Calculus - Course Diary

Global Governance Program

University of Rome Tor Vergata

Academic Year 2024/25 (Fall Semester)

Lesson diary

- **Lesson 1 [16/09/2024]** Functions: definition, domain, range. Examples. Functions used in economics (demand function, supply function, revenue, profit, etc.). Injectivity and Surjectivity. Graph of a function. Cartesian plane. Finding points of intersection. Linear functions and straight lines.
  Reference: Sections 1.1-2-3 in [1].

- **Lesson 2 [17/09/2024]** Linear functions and straight lines. Slope of a straight line. Equation of a line passing through two points. Parallel lines and perpendicular lines. Interception of two lines. Quadratic functions and parabola. Plotting parabolas: vertex of a parabola, concavity and intersection with the axes. Exponential function: definition and properties. Graph of the exponential function. Property of the exponential function and exponential rules.
  Reference: Sections 1.2-3 & 4.1 in [1].

- **Lesson 3 [18/09/2024]** Exponential rules. Compound interest and continuous compound interest. Napier's constant: e. Logarithmic functions: definition and main properties. Logarithmic rules. Graphs of logarithmic functions. Natural logarithm. Conversion formula for logarithms.
  Reference: Sections 4.1-2 in [1].

- **Lesson 4 [19/09/2024]** Exercises with exponentials and logarithms. Domain of a function and how to determine it (several examples). Introduction to the concept of limit.
  Reference: Sections 4.1-2, 1.1 and 1.5 in [1].

- **Lesson 5 [23/09/2024]** Concept of limit (both informal and more formal definition). Algebraic properties of limits. Limits of polynomials and quotient of polynomials (rational functions). Undetermined form $\frac{0}{0}$ and how to handle it.
  Reference: Sections 1.5 in [1].

- **Lesson 6 [24/09/2024]** Meaning of $\pm\infty$ and limits as x goes to $+\infty$ or $-\infty$. Undetermined form $\frac{\infty}{\infty}$ and how to handle it. One-sided limits.
  Reference: Sections 1.5-6 in [1].

- **Lesson 7 [25/09/2024]** Limits involving exponentials and logarithms. Definition of continuity of a function at one point. Polynomials and Rational functions are continuous in their domains of definition. Exponentials and Logarithms are continuous at their domains of definition. Concepts of horizontal and vertical asymptotes for a function.
  Reference: Sections 1.5-6 and 4.1-2 in [1].

- **Lesson 8 [26/09/2024]** Basic concepts of differentiation. Definition of derivative and interpretation as rate of change and the slope of the tangent line to a point of the graph. Relative rate of change and Percentage rate of change. Computation of derivatives of: constant function, linear and quadratic function. General rule for the derivative of the power of a function. Derivative of linear combination of functions. Derivative of product of functions. Derivative of quotient of functions. Relation between the sign of the derivative and the function being increasing/decreasing.
  Reference: Sections 2.1-2-3 in [1].

- **Lesson 9 [01/10/2024]** Relation between existence of the derivative and continuity. Examples in which the derivative does not exist ($\sqrt{x}$ and $|x|$ at $x = 0$) and geometric interpretation of these points. Chain rule. Rule for the derivative of the exponential, the logarithm, sine and cosine. Exercises.
  Reference: Sections 2.1-2-3, 4.3, 8.2 in [1].

- **Lesson 10 [02/10/2024]** Critical points of a function and classification (relative maxima or relative minima). Higher order derivatives and Second derivative (geometric interpretation: concavity of the function and points of inflection). Second derivative test for classifying critical points. Exercises and study of functions (curve sketching). Definition of horizontal, vertical and oblique asymptote. Marginal analysis and approximation of a function near a point by its Taylor polynomials of degree 1 and 2.
  Reference: Sections 3.1-2-3 in [1].

- **Lesson 11 [03/10/2024]** Exercise on the study of a function (curve sketching). Optimization problems. Find absolute maxima and minima of a continuous function on a closed interval (statement of Weierstrass theorem about their existence). Exercises.
  Reference: Sections 3.3-4 in [1].

- **Lesson 12 [07/10/2024]** More general optimization problems (not a closed bounded interval). Maximizing profit and average cost. Marginal analysis criterion for maximum profit and for minimal average cost. Evaluating Limits with L'Hp̂ital's rule.
  Reference: Sections 3.4 and A.3 in [1].

- **Lesson 13 [08/10/2024]** Exercised on L'Hôpital rule. Price elasticity of demand, levels of elasticity and effects on the revenue. Some applied optimization problems. Various exercises.
  Reference: Sections 3.4-5 and A.3 in [1].

- **Lesson 14 [10/10/2024]** Integration: antiderivative and indefinite integral. Properties of antiderivatives and rules of integration for some elementary functions. Algebraic rules for indefinite integration. Exercises and applied problems. Integration by substitution.
  Reference: Sections 5.1-2 in [1].

- **Lesson 15 [14/10/2024]** Integration by substitution: various examples and applications. Definite integrals and the fundamental theorem of calculus. Area under the graph of a function.
  Reference: Sections 5.2-3 in [1].

- **Lesson 16 [15/10/2024]** Rules for definite integrals. Evaluating definite integrals: various examples. Substitutions for definite integrals. Applications: finding net change; area between two curves.
  Reference: Section3 5.3-4 in [1].

- **Lesson 17 [16/10/2024]** Integration by parts (for indefinite and definite integrals). Applications of integrals: distribution of wealth (Lorenz curves and Gini index).
  Reference: Sections 5. 4 and 6.1 in [1].

- **Lesson 18 [17/10/2024]** Applications of integrals: average value of a function; net excess profit. future and present value of an income flow.
  Reference: Sections 5.4-5 in [1].

- **Lesson 19 [21/10/2024]** Revision in preparation to the midterm.

- **Lesson 20 [22/10/2024]** Revision in preparation to the midterm.

- **Lesson 21 [23/10/2024]** Revision in preparation to the midterm.

- **Lesson 22 [24/10/2024]** Revision in preparation to the midterm.

- **Lesson 23 [29/10/2024]** First Midterm.

- **Lesson 24 [11/11/2024]** Revision of the midterm. Improper integrals: definition and exercises.
  Reference: Sections 6.2 in [1].

- **Lesson 25 [14/11/2024]** Exercises on improper integrals and applications.
  Reference: Sections 6.2 in [1].

- **Lesson 26 [18/11/2024]** Introduction to differential equations. Main definitions and simple examples. General solution and Initial value problem/Cauchy problem. Antiderivative as a differential equation. Separable differential equations. Examples. Learning model.
  Reference: Sections 5.1 & 9.1 in [1] + Sections 1.1-2-3 in [2].

- **Lesson 27 [19/11/2024]** Separable differential equations. Examples. Logistic equation (Population growth) and its analysis.
  Reference: Section 9.1 in [1] + Sections 1.2-3 in [2].
- **Lesson 28 [20/11/2024]** Various exercises. First order linear differential equations: formula for its solution. Examples.
  Reference: Sections 9.1-2 in [1] + Sections 1.2-3 in [2].
- **Lesson 29 [08/01/2025]** Revision in preparation to the second midterm and the final exam.
- **Lesson 30 [25/11/2024]** Recap differential equations. Classification nomenclature (ordinary, explicit, implicit, autonomous, non-autonomous, linear, non-linear). Examples.
  Reference: Section 9.1-2 in [1] + Section 1.2-3 in [2]
- **Lesson 31 [26/11/2024]** Recap differential equations. Solution strategies (direct integration, general solution of linear problems). Applications.
  Reference: Section 9.1-2 in [1] + Section 1.2-3 in [2]
- **Lesson 32 [27/11/2024]** Long-term solutions in differential equations. Limiting behaviour in growth models. Applications.
  Reference: Section 9.1-2 in [1] + Section 1.2-3 in [2]
- **Lesson 33 [28/11/2024]** Stability in ordinary differential equations. Dependency on initial conditions. Reference: Section 9.1-2 in [1] + Section 1.2-4 in [2]
- **Lesson 34 [02/12/2024]** Equilibria and special solutions in differential equations. Perturbed logistic growth models. Stability depending on parameters. Applications. Reference: Section 9.1-2 in [1] + Section 1.2-4 in [2]
- **Lesson 35 [03/12/2024]** Solow Growth model. Cobb-Douglas function. Functions of several variables (domain and range). Applications. Reference: Section 7.1-2 in [1] + Section 1.2-4 in [2]
- **Lesson 36 [04/12/2024]** Functions of several variables (2D and 3D). Sketching functions in 3D. Applications. Reference: Section 7.1-2 in [1]
- **Lesson 37 [05/12/2024]** Functions of several variables, level curves, partial derivatives. Applications. Reference: Section 7.1-2 in [1]
- **Lesson 38 [09/12/2024]** Interpretation of partial derivatives of functions of several variables. Marginal analysis. Reference: Section 7.1-2 in [1]
- **Lesson 39 [10/12/2024]** Differential notation. Higher order partial derivatives. Chain rule. Exercises. Reference: Section 7.1-2 in [1]
- **Lesson 40 [11/12/2024]** Second order partial derivatives. Differentiation rules and Schwarz Lemma. Applications. Reference: Section 7.1-2 in [1]
- **Lesson 41 [12/12/2024]** Critical points of functions of 2 variables. Comparison with critical points in one variable. Relative minima and maxima on the surface. Saddle points. Examples. Reference: Section 7.1-2 in [1]
- **Lesson 42 [16/12/2024]** The method of least squares. Linear regression in data analysis - connection to optimization problems. Unconstrained optimization. Examples. Reference: Section 7.1-4 in [1]
- **Lesson 43 [17/12/2024]** Constrained optimization of functions in 2 variables. Usage of Lagrange multipliers. Applications. Reference: Section 7.1-5 in [1]

References

[1] Hoffmann, Laurence D., Bradley, Gerald L., Sobecki, David, & Price, Michael. *Applied Calculus: For Business, Economics, and the Social and Life Sciences, 11th Expanded Edition.* McGraw-Hill Education, 2011.

[2] Porretta, Alessio. *Applied Quantitative Analysis.* Lecture notes, 2014-15. See *Materiale didattico* in DIDA, Corso #2714.