Calculus - Course Diary

Global Governance Program
University of Rome Tor Vergata
Academic Year 2025/26 (Fall Semester)

Lesson diary

- **Lesson 1 [15/09/2025]** Functions: definition, domain, range. Examples. Injectivity and Surjectivity. Graph of a function. Cartesian plane. Finding points of intersection. Linear functions and straight lines. Quadratic functions and parabola.
  Reference: Sections 1.1-2-3 in [1].

- **Lesson 2 [16/09/2025]** Exponential function: definition and properties. Graph of the exponential function. Property of the exponential function and exponential rules. Compound interest and continuous compound interest. Napier's constant: e. Logarithmic functions: definition and main properties.
  Reference: Sections 1.2-3 & 4.1-2 in [1].

- **Lesson 3 [17/09/2025]** Logarithmic functions: definition and main properties. Logarithmic rules. Graphs of logarithmic functions. Natural logarithm. Conversion formula for logarithms. Exercises. Trigonometric functions (sine, cosine and tangent): definitions and main properties
  Reference: Sections 4.2 & 11.1 in [1].

- **Lesson 4 [18/09/2025]** Periodic functions: period of a periodic functions. Examples of periodic functions of a given period. Exercises. Domain of a function and how to determine it (several examples).
  Reference: Sections 1.5 & 11.1 in [1].

- **Lesson 5 [19/09/2024]** Domain of a function and how to determine it (several examples).
  Reference: Section 1.1 in [1].

- **Lesson 6 [22/09/2024]** Concept of limit (both informal and more formal definition). Algebraic properties of limits. Limits of polynomials and quotient of polynomials (rational functions). Undetermined form $\frac{0}{0}$ and how to handle it.
  Reference: Sections 1.5 in [1].

- **Lesson 7 [23/09/2025]** Meaning of $\pm\infty$ and limits of functions that tend to $\pm\infty$. One-sided limits. Examples.
  Reference: Sections 1.5-6 in [1].

- **Lesson 8 [24/09/2025]** Definition of continuity of a function at one point and on an interval. Polynomials and Rational functions are continuous in their domain of definition. Square root, Exponentials and Logarithms, Sine, Cosine and Tangent are continuous at their domains of definition. Orders of infinity.
  Reference: Sections 1.5-6, 4.1-2 and 11.1 in [1].

- **Lesson 9 [25/09/2025]** Review exercises on limits.
  Reference: Sections 1.5-6 and 4.1-2 in [1].

- **Lesson 10 [29/09/2025]** Basic concepts of differentiation. Definition of derivative and interpretation as rate of change and the slope of the tangent line to a point of the graph. Computation of derivatives of: constant function, linear and quadratic function. General rule for the derivative of the power of a function. Derivative of product of functions. Derivative of quotient of functions. Relation between the sign of the derivative and the function being increasing/decreasing.
  Reference: Sections 2.1-2-3 in [1].

- **Lesson 11 [01/10/2025]** Rule of differentiation. Derivative of linear combinations of functions. Derivative of product of functions. Derivative of quotient of functions. Relation between the sign of the derivative and the function being increasing/decreasing. Relation between existence of the derivative and continuity. Examples in which the derivative does not exist ($\sqrt{|x|}$ and $|x|$ at

$x = 0$) and geometric interpretation of these points. Rule for the derivative of the exponential, the logarithm, sine and cosine. Exercises.
Reference: Sections 2.1-2-3, 4.1-2 and 11.2 in [1].

- **Lesson 12 [06/10/2025]** Chain rule. Critical points of a function and classification (relative maxima or relative minima). Exercises and study of functions (curve sketching). Definition of horizontal, vertical and oblique asymptote.
Reference: Sections 2.4, 3.1-3 in [1].

- **Lesson 13 [08/10/2025]** Higher order derivatives and Second derivative (geometric interpretation: concavity of the function and points of inflection). Marginal analysis in economics.
Reference: Sections 2.5, 3.2-3 in [1].

- **Lesson 14 [09/10/2025]** Exercise on the study of a function (curve sketching). Evaluating Limits with L'Hôpital's rule.
Reference: Sections 3.3-4, A.3 in [1].

- **Lesson 15 [14/10/2024]** Exercise on the study of a function. Second derivative test for classifying critical points. Find global maxima and minima: case of a function with a unique critical point; case of a continuous function on a closed interval (statement of Weierstrass theorem about their existence).
Reference: Sections 3.3-4 in [1].

- **Lesson 16 [16/10/2025]** Optimization problems: various examples and exercises.
Reference: Sections 3.4-5 [1].

- **Lesson 17 [20/10/2025]** Integration: antiderivative and indefinite integral. Properties of antiderivatives and rules of integration for some elementary functions. Algebraic rules for indefinite integration.
Reference: Sections 5.1-2 in [1].

- **Lesson 18 [22/10/2025]** Exercises on integration. Integration by substitution and exercises. Integration by parts.

  Reference: Sections 5.2-3-4 in [1].

- **Lesson 19 [23/10/2025]** Revision for the first midterm.

- **Lesson 20 [27/10/2025]** Revision for the first midterm.

- **Lesson 21 [17/11/2025]** Definite integrals and the fundamental theorem of calculus. Area under the graph of a function.
Reference: Sections 5.2-3 in [1].

- **Lesson 22 [18/11/2025]**Rules for definite integrals. Evaluating definite integrals: various examples. Substitutions for definite integrals. Applications: finding net change; area between two curves.
Reference: Section3 5.3-4 in [1].

- **Lesson 23 [20/11/2025]** Exercises on Integrals
Reference: Sections 5. 4-5 and 6.1 in [1].

- **Lesson 24 [21/11/2025]** Applications of integrals: average value of a function, distribution of wealth (Lorenz curves and Gini index).
Reference: Sections 5.4-5 in [1].

- **Lesson 25 [24/11/2025]** Introduction to differential equations. Main definitions and simple examples. General solution and Initial value problem/Cauchy problem. Antiderivative as a differential equation. Separable differential equations. Examples.
Reference: Sections 5.1 & 9.1 in [1] + Sections 1.1-2-3 in [2].

- **Lesson 26 [25/11/2025]** Separable differential equations. Examples. Logistic equation (Population growth) and its analysis.
Reference: Section 9.1 in [1] + Sections 1.2-3 in [2].

- **Lesson 27 [26/11/2025]** Various exercises. First order linear differential equations: formula for its solution. Examples.
  Reference: Sections 9.1-2 in [1] + Sections 1.2-3 in [2].
- **Lesson 28 [27/11/2025]** Various exercises on differential equations.
  Reference: Sections 9.1-2 in [1] + Sections 1.2-3 in [2].
- **Lesson 29 [04/12/2025]** Various exercises on domain of functions in two variables, level sets and partial derivatives.
  Reference: Sections 7.1-2 in [1].
- **Lesson 30 [08/01/2026]** Revision for the second Midterm and the final exam.

References

[1] Hoffmann, Laurence D., Bradley, Gerald L., Sobecki, David, & Price, Michael. *Applied Calculus: For Business, Economics, and the Social and Life Sciences, 11th Expanded Edition.* McGraw-Hill Education, 2011.

[2] Porretta, Alessio. *Applied Quantitative Analysis.* Lecture notes, 2014-15. See *Materiale didattico* in DIDA, Corso #2714.