

Last Name:	First Name:	Student's ID:
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Instructions:

- You have 1 hour and 30 minutes to answer the following questions. Report your answers in the table provided below.
- No notes, books, or any other reference materials is allowed during the test.
- Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Answers

1	2	3	4	5	6	7	8	9

Questions

- 1.** Find the equilibria y_0 and their stability, (s)table or (u)nstable, for the differential equation:

$$y'(x) = y(x)(1 + y(x) - 17), \quad y(0) = y_0$$

- (a) $y_0 = 1$ (s), $y_0 = 17$ (u)
- (b) $y_0 = 0$ (s), $y_0 = 16$ (u)
- (c) $y_0 = 0$ (u), $y_0 = 16$ (s)
- (d) $y_0 = 1$ (u), $y_0 = 17$ (s)
- (e) $y_0 = 0$ (s), $y_0 = 16$ (s)

- 2.** Find the domain of the given function:

$$f(x, y) = \sqrt{x^2 - y^2}$$

- (a) $x \leq 9 - y$
- (b) $(x < 1 - y \wedge -y < x) \vee x > 1 - y$
- (c) $x \geq \sqrt{y^2} \vee x \leq -\sqrt{y^2}$
- (d) $x > -\frac{3y}{4} \vee x < -\frac{3y}{4}$
- (e) $x > 4 - y$

- 3.** Determine the solution of the following linear differential equation

$$y'(x) - \frac{4}{x}y(x) = (1+x)^2, \quad y(1) = 0$$

- (a) $y(x) = -\frac{1}{3}x + \frac{4}{3}x^2 - \frac{1}{3}x^3 + \frac{2}{3}x^4$
- (b) $y(x) = -\frac{1}{3}x - x^2 - x^3 + \frac{7}{3}x^4$
- (c) $y(x) = -\frac{1}{3}\log x - x^2 + x^4$
- (d) $y(x) = -\frac{1}{3}x^2 - \frac{4}{x^2} + \frac{2}{x} + \frac{1}{3}x^4$
- (e) $y(x) = -\frac{1}{3}x \log x + \frac{2}{3}\log x + x - x^4$

- 4.** Find the critical point of the function

$$f(x, y) = 2x^2 + y^2 + 2xy + 4x + 2y + 4$$

specifying wheter it is a relative minimum, maximum or saddle point:

- (a) $(1, -1)$, local maximum
- (b) $(0, -1)$, local minimum
- (c) $(-1, 0)$, local minimum
- (d) $(1, -1)$, saddle point
- (e) $(-1, 0)$, local maximum

5. Determine for which values of the constants m , n , the point $y_0 = -5$ is a stable equilibrium of the differential equation:

$$y'(x) = y(x)(1 + m(y(x) - n))$$

- (a) $m = 1, n = -4$
- (b) $m = 1, n = 4$
- (c) $m = -1, n = -6$
- (d) $m = -1, n = 0$
- (e) $m = -\frac{1}{2}, n = -7$

6. Find the second order partial derivative f_{xy} of the given function:

$$f(x, y) = e^{x^2 y}$$

- (a) $-\frac{4xy}{(x^2+y^2)^2}$
- (b) $2xe^{x^2 y} (x^2 y + 1)$
- (c) $e^x x(x + 2)$
- (d) $60x^3 y^2 + 2$
- (e) $-\frac{1}{(y-1)^2}$

7. Determine which of the following points belongs to the level curve $\{f(x, y) = C\}$, where

$$f(x, y) = x^2 - 4x - y \quad \text{and} \quad C = -4.$$

- (a) $(-6, -36)$
- (b) $(4, -2)$
- (c) $(5, 9)$
- (d) $(11, -5)$
- (e) $(2, 1)$

8. Determine the solution for the following separable differential equation:

$$y'(x) = \frac{y-1}{x+3}, \quad y(0) = -1$$

- (a) $y(x) = -\frac{x}{3}$
- (b) $y(x) = \frac{x+6}{3}$
- (c) $y(x) = -\frac{5x}{3} - 4$
- (d) $y(x) = -\frac{2x}{3} - 1$
- (e) $y(x) = -x - 2$

9. Find the partial derivative f_y of the given function:

$$f(x, y) = (x - 2y)^2 + (y - 3x)^2 + 5;$$

- (a) $-e^{-x} y + y^2 + e^{-2y}$
- (b) $2x$
- (c) $x^3 - 2$
- (d) $10(y - x)$
- (e) $\frac{3x}{(y-3x)^2} + \frac{1}{y-3x} + 1$