ES. 2 ESERCIZI PROVA 2

$N=60$

$y^{\prime}Ay = 22.242 \qquad (X_2^T A X_2)^{-1} = \begin{bmatrix} 0.023 & 0.001 \\ 0.001 & 0.020 \end{bmatrix}$

$X_2^T A y = \begin{bmatrix} 22.588 \\ 13.184 \end{bmatrix}$

(1) Da Frish-Waugh,

$b_2 = (X_2^T M_1 X_2)^{-1} X_2^T M_1 y$

dove $M_1 = I - X_1(X_1^T X_1)^{-1} X_1^T$

In questo caso, $X_1 = \mathbf{i}$, quindi

$M_1 = I - i(i^T i)^{-1} i^T = I - \frac{1}{n} i i^T = A$

Ne segue che

$b_2 = \begin{bmatrix} 0.023 & 0.001 \\ 0.001 & 0.020 \end{bmatrix} \cdot \begin{bmatrix} 22.588 \\ 13.184 \end{bmatrix} = \begin{pmatrix} 0.532708 \\ 0.286268 \end{pmatrix}$

(2) $e^T e = y^T y - y^T X b = y^T y - b^T X^T X b$

ma anche $A \cdot e = e$ perchè $\Sigma e_i = 0$

~~$e^T A e = e^T e = y^T A y$~~

Quindi usiamo la formula $TSS = y^T A y = RSS + ESS$

$= e^T e + \cancel{y^T A X} \; b^T X^T A X b$

$= e^T e + y^T A X b$

$= e^T e + y^T A X_2 b_2$

Da ciò segue:

$e^T e = 22.242 - [22.588 \; ; 13.184] \cdot \begin{bmatrix} 0.532708 \\ 0.286268 \end{bmatrix}$

$= 22.242 - 15.806966 = 6.43503$

$\Rightarrow \quad s^2 = \frac{6.43503}{57} = \; ?$

(3) $R^2 = 1 - \frac{e^Te}{y^TAy} = 0.71068$

(4) ~~L'ipotesi è~~ Vogliamo verificare la significatività del modello nel suo complesso.

La statistica F in questo caso si scrive

$$F = \frac{R^2/(k-1)}{(1-R^2)/(N-k)} = \frac{0.71068/2}{0.289319/57} = 70.007$$

$\boxed{\text{ESERCIZIO 1 ES. PROVA 3}}$

$$\hat{Y} = 4 + 0.4\, X_{1i} + 0.9\, X_{2i}$$

$R^2 = 2/15 \qquad e^Te = 520, \quad n = 29 \qquad X^TX = \begin{pmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{pmatrix}$

(a) $H_0: \; \beta_1 + \beta_2 = 1 \qquad R = [0 \;\; 1 \;\; 1] \quad q = 1$

$$Rb - q = 0.4 + 0.9 - 1 = 0.3$$

$$R(X^TX)^{-1}R^T = [0 \;\; 1 \;\; 1] \begin{pmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= (1 \;\; 1) \begin{pmatrix} 50 & 10 \\ 10 & 80 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} =$$

(N.B. non è sempre vero che l'inversa è così semplice questo è un caso particolare)

$$= (1 \;\; 1) \frac{1}{4000 - 100} \begin{pmatrix} 80 & -10 \\ -10 & 50 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{390} (8 - 1 - 1 + 5) = \frac{11}{390}$$

$$F = \frac{(Rb-q)^T \left[R(X^TX)^{-1}R^T\right]^{-1} (Rb-q)}{s^2} =$$

$$= \frac{(0.3)^2 \cdot \frac{390}{11}}{520/26} = \frac{0.09 \times \frac{390}{11}}{20} = 0.1595$$

p-value (right) = 0.692879

(b) Regressione vincolata con vincolo $Rb = 0$

$$R = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$$

$$b^* = b - (X^TX)^{-1} R^T \left[R(X^TX)^{-1} R^T \right]^{-1} Rb$$

$$= \begin{pmatrix} 4 \\ 0.4 \\ 0.9 \end{pmatrix} - \begin{pmatrix} 1/99 & 0 & 0 \\ 0 & 8/390 & -1/390 \\ 0 & -1/390 & 5/390 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \underbrace{\frac{390}{8}}_{[R(X^TX)^{-1}R^T]^{-1}} \cdot 0.4$$

$$= \begin{pmatrix} 4 \\ 0.4 \\ 0.9 \end{pmatrix} - \begin{pmatrix} 0 \\ 8/\cancel{390} \\ -1/\cancel{390} \end{pmatrix} \cdot \frac{\cancel{390}}{8} \cdot 0.4$$

$$= \begin{pmatrix} 4 \\ 0.4 - 0.4 \\ 0.9 + \frac{0.4}{8} \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0.95 \end{pmatrix}$$

$$e^{*T}e^* - e^Te = (Rb - q)^T \left[R(X^TX)^{-1} R^T \right]^{-1} (Rb - q)$$

$$= 0.4 \cdot \frac{390}{8} \cdot 0.4 = 7.8$$

$$F = \frac{(e^{*T}e^* - e^Te)/1}{e^Te/(N-K)} = \frac{7.8}{520/26} = \frac{7.8}{20} = 0.39$$

$$\text{p-value} = 0.537741$$

ES. 3 ESERCIZI PROVA 3

(a) $$\hat\beta = (X^TX)^{-1}X^Ty = \begin{pmatrix} 29 & 110 \\ 110 & 742 \end{pmatrix}^{-1} \begin{pmatrix} 40.63 \\ 103.29 \end{pmatrix} = \beta$$

$$= \frac{1}{9418}\begin{pmatrix} 742 & -110 \\ -110 & 29 \end{pmatrix}\begin{pmatrix} 40.63 \\ 103.29 \end{pmatrix} =$$

$$= \begin{pmatrix} 18785.56/9418 \\ -1473,89/9418 \end{pmatrix} = \begin{pmatrix} 1,9946 \\ -0.1565 \end{pmatrix}$$

$$\hat\sigma^2 = \frac{e^Te}{N} = \frac{e^Te}{29}$$

$$e^Te = 82.87 - y^TX\hat\beta = 82.87 - (40.63;\ 103.29)\begin{pmatrix} 1.9946 \\ -0.1565 \end{pmatrix}$$

$$= 82.87 - 64.8757 = 17.994$$

$$\hat\sigma^2 = \frac{17.994}{29} = 0.6894$$

(b) Test di Breush-Pagan:

$$LM = \frac{1}{2} v'AZ(Z^TAZ)^{-1}Z^TAv \quad \text{con} \quad Z = X$$

$$\sum v_i = \sum \frac{e_i^2}{\hat\sigma^2} = N = i^Tv = 29$$

per cui $$X^TAv = X^Tv - \frac{X^Tii^Tv}{N}$$

$$= \begin{pmatrix} 29 \\ 196.04 \end{pmatrix} - \frac{1}{\cancel{29}}\begin{pmatrix} 29 \\ 110 \end{pmatrix} \cdot \cancel{29}$$

$$= \begin{pmatrix} 0 \\ 86,04 \end{pmatrix}$$

$$X^TAX = \begin{pmatrix} 0 & 0 \\ 0 & 742 - (110)^2/29 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 324.759 \end{pmatrix}$$

$$LM = \frac{1}{2}(86.04)^2 \frac{1}{324.759} = \cancel{11.1380}\ 11.3975$$

p-value 0.000735