

CALCULUS
GLOBAL GOVERNANCE
A.Y. 2024/25

Tutorial nr. 6

Exercise 1.

Compute the following improper integrals:

$$(a) \int_1^{+\infty} \frac{x^2}{x^3+2} dx \quad (b) \int_{-\infty}^{+\infty} x e^{-|x|} dx \quad (c) \int_{-\infty}^0 \frac{1}{\sqrt[3]{2x-1}} dx.$$

Exercise 2. Find the general solution of these differential equations:

$$(a) \frac{dx}{dt} = \sqrt{t} + e^{-t} \quad (b) \frac{dx}{dt} = t e^{t-x} \quad (c) \frac{dx}{dt} = \frac{\ln t}{\ln x}$$
$$(d) t \frac{dx}{dt} + 2x = t e^{t^3} \quad (e) \frac{dx}{dt} + \frac{xt}{t^2+1} = t.$$

Exercise 3. The new US president elect Trump is known for his dynamic approval ratings, which change based on his policies, public events, and media coverage. Suppose his approval rating $P(t) \in [0, 100]$, as a percentage, follows the differential equation:

$$\frac{dP}{dt} = k(100 - P(t)) - mP(t),$$

where:

- $k > 0$ is a constant representing the rate of positive influence from his policies (*e.g.*, economic growth or foreign diplomacy success),
- $m > 0$ is a constant representing the rate of negative influence from controversies, gaffe or opposition,
- t is measured in months.

Initially, at $t = 0$, his approval rating is $P_0 \in [0, 100]$.

- (i) Determine for which values of P_0 , the solution of the corresponding initial value problem is constant (*i.e.*, it is an equilibrium solution).
- (ii) Find the solution $P(t)$ corresponding to the initial value $P(0) = P_0$. What is the limit as t goes to infinity of the approval rating? Discuss what happens to the approval rating if $k > m$, $k = m$, or $k < m$.
- (iii) Suppose that at $t = 0$, his approval rating is 50%. If $k = 0.02$ and $m = 0.01$, find $P(t)$ after 6 months and after 1 year. How could Trump adjust his policy or behavior (in terms of k and m) to improve his long-term approval rating?