CALCULUS (Global Governance) A.Y. 2025/26

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SECOND FINAL EXAM: 11^{th} FEBRUARY 2026

Last Name:	**First Name:**	**Student's ID:**

Instructions:

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Part I (Multiple choice questions)

Answers

1	2	3	4	5	6	7	8	9	10
C	B	B	D	A	C	A	D	D	B

1. Find the critical point of the function

$$f(x,y) = \exp(x^2 + xy + 3x + y)$$

specifying wheter it is a relative minimum, maximum or saddle point:

(a) $(1,-1)$, local maximum

(b) $(1,0)$, local minimum

▶ (c) $(-1,-1)$, saddle point

(d) $(1,1)$, local minimum

(e) $(-1,1)$, local maximum

2. A substance is released into a solution, and the concentration of the substance t seconds later, is given by $S(t)$ in grams per cubic centimeter (g/cm^3), where $S(t) = 0.3\left(1 + 3\,e^{-0.02t}\right)$. How much time (in seconds) will it take for the concentration to reach 0.53 g/cm^3 ?

(a) $25\ln(5)$

▶ (b) $50\ln(4)$

(c) $5\ln(2)$

(d) $\frac{10}{9}\ln(52.5) - \frac{1}{3}$

(e) It will never reach this concentration.

3. Compute the following limit

$$\lim_{x\to 2}\left(\frac{-\,e^{(-\,3x^2\,+\,15x\,-\,18)} + 1}{3x - 6}\right) =$$

(a) 0

▶ (b) -1

(c) $-\frac{2}{3}$

(d) $+\infty$

(e) It does not exist

4. Compute the following definite integral:

$$\int_0^1 (x+6)e^{4x}\,dx =$$

(a) $\frac{27\,e^{-4}+23}{16}$

(b) $\frac{28\,e^{-4}-25}{4}$

(c) $\frac{27\,e^{4}-23}{4}$

▶ (d) $\frac{27\,e^{4}-23}{16}$

(e) $\frac{3\,e^{4}+24}{16}$

5. At $x=4$, the function $g(x)=\log(17-x^2)$ is:

▶ (a) decreasing and concave

(b) neither increasing nor decrasing (i.e. $x=-1$ is a stationary point)

(c) decreasing and convex

(d) increasing and convex

(e) increasing and concave

6. Compute the following limit:

$$\lim_{x\to+\infty} e^{-(5x^2+2)}\left(\frac{2x^3+5x+2}{5x^3+5}\right)=$$

(a) $\frac{2}{5}$

(b) $+\infty$

▶ (c) 0

(d) $\frac{2}{5}\,e^{-2}$

(e) None of the above

7. Determine the solution to the following linear differential equation

$$y'(x)+\frac{1}{2x}y(x)=5,\ y(1)=0$$

▶ (a) $y(x)=\frac{10}{3}\frac{(x^{3/2}-1)}{\sqrt{x}}$

(b) $y(x)=\frac{10}{3}\frac{(x^{3/2}+1)}{\sqrt{x}}-\frac{10}{3}$

(c) $y(x)=\frac{5}{3}\frac{(x^{3/2}+1)}{\sqrt{x}}-\frac{5}{3}$

(d) $y(x)=\frac{5}{3}\frac{(x^{3/2}-1)}{\sqrt{x}}$

(e) $y(x)=\frac{5}{4}\frac{(x^{3/2}-1)}{\sqrt{x}}$

8. Which of the following best characterizes the domain of the function $f(x,y)=\frac{1}{\sqrt{1x^2-4y^2}}$?

(a) $x>2|y|$ and $x<-2|y|$

(b) $-2|y|<x<2|y|$

(c) $|y|>\frac{1}{2}$

▶ (d) $x>2|y|$ or $x<-2|y|$

(e) $-2|y|\le x\le 2|y|$

9. Find the second order partial derivative f_{yy} of the given function:

$$f(x,y)=\exp\left(3y+\sqrt[5]{x}\right)$$

(a) $3\exp\left(3y+\sqrt[5]{x}\right)$

(b) $\frac{\exp\left(3y+\sqrt[5]{x}\right)}{25}\left(x^{-\frac{8}{5}}-4x^{-\frac{9}{5}}\right)$

(c) $\frac{3}{5}\frac{\exp\left(3y+\sqrt[5]{x}\right)}{x^{\frac{4}{5}}}$

▶ (d) $9\exp\left(3y+\sqrt[5]{x}\right)$

(e) $\left(3y+\sqrt[5]{x}\right)^2\exp\left(3y+\sqrt[5]{x}\right)$

10. Determine the solution for the following separable differential equation:

$$y'(x)=\frac{e^{-9x}}{\sqrt{y-3}},\quad y(0)=4$$

(a) $y(x)=2\sqrt{3+(\frac{5+e^{-9x}}{6})^3}$

▶ (b) $y(x)=3+(\frac{7-e^{-9x}}{6})^{\frac{2}{3}}$

(c) $y(x)=5-(\frac{10-e^{-9x}}{9})^{\frac{3}{2}}$

(d) $y(x)=5-\frac{3\,e^{-9x}}{\sqrt{x+9}}$

(e) $y(x)=\sqrt[3]{4}\,(\frac{25-e^{-9x}}{6})^{\frac{2}{3}}$

Part II (Problems)

Problem 1. Study the following function and sketch its graph:

$$f(x) = \frac{2x+2}{x^2+8}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

Solution:

- **Domain and continuity**

DOMAIN: $\mathbb{R}$ (SINCE $x^2+8 \neq 0 \ \forall x \in \mathbb{R}$)

f IS CONTINUOUS AT EVERY POINT IN ITS DOMAIN

- **Intersection with axis and positivity/negativity**

- INTERSECTION WITH X-AXIS: $f(x) = 0 \iff 2x+2=0 \iff x=-1$ $\quad A=(-1,0)$
- INTERSECTION WITH Y-AXIS: $f(0) = \frac{1}{4}$: $B = (0, \frac{1}{4})$
- SIGN: $2x+2 \geq 0 \iff x \geq -1$

$x^2+8>0 \quad \forall x \in \mathbb{R}$

-1

f

$-$ $\quad$ $+$

$f>0$ IN $(-1,+\infty)$

$f<0$ IN $(-\infty,-1)$

- **Asymptotes**

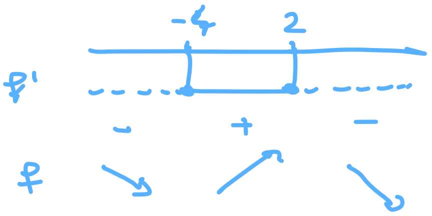
NO VERTICAL ASYMPTOTE

HORIZONTAL ASYMPTOTE: $\lim_{x \to \pm\infty} f(x) = 0 \longrightarrow y=0$ (BOTH AT $+\infty$ AND $-\infty$)

- $\boxed{\textbf{First derivative, interval of monotonicity, local maxima/minima}}$

$$f'(x) = \frac{2(x^2+8) - (2x+2)2x}{(x^2+8)^2} = \frac{2x^2+16-4x^2-4x}{(x^2+8)^2} = \frac{-2x^2-4x+16}{(x^2+8)^2} = \frac{-2(x^2+2x-8)}{(x^2+8)^2}$$

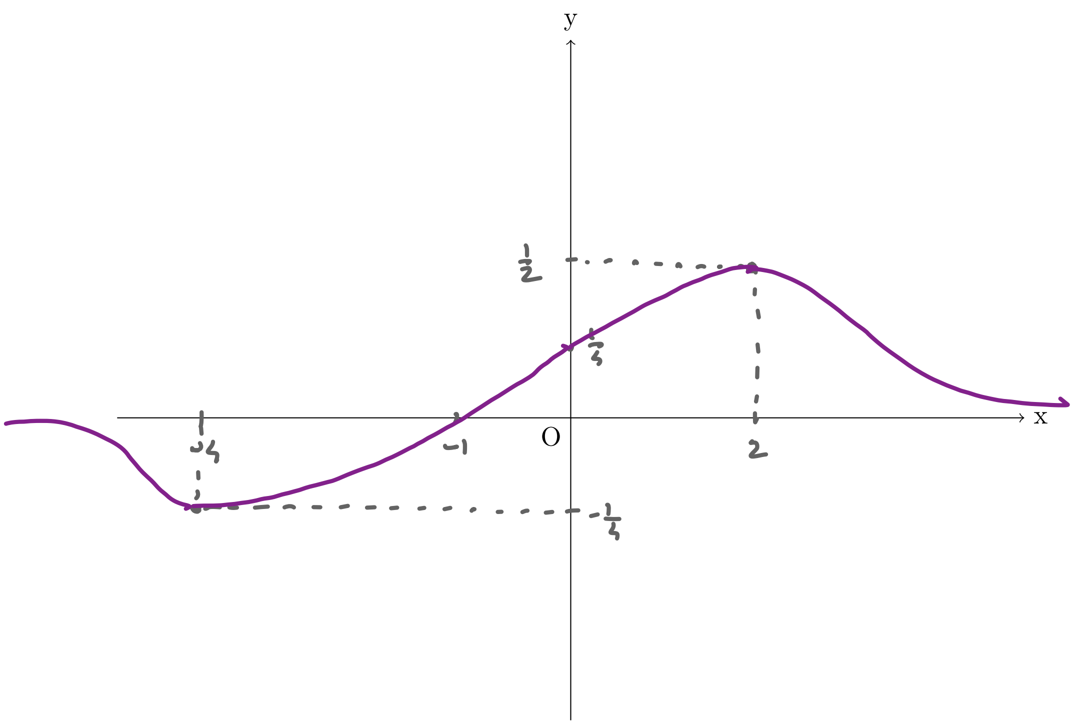
SIGN OF f': $\quad x^2+2x-8=0 \iff x = -1 \pm \sqrt{1+8} = -1 \pm 3 = \begin{cases} -4 \\ 2 \end{cases}$



$x=-4$ LOCAL MINIMUM $f(-4) = -\frac{1}{4}$

$x=2$ LOCAL MAXIMUM $f(2) = \frac{1}{2}$

- $\boxed{\textbf{Graph}}$



Problem 2. A shipping company models the operational efficiency $P(s)$ (in efficiency units) of delivering packages over a distance s hundred miles, where $0 \leq s \leq 32$.
The model is:

$$P(s) := -2s + 2 + \sqrt{5s^2 + 20}.$$

(a) Determine the shipping distance s (in hundreds of miles) that yields the maximum operational efficiency and the distance that yields the minimum operational efficiency within the company's service range $0 \leq s \leq 32$. Report both optimal distances and the corresponding values of $P(s)$.

(b) At what distance s does the operational efficiency increase most rapidly?

Solution

(A) $$P'(s) = -2 + \frac{5s}{\sqrt{5s^2+20}}$$

$$P'(s) = 0 \iff \frac{5s}{\sqrt{5s^2+20}} = 2 \iff \frac{25s^2}{5s^2+20} = 4 \iff$$

CRITICAL POINTS

$$\iff 25s^2 = 20s^2 + 80 \iff 5s^2 = 80 \iff s^2 = 16$$

$$\iff s = \pm 4$$ (WE CONSIDER ONLY $s=4$ SINCE $0 \leq s \leq 32$)

$$P(0) = 2 + \sqrt{20} = 2 + 2\sqrt{5} \simeq 6.47$$

$$P(4) = -8 + 2 + \sqrt{100} = 4$$ ← MINIMAL OPERATIONAL EFFICIENCY AT $s=4$

$$P(32) = -6 + 2\sqrt{1285} \simeq 9.69$$ ← MAXIMAL OPERATIONAL EFFICIENCY AT $s=32$

(B) $$P''(s) = \frac{5\sqrt{5s^2+20} - 5s\frac{5s}{\sqrt{5s^2+20}}}{(5s^2+20)} = \frac{5\left(\cancel{5s^2}+20-\cancel{5s^2}\right)}{(5s^2+20)^{3/2}} = \frac{100}{(5s^2+20)^{3/2}}$$

$P''(s) > 0 \Rightarrow P'$ IS INCREASING IN $0 \leq s \leq 32$

$\Rightarrow$ THE OPERATIONAL EFFICIENCY INCREASES MOST RAPIDLY AT $s = 32$