

Last Name:	First Name:	Student's ID:
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Instructions:

- You have 1 hour and 30 minutes to answer the following questions. Report your answers in the table provided below.
- No notes, books, or any other reference materials is allowed during the test.
- Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Answers

1	2	3	4	5	6	7	8

Questions

1. Where is the function $\ell(x) = x^3 - 6x^2 + 9x$ decreasing at the fastest rate when $-5 \leq x \leq 5$?

- (a) $x = 5$
- (b) $x = 2$
- (c) $x = 0$
- (d) $x = -5$
- (e) $x = \sqrt{2}$

2. In a certain region, the population of a species of birds is decreasing exponentially. The population $P(t)$ (in thousands) at time t (in years) is given by the equation:

$$P(t) = P_0 e^{-kt}$$

where P_0 is the initial population, and k is a positive constant that represents the rate of decrease. If the initial population of the birds is $P_0 = 150$ (thousands) and the rate of decrease $k = 0.03$ per year, determine how long it will take for the population to decrease to 50 thousand birds.

- (a) $\frac{1}{3} \ln(0.03)$
- (b) in the limit as $t \rightarrow +\infty$
- (c) $\frac{100}{3} \ln(3)$
- (d) $\frac{1}{3} \ln\left(\frac{100}{3}\right)$
- (e) $\frac{100}{3} \ln\left(\frac{1}{3}\right)$

3. Compute

$$\int_{-4}^{-2} \frac{x^2 + 2x + 2}{x + 1} dx$$

- (a) -4
- (b) $\ln 3 - 4$
- (c) $4 - \ln(-3)$
- (d) $4 \ln(3)$
- (e) $-4 - \ln(3)$

4. Find the antiderivatives of

$$g(x) = x(2x^2 + 3) \sqrt[3]{x^4 + 3x^2}$$

- (a) $\frac{3}{4} \ln |x^4 + 3x^2| + C$
- (b) $\frac{3}{32} x^2 (2x^2 + 3)^2 (x^4 + 3x^2)^{\frac{4}{3}} + C$
- (c) $\frac{1}{6} (x^4 + 3x^2)^{-\frac{2}{3}} + C$
- (d) $\frac{3}{8} (x^4 + 3x^2)^{\frac{4}{3}} + C$
- (e) $\frac{1}{4} (x^4 + 3x^2)^{\frac{1}{3}} + C$

5. Determine where $h(x) := \frac{e^{-2x}}{x^2-x}$ is decreasing:

- (a) $(-\infty, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, +\infty)$
- (b) $(-\infty, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, 1) \cup (1, +\infty)$
- (c) $[-\frac{1}{\sqrt{2}}, 0) \cup (0, \frac{1}{\sqrt{2}}]$
- (d) Never
- (e) $[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$

6. Compute the following limit:

$$\lim_{x \rightarrow -1} \frac{e^{(x^2-2x-3)} - \ln(2+x) - 1}{2x+2}$$

- (a) 0
- (b) $+\infty$
- (c) $-\frac{5}{2}$
- (d) $\frac{1}{2}$
- (e) It does not exist

7. Compute the following limit:

$$\lim_{x \rightarrow -\infty} \frac{5x^6 - 7x^4 + 3\sqrt{-x} - 2}{4x^5 - x^2 + \sqrt[3]{x} + 1}$$

- (a) It does not exist
- (b) $\frac{5}{4}$
- (c) 0
- (d) $-\infty$
- (e) $+\infty$

8. The domain of the function $f(x) := \frac{\ln(x+1)}{e^{\sqrt{x^2-4x+3}}}$ is

- (a) $(-1, 1] \cup [3, +\infty)$
- (b) $(-\infty, 1] \cup [3, +\infty)$
- (c) $(1, +\infty)$
- (d) $\mathbb{R} \setminus \{-1, 1, 3\}$
- (e) $[-1, 1) \cup (3, +\infty)$