

**CALCULUS**  
GLOBAL GOVERNANCE  
A.Y. 2024/25

**Tutorial nr. 3**

**Exercise 1 (Study of functions)**

Study the following functions and sketch their graphs. In particular determine: domain, intersection with axes, continuity, sign, asymptotes (vertical, horizontal or oblique), intervals of increase and decrease and concavity. For each function indicate their critical points (specifying whether they are relative/global maxima or minima) and their points of inflections.

(a)  $f(x) = x^4 - 9x^2$       (b)  $g(x) = \frac{x^4}{x^3 - 2}$       (c)  $h(x) = \ln\left(\frac{x^2}{1+x}\right)$       (d)  $\ell(x) = x^3 e^{-x}$ .

**Exercise 2 (GG1 Student population growth)**

It is estimated that  $t$  years from now, the number of students enrolled in the first year program of Global Governance (GG1 students) will be

$$S(t) = 200 - \frac{50}{\sqrt{2t^2 + 1}}.$$

- a) Derive a formula for the rate at which the GG1 population will be changing with respect to time  $t$  years from now.
- b) At what rate will the GG1 student population be growing 1 year from now? Use this information to get an approximation of how much the number of GG1 students will actually increase during the second year. By how much will the number of GG1 students actually increase during the second year?
- c) At what rate will the GG1 population be growing 9 years from now?
- d) What will happen to the rate of GG1 students' growth in the long run?

**Exercise 3 (Depreciation)**

The value  $V$  (in thousands of euros) of an industrial machine is modeled by

$$V(N) = \left(\frac{3N + 430}{N + 1}\right)^{2/3}$$

where  $N$  is the number of hours the machine is used each day. Suppose further that usage varies with time in such a way that

$$N(t) = \sqrt{t^2 - 10t + 45}$$

where  $t$  is the number of months the machine has been in operation.

- (a) How many hours per day will the machine be used 9 months from now? What will be the value of the machine at this time?
- (b) At what rate is the value of the machine changing with respect to time 9 months from now? Will the value be increasing or decreasing at this time?

- (c) Over what time interval is the value of the machine increasing? When is it decreasing?
- (d) At what time  $t$  is the value of the machine the largest? What is this maximum value?

**Exercise 4 (Political election)**

A poll indicates that  $x$  months after a particular candidate for the US presidency declares their candidacy, they will have the support of  $S(x)$  percent of the voters, where

$$S(x) = \frac{1}{29} (-x^3 + 6x^2 + 63x + 1080) \quad \text{for } 0 \leq x \leq 12.$$

Since the election is held on November 5th, when should have Kamala Harris announced her candidacy to maximize her support? If this was the case, should she expect to win (assuming that she wins if she gets at least 50% of the vote)?