CALCULUS
FUNCTIONS OF SEVERAL VARIABLES, ADDITIONAL EXERCISES NO.10

1. Partial derivatives

(1) Evaluate the partial derivatives f_x, f_y at the given point.:

$$f(x,y) = x^2 + 3y;\ (1,-1) \tag{1.1}$$

$$f(x,y) = x^3y - 2(x+y);\ (1,0) \tag{1.2}$$

$$f(x,y) = x + \frac{x}{y-3x};\ (1,1) \tag{1.3}$$

$$f(x,y) = (x-2y)^2 + (y-3x)^2 + 5;\ (0,-1) \tag{1.4}$$

$$f(x,y) = xy\log\frac{y}{x} + \log 2x - 3y;\ (1,1) \tag{1.5}$$

$$f(x,y) = xe^{-2y} + ye^{-x} + xy^2;\ (0,0) \tag{1.6}$$

(2) Find the second order derivatives (including mixed ones):

$$f(x,y) = 5x^4y^3 + 2xy \tag{1.7}$$

$$f(x,y) = \frac{x+1}{y-1} \tag{1.8}$$

$$f(x,y) = e^{x^2y} \tag{1.9}$$

$$f(x,y) = \log x^2 + v^2 \tag{1.10}$$

$$f(x,y) = x^2ye^x \tag{1.11}$$

(3) Use the chain rule to find dz/dt

$$z = 2x + 3y;\ x = t^2,\ y = 5t \tag{1.12}$$

$$z = \frac{3x}{y};\ x = t,\ y = t^2 \tag{1.13}$$

$$z = \frac{x+y}{x-y}; x = t^3 + 1,\ y = 1 - t^2 \tag{1.14}$$

$$z = xy;\ x = e^{2t},\ y = e^{-3t} \tag{1.15}$$

(4) Find the critical points of the given functions and classify each as a relative maximum, minimum, or saddle point.

$$f(x,y) = 5 - x^2 - y^2 \tag{1.16}$$

$$f(x,y) = 2x^2 - 3y^2 \tag{1.17}$$

$$f(x,y) = \frac{16}{x} + \frac{6}{y} + x^2 - 2y^2 \tag{1.18}$$

$$f(x,y) = xy^2 - 6x^2 - 3y^2 \tag{1.19}$$

$$f(x,y) = e^{-(x^2+y^2-6y)} \tag{1.20}$$

$$f(x,y) = \frac{1}{x^2 + y^2 + 3x - 2y + 1} \tag{1.21}$$

$$f(x,y) = \frac{x}{x^2 + y^2 + 4} \tag{1.22}$$

$$f(x,y) = x^2 - 6xy - 2y^3 \tag{1.23}$$

$$f(x,y) = xy \tag{1.24}$$

$$f(x,y) = (x-1)^2 + y^3 - 3y^2 + 9y + 5 \tag{1.25}$$