

CALCULUS
GLOBAL GOVERNANCE
A.Y. 2024/25

Tutorial nr. 4

Exercise 1 (de l'Hôpital's rule)

Compute these limits using de l'Hôpital's rule:

$$(a) \lim_{x \rightarrow \frac{1}{2}} \frac{8x^3 - 12x^2 + 6x - 1}{4x^4 - 4x^3 + 5x^2 - 4x + 1} \quad (b) \lim_{x \rightarrow -\infty} \frac{e^{-x} + 1}{e^{-2x} + 1} \quad (c) \lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{x}}.$$

Exercise 2 (Study productivity)

A GG1 student is preparing for the calculus exam. An efficiency study of students' average performance shows that, if they start studying at 8:00 a.m., their study productivity (measured as the percentage of the material that they manage to assimilate) after x hours is modeled by the function:

$$f(x) = -x^3 + 6x^2 + 15x.$$

However, the study also shows that after a 15-minute break, the student's productivity is described by a different function:

$$g(x) = -\frac{1}{3}x^3 + x^2 + 23x$$

where x denotes the number of hours passed that have passed since the break. Determine the optimal time between 8:00 a.m. and noon for the student to take the 15-minute coffee break in order to maximize the percentage of assimilated material by 12:15 p.m.

Exercise 3 (Hateful agenda versus protests)

Matthew Salwyn, a controversial politician known for his racist and xenophobic views, is planning a series of public speeches across the country. The longer he speaks, the more people become aware of his harmful rhetoric and actively protest against him. However, the opposition response is not linear: while initially the number of protesters increases as Mr. Salwyn speaks, eventually, diminishing returns set in as people grow fatigued by his repetitive speeches.

The percentage of active protesters among the population, $P(t)$, as a function of the length of Mr. Salwyn speech t (in hours), is modeled by the following function:

$$P(t) := \frac{450 + 90t}{\sqrt{100 + 9t^2}}.$$

- (a) Determine the value of t that maximizes the protest against his hateful agenda and calculate the percentage of active protesters at this time.
- (b) What should Mr. Salwyn's strategy be in order to minimize the size of the protest?

Exercise 4 (Indefinite integrals)

(i) Compute the following indefinite integrals:

$$\text{(a)} \int \left(\frac{1}{3x} - 2x\sqrt[3]{x} + 1 \right) dx \quad \text{(b)} \int \left(e^{\frac{t}{2}} + 2 \right)^2 dt \quad \text{(c)} \int \left(3 \sin y + \frac{y^2 + 1}{2\sqrt{y}} \right) dy.$$

(ii) Compute the following indefinite integrals by substitution:

$$\text{(a)} \int 3y \sqrt{y^2 + 9} dy \quad \text{(b)} \int (3x^2 - 1)e^{(x^3 - x)} dx \quad \text{(c)} \int \frac{z \ln(1 + z^2)}{z^2 + 1} dz.$$