# CALCULUS (Global Governance) A.Y. 2025/26

**Prof. Rafael L. Greenblatt & Alfonso Sorrentino**

Sample Final Exam: 19th January 2026

| **Last Name:** | **First Name:** | **Student's ID:** |
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**Instructions:**

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

## Part I (Multiple choice questions)

### Answers

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
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**1.** Find the critical point of the function

$$f(x, y) = \exp(xy + x - y)$$

specifying wheter it is a relative minimum, maximum or saddle point:

(a) $(1, -1)$, saddle point

(b) $(1, 1)$, saddle point

(c) $(1, -1)$, local minimum

(d) $(1, -1)$, local maximum

(e) $(-1, 0)$, local minimum

**2.** Compute

$$\int_{-4}^{-2} \frac{x^2 + 2x + 2}{x + 1}\, dx$$

(a) $-4$

(b) $-4 - \ln(3)$

(c) $\ln 3 - 4$

(d) $4 \ln(3)$

(e) $4 - \ln(-3)$

**3.** Compute the following limit:

$$\lim_{x \to -\infty} \frac{5x^6 - 7x^4 + 3\sqrt{-x} - 2}{4x^5 - x^2 + \sqrt[3]{x} + 1}$$

(a) It does not exist

(b) $\frac{5}{4}$

(c) $-\infty$

(d) $+\infty$

(e) $0$

**4.** Determine the solution for the following separable differential equation:

$$y'(x) = \frac{y - 1}{x + 3}, \quad y(0) = -1$$

(a) $y(x) = -\frac{x}{3}$

(b) $y(x) = -x - 2$

(c) $y(x) = \frac{x+6}{3}$

(d) $y(x) = -\frac{2x}{3} - 1$

(e) $y(x) = -\frac{5x}{3} - 4$

**5.** In a certain region, the population of a species of birds is decreasing exponentially. The population $P(t)$ (in thousands) at time $t$ (in years) is given by the equation:

$$P(t) = P_0 e^{-kt}$$

where $P_0$ is the initial population, and $k$ is a positive constant that represents the rate of decrease. If the initial population of the birds is $P_0 = 150$ (thousands) and the rate of decrease $k = 0.03$ per year, determine how long it will take for the population to decrease to 50 thousand birds.

(a) $\frac{1}{3}\ln\left(\frac{100}{3}\right)$

(b) $\frac{100}{3}\ln\left(\frac{1}{3}\right)$

(c) in the limit as $t \to +\infty$

(d) $\frac{1}{3}\ln(0.03)$

(e) $\frac{100}{3}\ln(3)$

**6.** Where is the function $\ell(x) = x^3 - 6x^2 + 9x$ decreasing at the fastest rate when $-5 \le x \le 5$?

(a) $x = 2$

(b) $x = 0$

(c) $x = \sqrt{2}$

(d) $x = 5$

(e) $x = -5$

**7.** Compute the following limit:

$$\lim_{x\to-1} \frac{e^{(x^2-2x-3)} - \ln(2+x) - 1}{2x+2}$$

(a) 0

(b) $-\frac{5}{2}$

(c) $+\infty$

(d) $\frac{1}{2}$

(e) It does not exist

**8.** Find the second order partial derivative $f_{xy}$ of the given function:

$$f(x, y) = y\ln(x^2 + y)$$

(a) $\frac{y}{x^2+y} + \ln(x^2 + y)$

(b) $\frac{2x^3}{(x^2+y)^2}$

(c) $2\frac{2x^2+y}{(x^2+y)^2}$

(d) $\frac{2xy}{x^2+y}$

(e) $2\frac{y^2-x^2y}{(x^2+y)^2}$

**9.** Which of the following best characterizes the domain of the function $f(x, y) = \sqrt{x^2 - y^2 - 4}$?

(a) $y \in [-2, 2]$

(b) $x \in (-\infty, -2] \cup [2, \infty)$

(c) $x \ge \sqrt{y^2 + 4}$ or $x \le -\sqrt{y^2 + 4}$

(d) $x^2 + y^2 \le 4$

(e) $-\sqrt{y^2 + 4} \le x \le \sqrt{y^2 + 4}$

**10.** Determine the solution of the following linear differential equation

$$y'(x) - \frac{4}{x}y(x) = (1 + x)^2, \; y(1) = 0$$

(a) $y(x) = -\frac{1}{3}x + \frac{4}{3}x^2 - \frac{1}{3}x^3 + \frac{2}{3}x^4$

(b) $y(x) = -\frac{1}{3}x\log x + \frac{2}{3}\log x + x - x^4$

(c) $y(x) = -\frac{1}{3}x^2 - \frac{4}{x^2} + \frac{2}{x} + \frac{1}{3}x^4$

(d) $y(x) = -\frac{1}{3}\log x - x^2 + x^4$

(e) $y(x) = -\frac{1}{3}x - x^2 - x^3 + \frac{7}{3}x^4$

# Part II (Problems)

**Problem 1.** Study the following function and sketch its graph:

$$f(x) = \frac{x^4}{x^3 - 2}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

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**Solution:**

- **Domain and continuity**

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- **Intersection with axis and positivity/negativity**

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- **Asymptotes**

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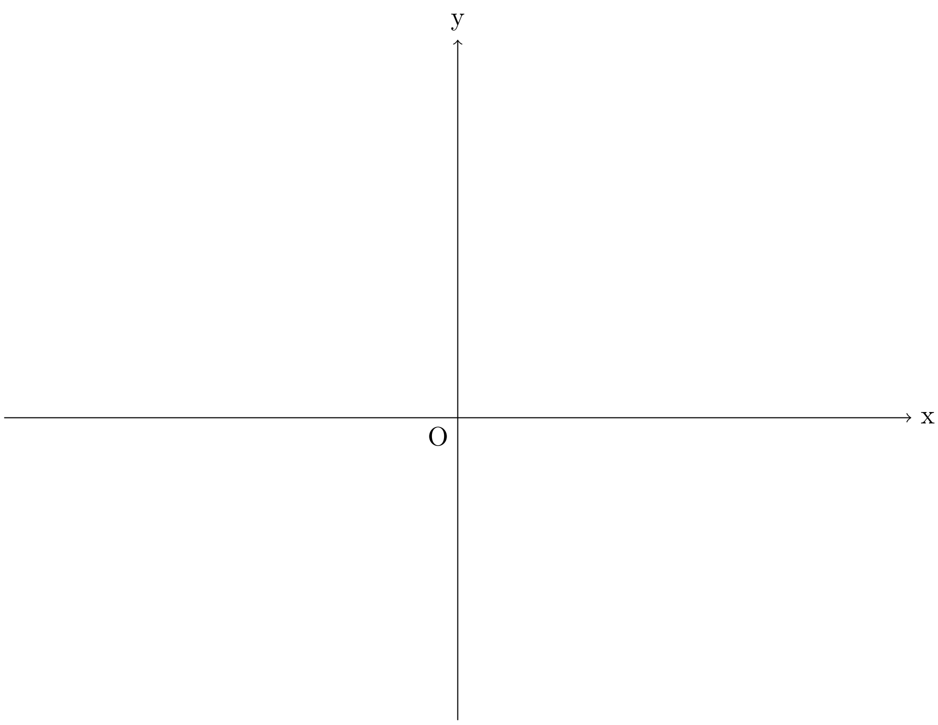
- **First derivative, interval of monotonicity, local maxima/minima**

- $\boxed{\textbf{Second derivative and concavity}}$

- $\boxed{\textbf{Graph}}$



**Problem 2.** The value $V$ (in thousands of euros) of an industrial machine is modeled by

$$V(N) = \left(\frac{3N + 430}{N + 1}\right)^{2/3}$$

where $N$ is the number of hours the machine is used each day. Suppose further that usage varies with time in such a way that

$$N(t) = \sqrt{t^2 - 10t + 45}$$

where $t$ is the number of months the machine has been in operation.

**(a)** How many hours per day will the machine be used 9 months from now? What will be the value of the machine at this time?

**(b)** At what rate is the value of the machine changing with respect to time 9 months from now? Will the value be increasing or decreasing at this time?

**(c)** At what time $t$ is the value of the machine the largest? What is this maximum value?

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$\boxed{\textbf{Solution}}$