CALCULUS (Global Governance) A.Y. 2024/25

Prof. Alfonso Sorrentino

FAKE MIDTERM TEST: 23^{rd} OCTOBER 2024

Last Name:	First Name:	Student's ID:

Instructions:

- You have 1 hour and 30 minutes to answer the following questions. Report your answers in the table provided below.
- No notes, books, or any other reference materials is allowed during the test.
- Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Answers

1	2	3	4	5	6	7	8

Questions

1. Where is the function $\ell(x) = x^3 - 6x^2 + 9x$ decreasing at the fastest rate when $-5 \leq x \leq 5$?

(a) $x = 5$

▶ (b) $x = 2$

(c) $x = 0$

(d) $x = -5$

(e) $x = \sqrt{2}$

2. In a certain region, the population of a species of birds is decreasing exponentially. The population $P(t)$ (in thousands) at time t (in years) is given by the equation:

$$P(t) = P_0 e^{-kt}$$

where P_0 is the initial population, and k is a positive constant that represents the rate of decrease. If the initial population of the birds is $P_0 = 150$ (thousands) and the rate of decrease $k = 0.03$ per year, determine how long it will take for the population to decrease to 50 thousand birds.

(a) $\frac{1}{3}\ln(0.03)$

(b) in the limit as $t \to +\infty$

▶ (c) $\frac{100}{3}\ln(3)$

(d) $\frac{1}{3}\ln\left(\frac{100}{3}\right)$

(e) $\frac{100}{3}\ln\left(\frac{1}{3}\right)$

3. Compute

$$\int_{-4}^{-2} \frac{x^2 + 2x + 2}{x + 1}\, dx$$

(a) -4

(b) $\ln 3 - 4$

(c) $4 - \ln(-3)$

(d) $4\ln(3)$

▶ (e) $-4 - \ln(3)$

4. Find the antiderivatives of

$$g(x) = x(2x^2 + 3)\sqrt[3]{x^4 + 3x^2}$$

(a) $\frac{3}{4}\ln|x^4 + 3x^2| + C$

(b) $\frac{3}{32}x^2(2x^2 + 3)^2(x^4 + 3x^2)^{\frac{4}{3}} + C$

(c) $\frac{1}{6}(x^4 + 3x^2)^{-\frac{2}{3}} + C$

▶ (d) $\frac{3}{8}(x^4 + 3x^2)^{\frac{4}{3}} + C$

(e) $\frac{1}{4}(x^4 + 3x^2)^{\frac{1}{3}} + C$

5. Determine where $h(x) := \frac{e^{-2x}}{x^2-x}$ is decreasing:

(a) $(-\infty, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, +\infty)$

▶ (b) $(-\infty, -\frac{1}{\sqrt{2}}] \cup [\frac{1}{\sqrt{2}}, 1) \cup (1, +\infty)$

(c) $[-\frac{1}{\sqrt{2}}, 0) \cup (0, \frac{1}{\sqrt{2}}]$

(d) Never

(e) $[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$

6. Compute the following limit:

$$\lim_{x \to -1} \frac{e^{(x^2-2x-3)} - \ln(2+x) - 1}{2x+2}$$

(a) 0

(b) $+\infty$

▶ (c) $-\frac{5}{2}$

(d) $\frac{1}{2}$

(e) It does not exist

7. Compute the following limit:

$$\lim_{x \to -\infty} \frac{5x^6 - 7x^4 + 3\sqrt{-x} - 2}{4x^5 - x^2 + \sqrt[3]{x} + 1}$$

(a) It does not exist

(b) $\frac{5}{4}$

(c) 0

▶ (d) $-\infty$

(e) $+\infty$

8. The domain of the function $f(x) := \frac{\ln(x+1)}{e^{\sqrt{x^2-4x+3}}}$ is

▶ (a) $(-1, 1] \cup [3, +\infty)$

(b) $(-\infty, 1] \cup [3, +\infty)$

(c) $(1, +\infty)$

(d) $\mathbb{R} \setminus \{-1, 1, 3\}$

(e) $[-1, 1) \cup (3, +\infty)$