

CALCULUS
GLOBAL GOVERNANCE
A.Y. 2025/26

Recitation I

Exercise 1 (Domain of a function)

Determine the domain of the following functions:

$$(a) f(x) = 2\sqrt{\frac{x^2-1}{x^2-5x+4}} \quad (b) g(x) = \frac{\sqrt{x^3-5x^2+6x}}{\ln(x^2-4)} \quad (c) h(x) = \sin(x^2) + \log_2\left(\frac{x+1}{x-5}\right).$$

Exercise 2 (Limits)

I. Compute, if they exist, the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^3 - 5x + 4}{x^3 - 1} \quad (b) \lim_{x \rightarrow 0} \frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x} \quad (c) \lim_{x \rightarrow -2} \frac{3x + 6}{(x+1)(x^2 + 4x + 4)}.$$

In the cases in which the limit does not exist, discuss the existence of the one-sided limit (from the left and from the right).

II. Compute, if they exist, the following limits:

$$(a) \lim_{x \rightarrow +\infty} \frac{x^3 - 5x + 4}{x^3 - 1} \quad (b) \lim_{x \rightarrow -\infty} \frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x} \quad (c) \lim_{x \rightarrow +\infty} \frac{3x + 6}{(x+1)(x^2 + 4x + 4)}.$$

III. Compute, if they exist, the following limits:

$$(a) \lim_{x \rightarrow +\infty} \sqrt{\frac{x^3 - 5x + 4}{x^3 - 1}} \quad (b) \lim_{x \rightarrow -\infty} e^{\frac{x^4 - 4x^3 + x^2}{x^3 + x^2 + x}} \quad (c) \lim_{x \rightarrow +\infty} \log\left(\frac{3x + 6}{(x+1)(x^2 + 4x + 4)}\right).$$

Exercise 3

In 1389, Pierre d'Arcis, the Bishop of Troyes, wrote a memo to the Pope, accusing a colleague of passing off “*a certain cloth, cunningly painted*”, as the burial shroud of Jesus Christ. Despite this early testimony of forgery, the image on the cloth is so compelling that many people regard it as a sacred relic. Known as the *Shroud of Turin*, the cloth was subject to carbon dating in 1988: this test consists in measuring the amount of the radioactive isotope ^{14}C (carbon-14) contained on the Shroud.

- (a) Denote by $R(t)$ the amount of ^{14}C contained on the Shroud after t years and assume that it decays as

$$R(t) = R_0 e^{-kt}$$

where $R_0 > 0$ is the initial amount of ^{14}C (at time $t = 0$) and $k > 0$.

It is known that after 5,730 years the amount of ^{14}C becomes $\frac{R_0}{2}$ (i.e., it halves). Determine the value of k .

- (b) If the Shroud were actually 1,960 years old in 1988, what percentage of ^{14}C would have remained?

- (c) Scientists determined that 92.3% of the Shroud's original ^{14}C remained in 1988. Based on this information alone, what was the likely age of the Shroud in 1988?

Exercise 4 The magnitude formula for the Richter scale is

$$R(E) = \frac{2}{3} \log_{10} \left(\frac{E}{E_0} \right),$$

where E is the energy released by the earthquake (in joules), and $E_0 = 10^{4.4}$ joules is the energy released by a small reference earthquake used as a standard of measurement.

- (a) The 1906 San Francisco earthquake released approximately 5.96×10^{16} joules of energy. What was its magnitude on the Richter scale?
- (b) How much energy was released by the Indian earthquake of 1993, which measured 6.4 on the Richter scale?

If I denotes the intensity of the earthquake, then

$$R(I) = \frac{\ln I}{\ln 10}.$$

- (c) Determine the relation between E and I , *i.e.*, E as a function of I and vice versa.
- (d) Find the intensity of the 1906 San Francisco earthquake.
- (e) How much more intense was the San Francisco earthquake of 1906 than the devastating 1995 earthquake in Kobe, Japan, which measured 7.1 on the Richter scale?