

Last Name:	First Name:	Student's ID:
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Instructions:

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Part I (Multiple choice questions)

Answers

1	2	3	4	5	6	7	8	9	10

- 1.** Find the critical point of the function

$$f(x, y) = 2x^2 + y^2 + 2xy + 4x + 2y + 4$$

specifying wheter it is a relative minimum, maximum or saddle point:

- (a) $(1, -1)$, saddle point
- (b) $(-1, 0)$, local minimum
- (c) $(-1, 0)$, local maximum
- (d) $(1, -1)$, local maximum
- (e) $(0, -1)$, local minimum

- 2.** Compute the following limit:

$$\lim_{x \rightarrow -1} \frac{e^{(x^2-2x-3)} - \ln(2+x) - 1}{2x+2}$$

- (a) $+\infty$
- (b) It does not exist
- (c) $-\frac{5}{2}$
- (d) 0
- (e) $\frac{1}{2}$

- 3.** Determine the solution of the following linear differential equation

$$y'(x) - \frac{4}{x}y(x) = (1+x)^2, \quad y(1) = 0$$

- (a) $y(x) = -\frac{1}{3} \log x - x^2 + x^4$
- (b) $y(x) = -\frac{1}{3}x + \frac{4}{3}x^2 - \frac{1}{3}x^3 + \frac{2}{3}x^4$
- (c) $y(x) = -\frac{1}{3}x^2 - \frac{4}{x^2} + \frac{2}{x} + \frac{1}{3}x^4$
- (d) $y(x) = -\frac{1}{3}x \log x + \frac{2}{3} \log x + x - x^4$
- (e) $y(x) = -\frac{1}{3}x - x^2 - x^3 + \frac{7}{3}x^4$

- 4.** Where is the function $\ell(x) = x^3 - 6x^2 + 9x$ decreasing at the fastest rate when $-5 \leq x \leq 5$?

- (a) $x = \sqrt{2}$
- (b) $x = -5$
- (c) $x = 5$
- (d) $x = 2$
- (e) $x = 0$

5. Compute

$$\int_{-4}^{-2} \frac{x^2 + 2x + 2}{x + 1} dx$$

- (a) -4
- (b) $-4 - \ln(3)$
- (c) $\ln 3 - 4$
- (d) $4 \ln(3)$
- (e) $4 - \ln(-3)$

6. Compute the following limit:

$$\lim_{x \rightarrow -\infty} \frac{5x^6 - 7x^4 + 3\sqrt{-x} - 2}{4x^5 - x^2 + \sqrt[3]{x} + 1}$$

- (a) 0
- (b) $\frac{5}{4}$
- (c) $-\infty$
- (d) $+\infty$
- (e) It does not exist

7. The domain of the function $f(x) := \frac{\ln(x+1)}{e^{\sqrt{x^2-4x+3}}}$ is

- (a) $\mathbb{R} \setminus \{-1, 1, 3\}$
- (b) $(1, +\infty)$
- (c) $(-\infty, 1] \cup [3, +\infty)$
- (d) $(-1, 1] \cup [3, +\infty)$
- (e) $[-1, 1) \cup (3, +\infty)$

8. Find the second order partial derivative f_{xy} of the given function:

$$f(x, y) = e^{x^2 y}$$

- (a) $60x^3 y^2 + 2$
- (b) $2xe^{x^2 y} (x^2 y + 1)$
- (c) $-\frac{4xy}{(x^2 + y^2)^2}$
- (d) $e^x x(x + 2)$
- (e) $-\frac{1}{(y-1)^2}$

9. Find the equilibria y_0 and their stability, (s)table or (u)nstable, for the differential equation:

$$y'(x) = y(x)(1 + y(x) - 17), \quad y(0) = y_0$$

- (a) $y_0 = 1$ (s), $y_0 = 17$ (u)
- (b) $y_0 = 1$ (u), $y_0 = 17$ (s)
- (c) $y_0 = 0$ (s), $y_0 = 16$ (s)
- (d) $y_0 = 0$ (s), $y_0 = 16$ (u)
- (e) $y_0 = 0$ (u), $y_0 = 16$ (s)

10. Determine the solution for the following separable differential equation:

$$y'(x) = \frac{y-1}{x+3}, \quad y(0) = -1$$

- (a) $y(x) = -\frac{x}{3}$
- (b) $y(x) = -\frac{5x}{3} - 4$
- (c) $y(x) = -x - 2$
- (d) $y(x) = \frac{x+6}{3}$
- (e) $y(x) = -\frac{2x}{3} - 1$

Part II (Problems)

Problem 1. Study the following function and sketch its graph:

$$f(x) = \frac{x^4}{x^3 - 2}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

Solution:

Problem 2. The value V (in thousands of euros) of an industrial machine is modeled by

$$V(N) = \left(\frac{3N + 430}{N + 1} \right)^{2/3}$$

where N is the number of hours the machine is used each day. Suppose further that usage varies with time in such a way that

$$N(t) = \sqrt{t^2 - 10t + 45}$$

where t is the number of months the machine has been in operation.

- (a) How many hours per day will the machine be used 9 months from now? What will be the value of the machine at this time?
- (b) At what rate is the value of the machine changing with respect to time 9 months from now? Will the value be increasing or decreasing at this time?
- (c) At what time t is the value of the machine the largest? What is this maximum value?

Solution: