.e., paths with the fewest number of links)
  - Among which Dijkstra algorithm
- Used by Google Maps to find the shortest route
  - Shortest routes depend upon current traffic conditions

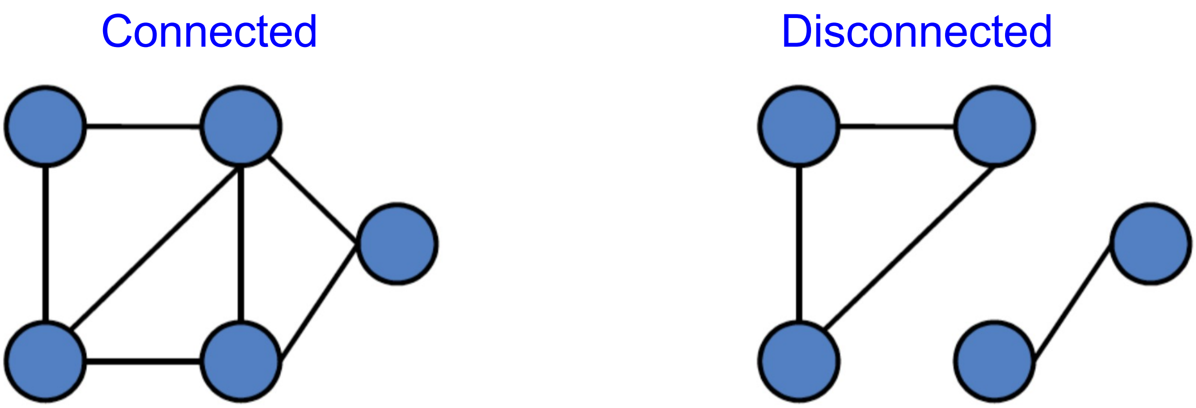
Graph theory: cycle

- Cycle or loop: a path in which the first and last nodes are the same, and all the other nodes are distinct



Graph theory: connectivity

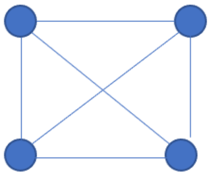
- An undirected graph is *connected* if, *for* every pair of nodes, there is a path between them



Graph theory: connectivity

- In a *fully connected* undirected graph with N nodes, the number of links is equal to

$$\frac{N \times (N-1)}{2}$$



Example: *N*=4

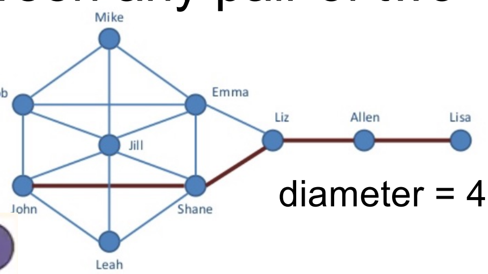
Number of links is $\frac{4 \times 3}{2} = 6$

Social network analysis

- **Social network analysis** (**SNA**) represents how individuals connect to each other and shows the flow of information in a network
- Can help identify influencers, key players, disconnected individuals and communities
- Consists in models that define relationships and interactions between individuals, as well as patterns that emerge from those relationships and interactions
- Importance of SNA in different fields
  – Sociology, business, engineering, economics, politics, computer science, mathematics, physics, biology, security

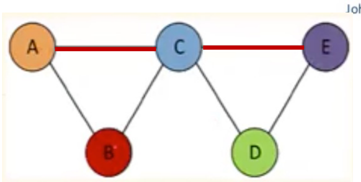
SNA: network measures

- Let's analyze some network measures that allow us to characterize its structure
- Let's first consider network size
  – Diameter
  – Average shortest path
- Diameter: length of the longest shortest path through the network between any pair of two nodes
  – Diameter marked in red



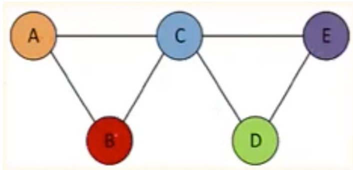
diameter = 4

diameter = 2



SNA: network measures

- Average shortest path length: shortest path length (SPL) between two nodes averaged over all pairs of nodes
  - Not as extreme as diameter
  - Example:
    - SPL from A to B: 1
    - SPL from A to C: 1
    - SPL from A to D: 2
    - SPL from A to E: 2
    - SPL from B to C: 1
    - SPL from B to D: 2
    - SPL from B to E: 2
    - SPL from C to D: 1
    - SPL from C to E: 1
    - SPL from D to E: 1

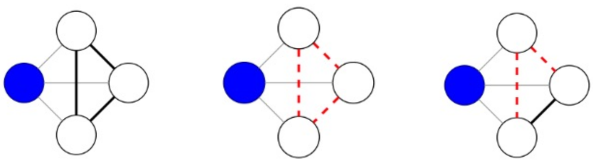


Average SPL is:

$$\frac{1+1+2+2+1+2+2+1+1+1}{10} = 1.4$$

SNA: network measures

- Clustering coefficient: measure of the degree to which nodes in a network tend to cluster together; can be local or global
  - *Clustering coefficient of a node:* number of links between neighbors of that node divided by the maximum number of possible links between them (if node *v* has m_v neighbors, max = $m_v*(m_v-1)/2$)
  - *Average clustering coefficient*: averaged over all nodes of the network
  - E.g., clustering coefficient of blue node



c = 1 c = 0 c = 1/3

3 neighbors

max number of links: 3*2/2 = 3

How to build a social network?

- Network science aims to build models that reproduce properties of real networks
- Goal: how to build a model for network formation
- We consider 3 models
  - Erdos-Renyi model
  - Small-world model (or Watts-Strogatz)
  - Scale-free model (or Barabási-Albert)
- Properties of networks we consider
  - *Average clustering coefficient*
  - *Average shortest path length*

Network models: Erdos-Renyi

- **Erdos-Renyi** is the classical random network: nodes are randomly connected to each other (i.e., choices independent of current network structure)
  - *N*: number of nodes in the network
  - *p*: probability that two nodes are connected by a link
  - Node degree follows binomial distribution

    $P(deg(v) = k)$: probability that node *v* has degree *k*

$$P(deg(v) = k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

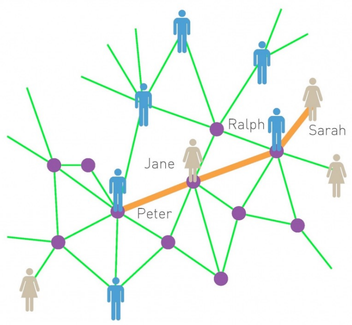
  - Average clustering coefficient: *p*
  - Average shortest path length: $\frac{log(N)}{log((N-1)p)}$

Network models: Erdos-Renyi

- Erdos-Renyi properties
  - Small average shortest path length and low clustering coefficient
  - Homogeneous network, i.e., all nodes are similar
  - But low clustering and homogeneity are not observed in many real-world networks!
- Properties observed in real-world networks:
  - High clustering coefficient
  - Presence of hubs (high-degree nodes), i.e., non-homogeneous network
  - Small-world property: average distance between two nodes depends logarithmically on N

Network models: Watts-Strogatz

- **Watts-Strogatz**: *small-world* random network
- *Small-world phenomenon*, also known as *six degrees of separation*, states that if you choose any two individuals anywhere on Earth, you will find a path of at most six acquaintances between them
  - The distance between any two nodes in a network is unexpectedly small



Small world: experimental confirmation

- The first small-world study comprised several experiments conducted in 1967 by Stanley Milgram to measure the average path length for social networks of people in the US
  - Letters sent from randomly selected residents of Wichita (Kansas) and Omaha (Nebraska) to two target persons in Boston
  - Senders were asked to forward the letter to a friend, relative or acquaintance who is most likely to know the target person
- The research was groundbreaking in that it suggested that human society is a small-world network characterized by short path lengths
- The experiments are often associated with the phrase “six degrees of separation”, although Milgram did not use this term himself

Small world: experimental confirmation

One possible path of a message in the “Small World” experiment by Stanley Milgram

https://en.wikipedia.org/wiki/Small-world_experiment

Small world: experiment repeated

- Global, Internet-based social search experiment
  - 18 target persons in 13 different countries
  - 60,000+ participants: by email pass the message to a social acquaintance whom they considered “closer” than themselves to the target
  - 24,163 message chains
  - 384 reached their targets
  - average path length 4.0
  - Actual success depends sensitively on individual incentives

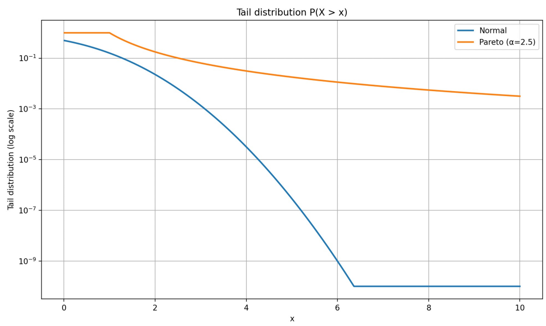
Dodds, Muhamad, Watts, An Experimental Study of Search in Global Social Networks, *Science,* 2003
https://www.science.org/doi/pdf/10.1126/science.1081058

Network models: Watts-Strogatz

- **Watts-Strogatz**: *small-world* random network
- Watts-Strogatz properties:
  - Small average shortest path length
  - High clustering coefficient, independent of the number of vertices
- Most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops
  - Example: social networks
- Limitation: does not account for the formation of hubs

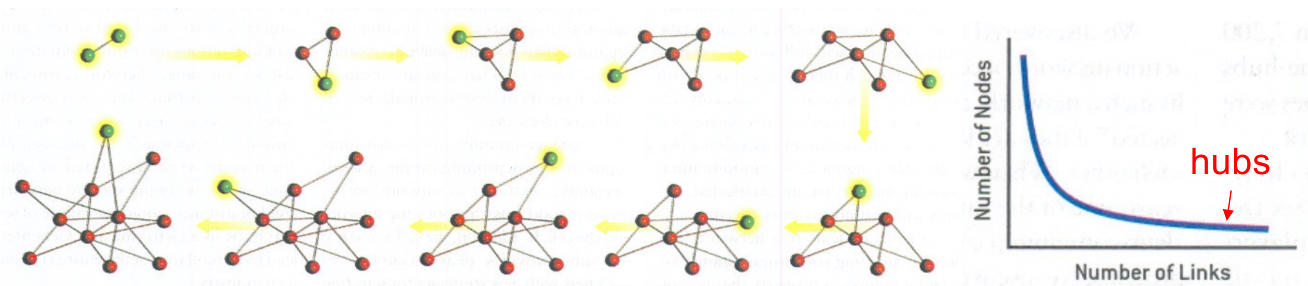
Network models: Barabási-Albert

- **Barabási-Albert**: a *scale-free* network
  - Node degree follows a power law distribution
    $$P(deg(v) = k) \sim k^{-\alpha} \qquad \text{where} \quad 2 < \alpha < 3$$
- Power law: event frequency varies as a power of some attribute of that event
  - Examples: distribution of wealth among individuals, number of cities with a certain population size
  - A property of power law is scale invariance https://en.wikipedia.org/wiki/Scale_invariance
  - Power law is also *heavy-tailed*
    - Tail distribution contains a great deal of probability (i.e., heavier tail than that of normal distribution)



Network models: Barabási-Albert

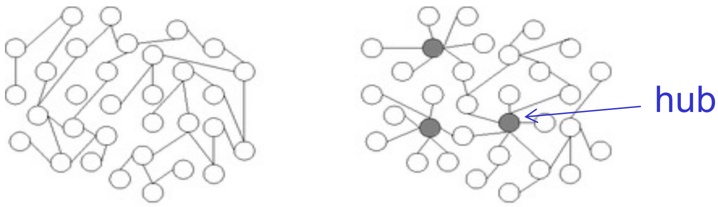
- How do new nodes connect to existing nodes? On the basis of preferential attachment
  - The more connected a node is, the more likely it is to receive new links (i.e., rich-gets-richer process): thanks about connections in a social network!
  - As a result, there are a few hubs (nodes with a very high number of links), while most nodes have just a few links (power law behavior)



Source: https://www.jstor.org/stable/26060284

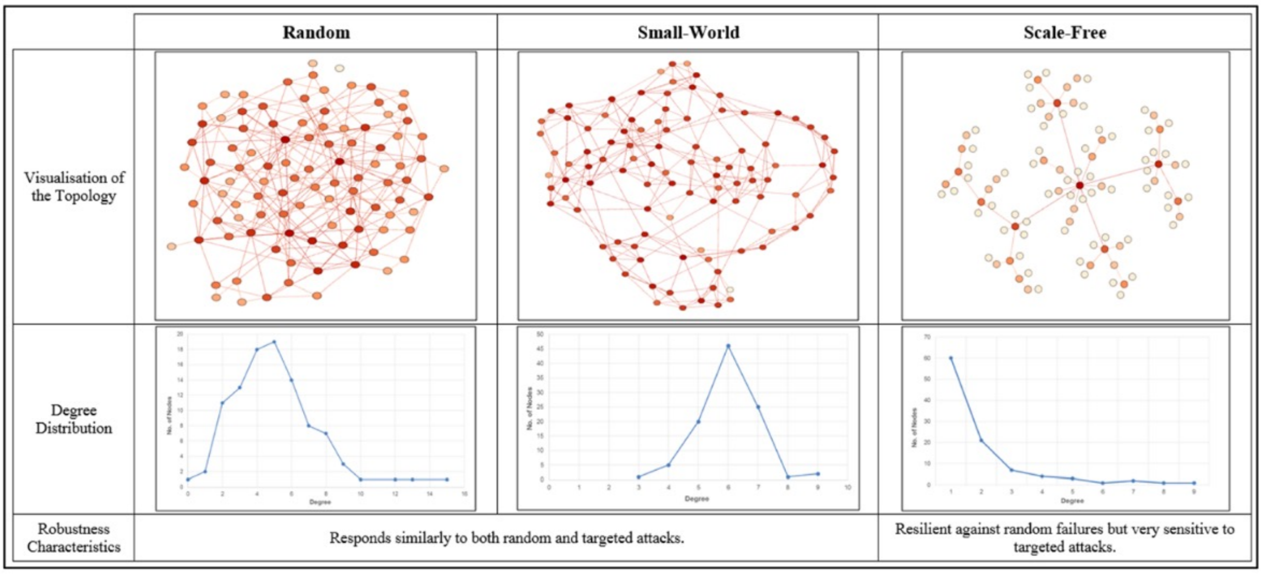
Network models: Barabási-Albert

- Scale-free property: network diameter ~ ln (ln N)
- Scale-free networks are robust against accidental node failures, but vulnerable to coordinated attacks against their hubs
- Many networks are conjectured to be scale-free: Internet, Web, social networks, power grids



(a) Random network (b) Scale-free network

Network models: comparison



	Random	Small-World	Scale-Free
Visualisation of the Topology			
Degree Distribution			
Robustness Characteristics	Responds similarly to both random and targeted attacks.		Resilient against random failures but very sensitive to targeted attacks.

Source: https://appliednetsci.springeropen.com/articles/10.1007/s41109-017-0053-0

SNA measures: centrality

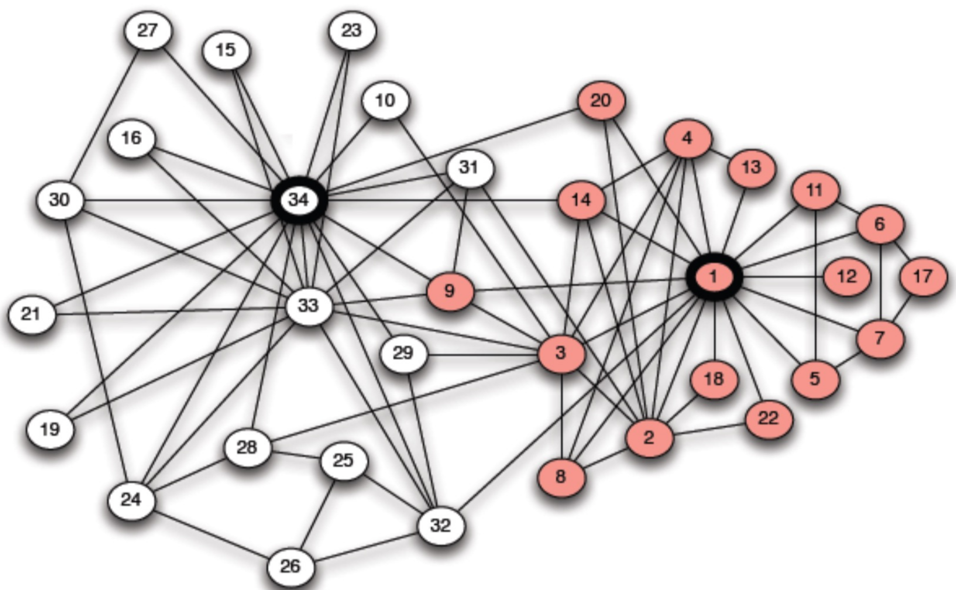
- Let's analyze **centrality** measures

  Which are the most important nodes within the network?

- Useful to identify:
  - Most influential persons in a social network
  - Key infrastructure nodes in Internet or urban networks
  - Super-spreaders of disease

SNA measures: centrality

- The social network of friendships within a 34-person karate club provides clues to the fault lines that eventually split the club apart

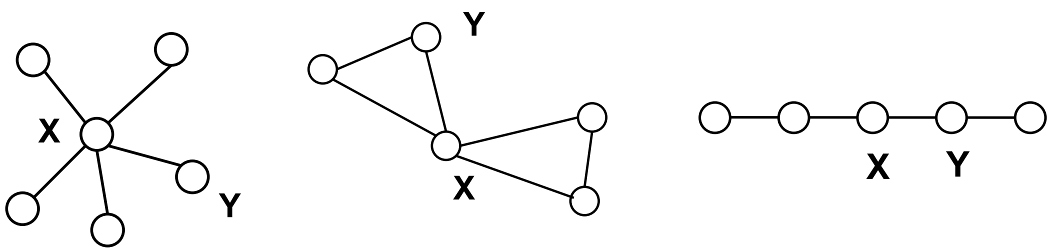


SNA measures: centrality

- Which nodes are most "central"?
- Definition of central varies by context/purpose
- Local measure:
  - Degree centrality
- Relative to rest of network:
  - Betweenness centrality
  - Closeness centrality
  - PageRank centrality
- For each measure: the higher the score, the more central the node

Centrality: who's important based on its network position

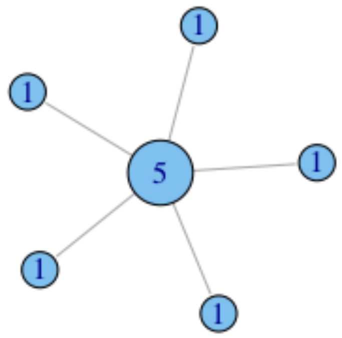
- In each of the following networks, X has higher centrality than Y according to a particular measure



degree betweenness closeness

Degree centrality

- Idea: a node is important if it has many neighbors
- Count how many neighbors a node has



Degree centrality of a node: the degree of that node

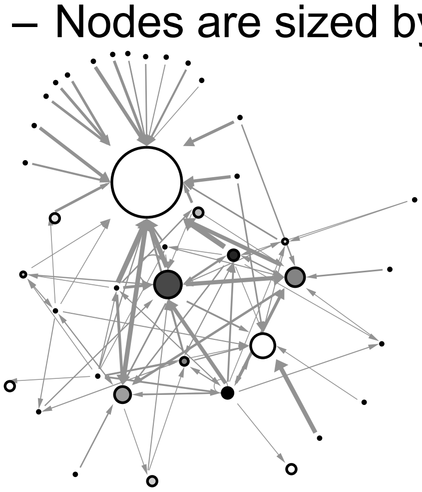
- Node degree centrality $C_D(i)$ can be extended to whole graph: C_D

$$C_D = \frac{\sum_{i=1}^{N}(C_D(n^*) - C_D(i))}{(N-1)(N-2)}$$

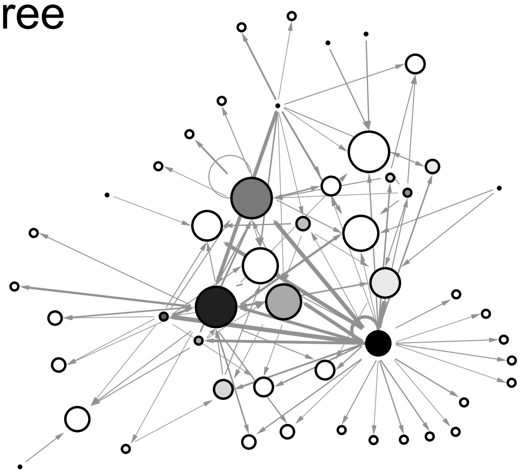
$C_D(n^*)$ is the maximum node degree

Degree centrality: examples

- Examples of financial trading networks
  - Nodes are sized by degree



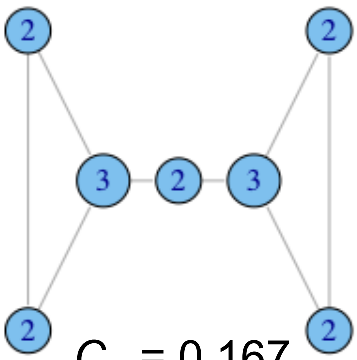
High centralization: one node trading with many others



Low centralization: trades are more evenly distributed

When degree is not everything

- In what ways does degree fail to capture centrality in the networks below?



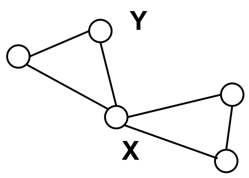
$C_D = 0.167$

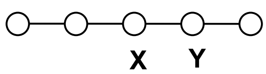
$C_D = 0.167$

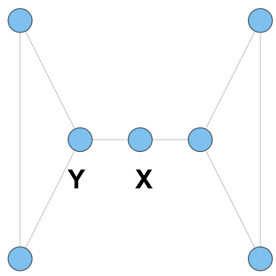
- Ability to broker between groups
- Likelihood that information originating anywhere in the network reaches you

Betweenness: another centrality measure

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between each pair of nodes
- Who has higher betweenness, X or Y?





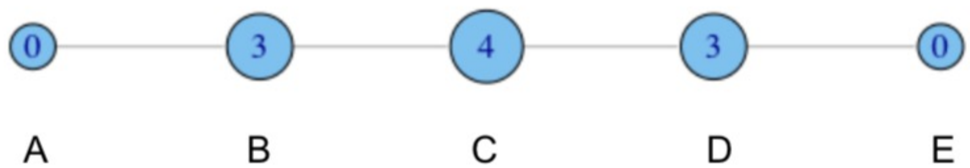


Betweenness: definition

- $C_B(i)$, betweenness of node i:
  - V: set of nodes
  - $\sigma_{jk}(i)$: number of shortest paths from j to k that pass through i
  - σ_{jk} : total number of shortest paths from j to k

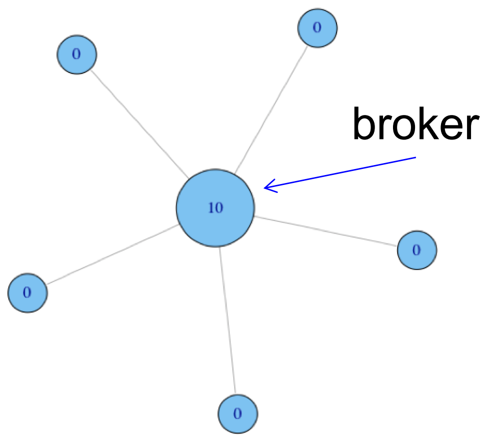
$$C_B(i) = \sum_{i \neq j \neq k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$$

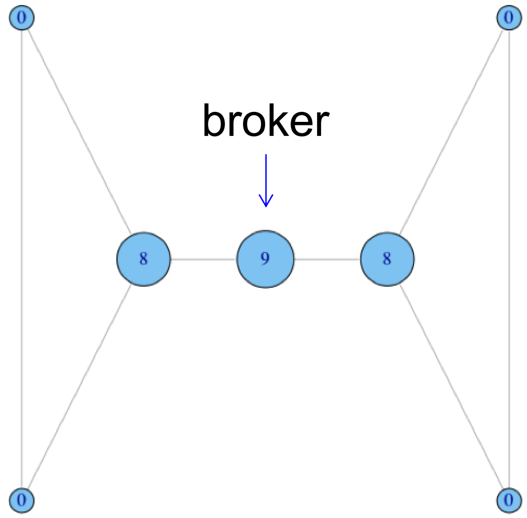
Betweenness: example



- $C_B(A) = C_B(E) = 0$: A (or E) never acts as a bridge
- $C_B(B) = 3$: B acts as a bridge 3 times along the shortest paths between A and C, A and D, A and E
- $C_B(C)= 4$: C acts as a bridge 4 times along the shortest paths between A and D, A and E, B and D, B and E
- There is only one shortest path between each pair of nodes, so denominator is equal to 1

Betweenness: example

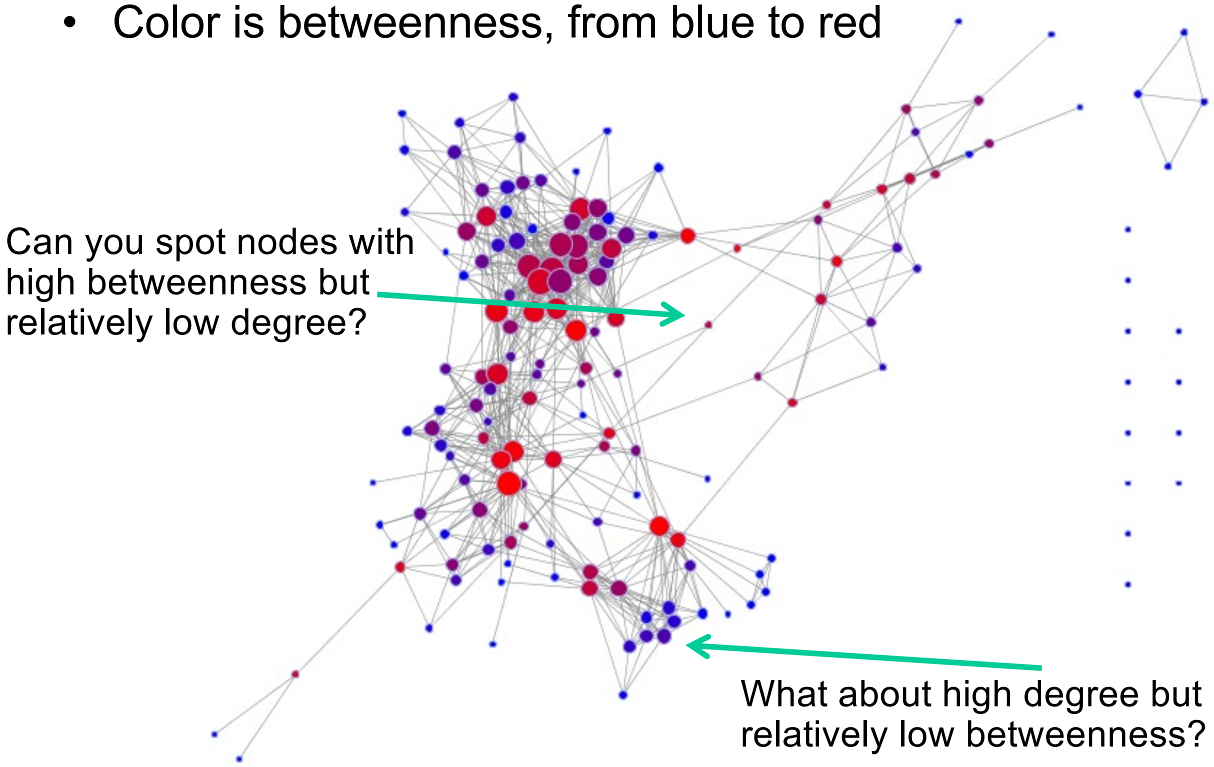




- If broker is removed, the entire connection would be completely collapsed with the rest of the community, and so you will notice separated subgroups

Betweenness: example

- Size is degree
- Color is betweenness, from blue to red



Closeness: another centrality measure

- What if it is not so important to have many direct friends (i.e., degree)?
- Or be "between" others (i.e., betweenness)?
- But one still wants to be in the "middle" of things, not too far from the center

Closeness: definition

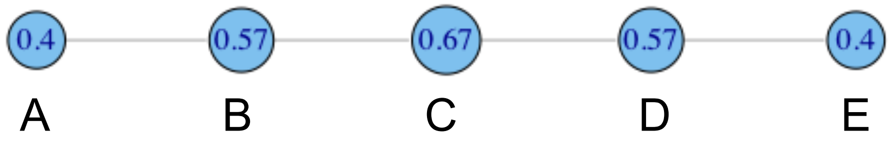
- Closeness of a node is based on the average length of the shortest paths between that node and all other nodes in the network
  - The more central a node is, the closer it is to all other nodes
- $C_C(i)$, closeness for node i:
  - $d(i,j)$: length of shortest path between i and j

$$C_C(i) = \Big(\sum_{j=1}^{N} d(i,j) \Big)^{-1}$$

- $C'_C(i)$, normalized closeness for node i

$$C'_C(i) = (N-1)C_C(i) = \frac{N-1}{\sum_{j=1}^{N} d(i,j)}$$

Closeness: example

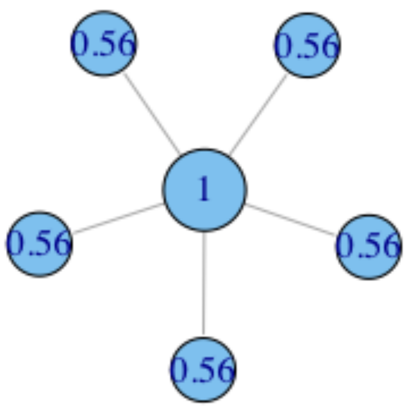


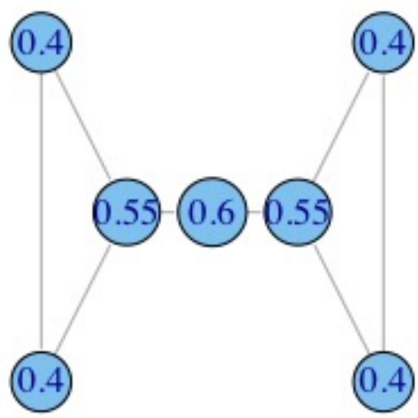
$$C'_C(A) = \frac{N-1}{\sum_{j=1}^{N} d(i,j)} = \frac{5-1}{0+1+2+3+4} = \frac{4}{10} = 0.4$$

$$C'_C(B) = \frac{N-1}{\sum_{j=1}^{N} d(i,j)} = \frac{5-1}{1+0+1+2+3} = \frac{4}{7} = 0.57$$

$$C'_C(C) = \frac{N-1}{\sum_{j=1}^{N} d(i,j)} = \frac{5-1}{2+1+0+1+2} = \frac{4}{6} = 0.67$$

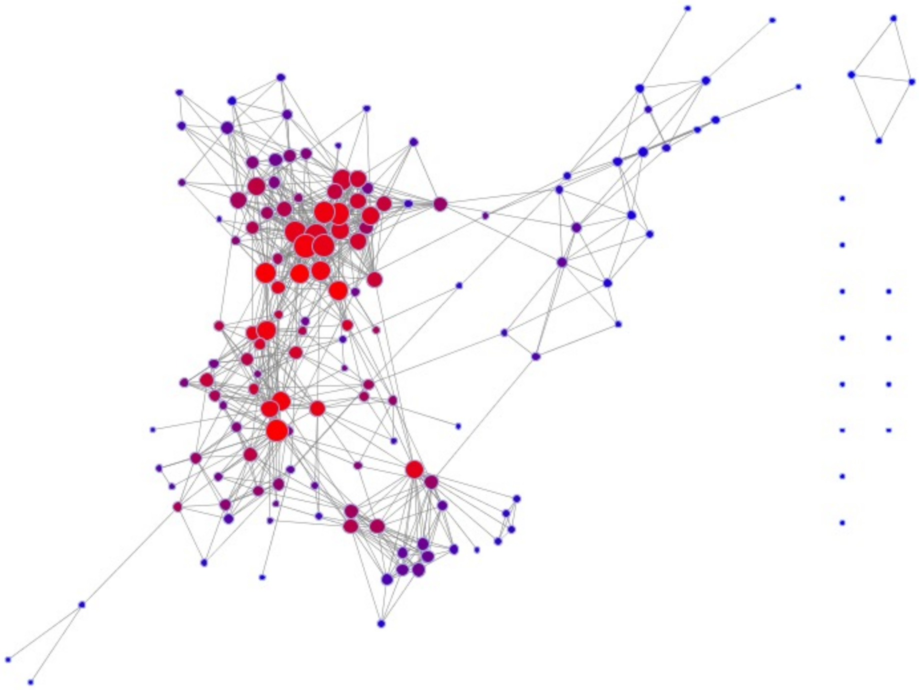
Closeness: examples





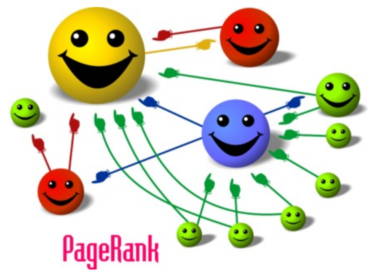
How closely does degree correspond to closeness?

- Size is degree
- Color is closeness, from blue to red



PageRank as centrality measure

- PageRank idea: *a node is important if it is linked from other important or if it is highly linked*

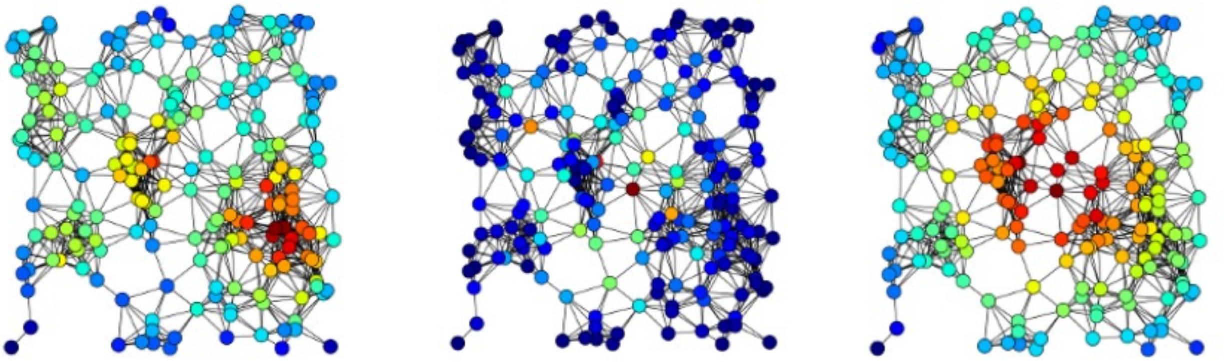


- PageRank can be used to measure centrality
  – Differently from the other centrality measures, it accounts for link direction
    - Each node in a network is assigned a score based on its number of incoming links; these links are also weighted depending on the relative score of its originating node
    - The result is that nodes with many incoming links are influential, and nodes to which they are connected share some of that influence

Wrap-up on centrality

- We have looked at some popular centrality measures

"There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement.” (Freeman, 1979)

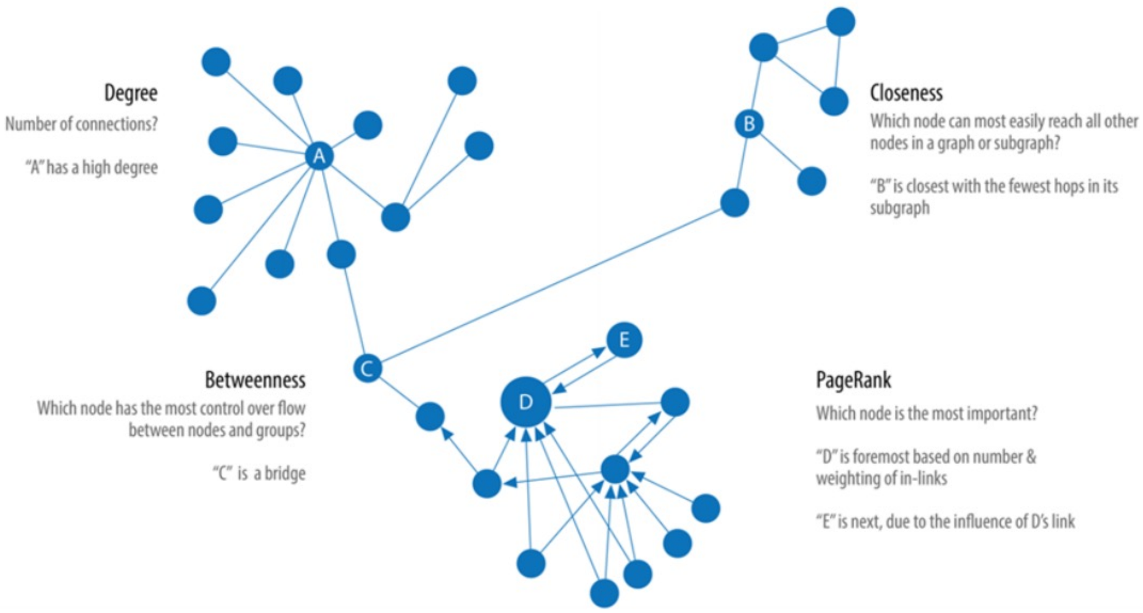


Degree Betweenness Closeness

Color from minimum (blue) to maximum (red)

Wrap-up on centrality

- Different centrality measures can produce significantly different results based on what they were created to measure



Wrap-up on centrality

- Degree centrality
  - *What it tells us*: number of links each node has to other nodes in the network
  - *When to use it*: to find very connected individuals, popular individuals, individuals who are likely to hold most information or individuals who can quickly connect with the wider network

- Betweenness centrality
  - *What it tells us*: which nodes are “bridges” between nodes in a network; measures the number of times a node lies on the shortest path between other nodes
  - *When to use it*: to find individuals who influence the flow around a network

Wrap-up on centrality

- Closeness centrality
  - *What it tells us*: scores each node based on its “closeness” to all other nodes in the network; calculates the shortest paths between all nodes, then assigns each node a score based on its sum of shortest path lengths
  - *When to use it*: to find individuals who are best placed to influence the entire network most quickly

- PageRank
  - *What it tells us*: node’s importance from its linked neighbors and their neighbors; uncovers nodes whose influence extends beyond their direct links into the wider network
  - *When to use it*: to understand authority and citations

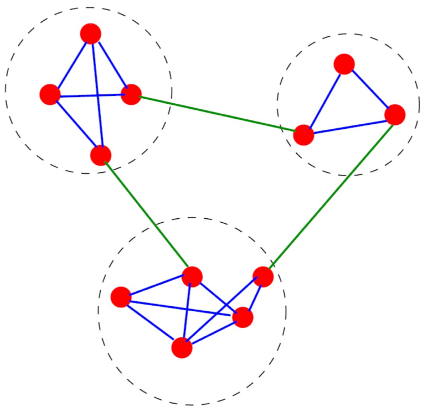
What is a community?

- Set of nodes that share something:
  - Affiliation (friends, colleagues, club, …)
  - Similar interests (tagging systems, …)
  - Similar contents (movies, books, products, …)

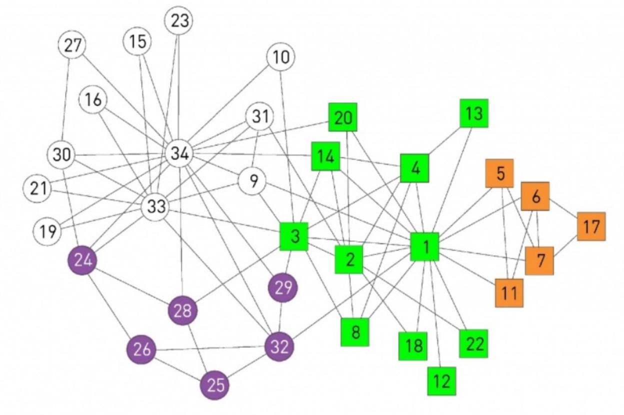
- What is the connection with the network structure?
- Intuition: more densely connected inside than outside

SNA: Community detection

- Goal: identify automatically communities with statistically significantly more links between nodes in the same group than nodes in different groups



Community detection: example



- Zachary observed 34 members of a karate club over 2 years. Links connect individuals who were observed to interact outside the club activities

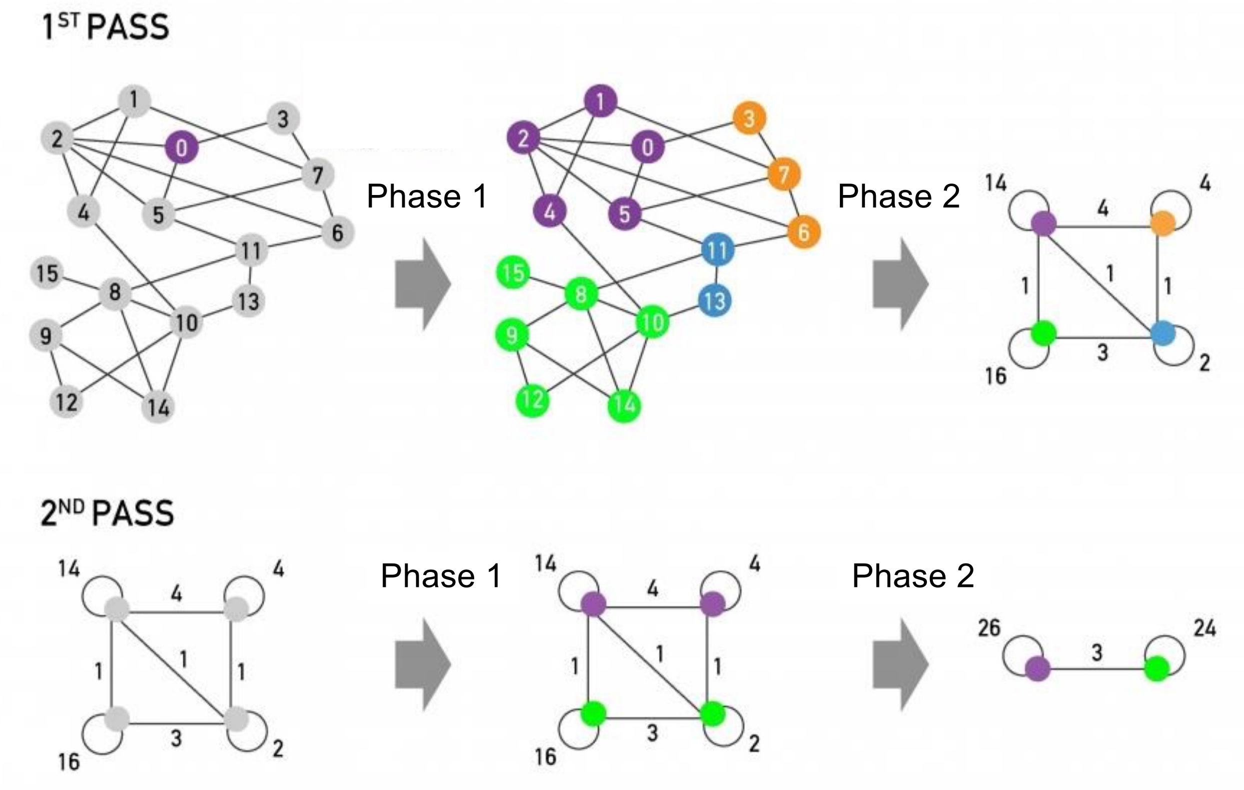
  During the course of observation, the club members split into 2 groups because of the disagreement between the club administrator and the club instructor (nodes 1 and 34), and the members of one group left to start their own club

https://en.wikipedia.org/wiki/Zachary%27s_karate_club

Community detection: Louvain algorithm

- Louvain algorithm: greedy optimization method that aims to maximize modularity
  - Modularity: value in [-1, 1] that measures the density of links inside communities compared to links between communities
  - Idea: 2 phases that are repeated iteratively
    - In phase 1, each node is first assigned to its own community. Then, for each node *i* the change in modularity is calculated by removing *i* from its own community and placing it into the community of each neighbor *j*. *i* is placed into the community that resulted in the greatest modularity increase
    - In phase 2, the algorithm groups all of the nodes in the same community and builds a new network whose nodes are the communities from previous phase. Then, phase 1 is re-applied to the new network until a maximum of modularity is attained
  - Running time: O(*N log N*), being *N* the number of nodes

Community detection: Louvain algorithm



References

- D. Easley and J. Kleinberg, Networks, Crowds, and Markets: Reasoning About a Highly Connected World, chapters 1 and 2, Cambridge University Press, 2010. http://www.cs.cornell.edu/home/kleinber/networks-book
- A.-L. Barabási, Network Science, chapters 1, 2, 3, and 9, Cambridge University Press, 2016. https://networksciencebook.com