# CALCULUS

GLOBAL GOVERNANCE

A.Y. 2024/25

## Tutorial nr. 2

### Exercise 1 (Domain of a function)

Determine the domain of the following functions:

**(a)** $f(x) = 2^{\frac{x^2-1}{\sqrt{x^2-5x+4}}}$ **(b)** $g(x) = \dfrac{\sqrt{x^3 - 5x^2 + 6x}}{\ln(x^2 - 4)}$ **(c)** $h(x) = \sin(x^2) + \log_2\left(\dfrac{x+1}{x-5}\right)$.

### Exercise 2 (Limits)

**I.** Compute, if they exist, the following limits:

**(a)** $\displaystyle\lim_{x\to 1} \frac{x^3 - 5x + 4}{x^3 - 1}$ **(b)** $\displaystyle\lim_{x\to 0} \frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x}$ **(c)** $\displaystyle\lim_{x\to -2} \frac{3x + 6}{(x+1)(x^2 + 4x + 4)}$.

In the cases in which the limit does not exist, discuss the existence of the one-sided limit (from the left and from the right).

**II.** Compute, if they exist, the following limits:

**(a)** $\displaystyle\lim_{x\to +\infty} \frac{x^3 - 5x + 4}{x^3 - 1}$ **(b)** $\displaystyle\lim_{x\to -\infty} \frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x}$ **(c)** $\displaystyle\lim_{x\to +\infty} \frac{3x + 6}{(x+1)(x^2 + 4x + 4)}$.

**III.** Compute, if they exist, the following limits:

**(a)** $\displaystyle\lim_{x\to +\infty} \sqrt{\frac{x^3 - 5x + 4}{x^3 - 1}}$ **(b)** $\displaystyle\lim_{x\to -\infty} e^{\frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x}}$ **(c)** $\displaystyle\lim_{x\to +\infty} \log\left(\frac{3x + 6}{(x+1)(x^2 + 4x + 4)}\right)$.

### Exercise 3 (Exponential and Logarithm)[1]

In 1389, Pierre d'Arcis, the Bishop of Troyes, wrote a memo to the Pope, accusing a colleague of passing off *"a certain cloth, cunningly painted "*, as the burial shroud of Jesus Christ. Despite this early testimony of forgery, the image on the cloth is so compelling that many people regard it as a sacred relic. Known as the *Shroud of Turin*, the cloth was subject to carbon dating in 1988: this test consists in measuring the amount of the radioative isotope $^{14}$C (carbon-14) contained on the Shroud.

**(a)** Denote by $R(t)$ the amount of $^{14}$C contained on the Shroud after $t$ years and assume that it decays as

$$R(t) = R_0 e^{-kt}$$

where $R_0 > 0$ is the initial amount of $^{14}$C (at time $t = 0$) and $k > 0$.
It is known that after $5,730$ years the amount of $^{14}$C becomes $\frac{R_0}{2}$ (i.e., it halves). Determine the value of $k$.

**(b)** If the Shroud were actually 1,960 years old in 1988, what percentage of $^{14}$C would have remained?

---

[1]See: Exercise 64 page 327 in the textbook.

**(c)** Scientists determined that 92.3% of the Shroud's original $^{14}$C remained in 1988. Based on this information alone, what was the likely age of the Shroud in 1988?

**Exercise 4 (Derivatives)**[2]

It is estimated that $t$ years from now, the circulation of a local newspaper will be

$$C(t) = 100t^2 + 400t + 5,000.$$

**(a)** Derive an expression for the rate at which the circulation will be changing with respect to time $t$ years from now.

**(b)** At what rate will the circulation be changing 5 years from now? How much will the circulation actually change during the sixth year? Compare it with the rate of change found for the fifth year.

**(c)** Determine when the circulation will be increasing or decreasing with respect to time.

---

[2]See: Exercise 56 page 130 in the textbook.