CALCULUS (Global Governance) A.Y. 2025/26

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FINAL EXAM: 31st AUGUST 2026

Last Name:	**First Name:**	**Student's ID:**

Instructions:

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Part I (Multiple choice questions)

Answers

1	2	3	4	5	6	7	8	9	10

1. A substance is released into a solution, and the concentration of the substance t seconds later, is given by $S(t)$ in grams per cubic centimeter (g/cm^3), where $S(t) = 0.3\left(1 + 6\,e^{-0.03t}\right)$. How much time (in seconds) will it take for the concentration to reach 0.75 g/cm^3 ?

(a) $\frac{10}{3}\ln(3)$

(b) $\frac{100}{3}\ln(4)$

(c) $\frac{5}{9}\ln(75) - \frac{1}{6}$

(d) $\frac{50}{3}\ln(5)$

(e) It will never reach this concentration.

2. Determine the solution for the following separable differential equation:

$$y'(x) = \frac{e^{-9x}}{\sqrt{y-4}}, \quad y(0) = 5$$

(a) $y(x) = \sqrt[3]{5}\,(\frac{31-e^{-9x}}{6})^{\frac{2}{3}}$

(b) $y(x) = 6 - \frac{4\,e^{-9x}}{\sqrt{x+16}}$

(c) $y(x) = \frac{5}{2}\sqrt{3 + (\frac{5+e^{-9x}}{6})^3}$

(d) $y(x) = 4 + (\frac{7-e^{-9x}}{6})^{\frac{2}{3}}$

(e) $y(x) = 6 - (\frac{10-e^{-9x}}{9})^{\frac{3}{2}}$

3. Determine the solution to the following linear differential equation

$$y'(x) + \frac{1}{x}y(x) = 2,\ y(1) = 0$$

(a) $y(x) = \frac{x-1}{x}$

(b) $y(x) = \frac{x^2+1}{x} - 2$

(c) $y(x) = \frac{x+1}{x} - 2$

(d) $y(x) = \frac{x^3-1}{x}$

(e) $y(x) = \frac{x^2-1}{x}$

4. Find the second order partial derivative f_{xx} of the given function:

$$f(x,y) = \log\left(4x^2 + y^5\right)$$

(a) $-\frac{40xy^4}{(4x^2+y^5)^2}$

(b) $\frac{8}{(4x^2+y^5)^2}$

(c) $\frac{80x^2y^3-5y^8}{(4x^2+y^5)^2}$

(d) $\frac{8x}{4x^2+y^5}$

(e) $\frac{-32x^2+8y^5}{(4x^2+y^5)^2}$

5. Compute the following definite integral:

$$\int_0^1 (x+4)e^{6x}\,dx =$$

(a) $\frac{30\,e^{-6}-25}{6}$

(b) $\frac{29\,e^{6}-23}{36}$

(c) $\frac{5\,e^{6}+24}{36}$

(d) $\frac{29\,e^{-6}+23}{36}$

(e) $\frac{29\,e^{6}-23}{6}$

6. Compute the following limit

$$\lim_{x\to-\infty} \frac{7x^6 + 2x^5 - 6x + 14}{12x^6 - 2x^4 + 6} =$$

(a) $-\infty$

(b) $\frac{7}{3}$

(c) 0

(d) - $\frac{7}{12}$

(e) $\frac{7}{12}$

7. Find the critical point of the function

$$f(x,y) = x^4 + 4x^3 + 6x^2 + y^2 + 4x + 2y$$

specifying wheter it is a relative minimum, maximum or saddle point:

(a) $(-1,-1)$, local maximum

(b) $(-1,1)$, saddle point

(c) $(-1,-1)$, local minimum

(d) $(-1,1)$, local maximum

(e) $(-1,1)$, local minimum

8. At $x = 3$, the function $g(x) = x^2\sin(\pi x)$ is:

(a) neither increasing nor decrasing (i.e. $x = 3$ is a stationary point)

(b) decreasing and concave (concave down)

(c) increasing and concave (concave down)

(d) increasing and convex (concave up)

(e) decreasing and convex (concave up)

9. Compute the following limit

$$\lim_{x\to-3}\left(\frac{3e^{(2x^2\,+\,16x+\,30)}-3}{5x+15}\right) =$$

(a) $\frac{8}{5}$

(b) $\frac{12}{5}$

(c) 0

(d) $+\infty$

(e) It does not exist

10. Which of the following best characterizes the domain of the function $f(x,y) = \frac{1}{\log(9y^2-x^2)}$?

(a) $x \geq 3|y|$ and $x \leq -3|y|$

(b) $x > 3|y|$ or $x < -3|y|$

(c) $|y| > \frac{1}{3}$

(d) $-3|y| < x$ and $x < 3|y|$

(e) $-3|y| \leq x \leq 3|y|$

Part II (Problems)

Problem 1. Study the following function and sketch its graph:

$$f(x) = \frac{3x + 19 - x^2}{x + 4}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

Solution:

- **Domain and continuity**

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- **Intersection with axis and positivity/negativity**

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- **Asymptotes**

- **First derivative, interval of monotonicity, local maxima/minima**

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- **Graph**