CALCULUS
FUNCTIONS OF SEVERAL VARIABLES, ADDITIONAL EXERCISES NO.11

1. Linear regression

(1) i) Plot the points (x, y) and use the method of linear regression to find the line $y = mx + b$ that minimizes the distance to the points. ii) predict the value of yk for given value of xk for the solution of the parameters m, b.

$$(0, 1),\ (2, 3),\ (4, 2);\ xk = 3 \tag{1.1}$$

$$(1, 2),\ (2, 4),\ (4, 4),\ (5, 5);\ xk = 3 \tag{1.2}$$

$$(1, 1),\ (2, 2),\ (6, 0);\ xk = 4 \tag{1.3}$$

2. Unconstrained optimization

(1) Find the relative maximum and minimum values of the function $f(x, y) = e^{xy}$.

(2) Find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$.

(3) Find the minimum value of the function $f(x, y) = 2x^2 + y^2 + 2xy + 4x + 2y + 7$.

3. Constrained optimization

(1) Find the maximum value of $f(x, y) = xy$ subject to the contraint $x + y = 1$.

(2) Find the minimum value of $f(x, y) = x^2 + y^2$ subject to the contraint $xy = 1$.

(3) Find the maximum and minimum values of $f(x, y) = xy$ subject to the contraint $x^2+y^2 = 1$.