4 ELEMENTARY GAMES. EQUILIBRIUM VS PARETO STRATEGIES

1 MAIN TYPES OF PURE ONE-STAGE STRATEGIES. 2×2 PAYOFF MATRICES

Dominant and equilibrium strategies

Consider first F4.1. The figure shows the different types of **pure strategies** in the *simplest framework* of **one-stage games** with 2 players: Row (**Ro**) and Column (**Co**), and 2 choices for each player: Top (**T**), Bottom (**B**) and Left (**L**), Right (**R**), respectively. This simplest framework can be represented in the form of a **2×2 payoff matrix**. The numbers in the matrix cells represent the players' 'utility' (payoff) levels: e.g. in the top left cell of the top matrix the numbers **1,2** mean that that particular strategy gives **Ro** a 'utility' of 1 and **Co** a utility of 2.

The **top matrix** represents the case where there exists a **Dominant equilibrium strategy**, DS, which we identify by the colour **blue**. if **Ro** chooses **T** then **Co** prefers **L**, but if **Ro** chooses **B** then **Co** still prefers **L**. On the other hand, if **Co** chooses **L** then **Ro** prefers **B**, but if **Co** chooses **R** then **Ro** still prefers **B**. We say that the strategy **B-L = (2,1)** is dominant in this game, because the preferred choice of **Ro** is always **B** whatever the choice by **Co**, and the preferred choice of **Co** is always **L** whatever the choice by **Ro**. Generalizing to **many players and many choices**, a dominant strategy is one where the preferred choice of each player is the *same* for whatever choices are made by **all the others**. By construction a dominant strategy is also an **equilibrium strategy**, ES: *whatever* choices are made by all other players, *no player would like to change his own choice*. A dominant strategy may, and mostly does, not exist, but if it exists at all, then by construction it must be **unique**.

Pareto strategies

Now **Pareto strategies**, PS. The two left cells in the top matrix are indeed **Pareto strategies**, which we identify by the **orange** colour: in each of these cells the individual payoffs are such that there are no other cells (strategies) where at least one player gains and no player looses, i.e there are no other strategies (cells) **Pareto superior** to it. Conversely, a strategy is **not** PS when there are other strategies that are Pareto superior to it. A number of facts are worth highlighting in this simple game. **1)** there are **more than one** PS, **2)** there is a (unique) DS**, which is also** PS, **3)** the DS **is also, by definition, a Nash strategy,** NS (defined in the next matrix). When a game has equilibrium strategies that are also PS we may say that it doesn't present a **coordination failure** (see § below).

Nash strategies

The **second matrix** represents the case of **Nash strategies**, NS, identified by the colour **green**. It actually exhibits **two** NSs. The **T-L** cell **(2,1)** is such that if **Ro** chooses **T**, then **Co** has an incentive to stay in **L**, and if **Co** chooses **L** then Ro has an incentive to stay in **T**. This is the meaning of a Nash strategy. It is an **equilibrium strategy** in the sense that, **given the choices made by all players, forming a particular strategy**, no player has an incentive to change his own choice, which means that the strategy has a degree - however weak - of equilibrium. Instead at **T-R**, **Ro** has an incentive to move to **B**, because if he does so **Co** has an incentive to stay at **R**. Similarily, at **B-L**, **Co** has an incentive to move to **R** because if he does so **Ro** will stay at **B**. We can see that **T-L** is not the only NS. The **B-R** cell **(1,2)** is also a NS, because it possesses the same property of **T-L**. Two further facts in this particular game are worth mentioning: 1) there is **no** DS, 2) the **two equilibrium** NSs are both PSs. Therefore this game is not a case of *coordination failure* (see next §).

Notice two general properties of NSs:

1) a DS is also, by construction, a NS, but the reverse is not true: a NS needs not be, and usually is not, a DS.

2) NSs, like DSs, may **not exist** (see the *third matrix* below), but, contrary to DSs, if they exist they **need not be unique** (as in this case).

Strategic coordination failures

The **third matrix** introduces the concept of **Strategic Coordination Failure**, SCF. Although there exist PSs **B-L (1,0)** and **B-R (-1,3)**, neither of them is an *equilibrium* (Dominant or Nash). This is a game **without equilibrium strategies**. Therefore, in particular, it has no drive towards the efficient PSs. Suppose they are at **T-L (0,0)**. Why don't they go to **B-L (1,0)** where **Ro** gains and **Co** doesn't loose? Because **B-L** is not an equilibrium: if **Ro** chose **B** then **Co** would have an incentive to change from **L** to **R**. Alternatively, suppose they are at **T-R (0,-1)**. Why don't they go again to **B-L (1,0)** where they would *both gain*? Because **B-L** is not an equilibrium. This game is an example of SCF in the **weak sense**: **no existing** PS **is an equilibrium**.

The game has also the particular feature of not containing **any equilibrium strategy**.

2 THE STANDARD CASE OF STRATEGIC COORDINATION FAILURE: THE PRISONERS' GAME

Here we discuss the standard case of full SCF. The **fourth matrix** represents a case which has a special place in the theory of the *coordination failure of individual actions*, and is known as the *Prisoners' game,* PG, or *Prisoners' dilemma,* PD. It shows - in the simplest of formats - what theorists mean when they talk of *strategic coordination failure* in its full **strong sense**. We can check that the **C-C** (**-3,-3**) cell is a DS, and therefore also a NS. But we can also check that it is **not a** PS because there is another cell **NC-NC (-1,-1)** which is *Pareto superior* to it. This latter cell is a PS, and so are the remaining two cells, because also **C-NC (0,-6)** and **NC-C (-6,0)** have no Pareto superior alternatives. Although these three strategies are all PS, the **NC-NC (-1,-1)** has the *special status* of being the strategy that makes the DS **C-C (-3,-3)** into a *non Pareto* one. In other words it is the existence of the strategy **NC-NC (-1,-1)** that causes the DS **C-C** (**-3,-3**) to be inefficient (non PS). The *Prisoners' game* is a case of **full strategic coordination failure** in the precise sense that there are equlibrium strategies (actually only one, which is also dominant) that drive players to stay away from PSs, which are non-equilibrium.

Now, the state of things represented by this type of game raises a *truly fundamental question*: how is it possible for an ES, dominant or Nash, where no player would like to change his choice because he is happy with it, not to be also a PS? The matrix shows that the two players have an incentive to choose **C-C**, where they get **(-3,-3)**, while they have *no incentive* to move from **C-C** to **NC-NC**, where they get (**-1,-1**), in spite of the fact that the latter is *Pareto superior* to the former because by doing so they would both gain. They also have no incentive to move to **NC-C**, nor to **C-NC**, but these alternative strategies are *not Pareto superior* to the DS **C-C**.

How are strategic coordination failures possible? A wider critical reflection

Faced with a state of things like this we must ask ourselves: **what is it that makes it possible for the players to have no incentive to move to where both would gain?** The question of SCFs raised by pure strategies of the PG type is of **great social importance**, and also a difficult one, because it requires exiting the narrow field of economics and entering into those of **philosophy**, **psychology, sociology** and **anthropology**. Although we are not qualified to enter into such fields in a systematic way, in Chapter 6 on the *Nash-Lindahl* theory we shall nevertheless develop some concepts relevant for such an investigation, in connection with the special topic of the *failure of free cooperation in the pursuit of shared/public interests*, a topic having a central place in the study of the **public economy**.

Here we use the *Prisoners game* of F4.1 to show how standard concepts of economics and game theory help in bringing into focus the basic cause of strategic coordination failures. In order for the two players in the *Prisoners game* to move from **C-C (-3.-3)** to **NC-NC (-1,-1)** they need to **negotiate**, but in order for them to negotiate two conditions are needed:

1) the players must be able to *interact with each other.* This condition is **trivial**. Clearly if they can't interact they also can't negotiate. But the *possibility* of negotiating is *not sufficient* for driving them to actually negotiate.

2) in order to have an *incentive to negotiate*, each player must be *confi-*

dent that if he **negotiates a joint action agreement** the other player will **stick to it**. This is known as the condition of **The Possibility of Binding Agreements**, PBA.

Notice that **negotiating** is not the same thing as **cooperating**, because cooperating means sharing the costs and benefits of a **collective action**, while negotiating doesn't necessarily imply such sharing.

The assumption underlying the identification of the *non-Pareto* strategy **C-C** as an *equilibrium* one is not that the players cannot talk to each other. Surely, if they cannot communicate they cannot negotiate any kind of agreement. However, even if they can communicate, each player simply cannot trust the other to stick to whatever agreement were reached. As we see in the matrix, if they do make a bargain for the **NC-NC** Pareto superior strategy, player **Ro** finds that, if player **Co** does choose **NC** as agreed, his own best choice would not longer be **NC** but **C** because by doing so his payoff would increase from **-1** to **0**. In other words, if **Co** chooses **NC** then **Ro** has an incentive to move from **NC** to **C:** he has an **incentive to disattend the agreement**. The same thing happens to player **Co**. If **Ro** does choose **NC** as agreed, then **Co** would have an incentive to move from **NC** to **C** because again by doing so his payoff would increase from **-1** to **0**. In so far as the players think that they are under **no obligation to obey agreements**, the PS **(-1.-1)** cannot be an equilibrium. On the other hand, if players knew for sure (no matter for what reasons) that if they negotiated an agreement, then both players would stick to it, then the **(-3,-3)** strategy would cease to be an equilibrium because both players would have an incentive to move to the **(-1,-1)** PS, which would become the *new - and only - equilibrium strategy* of the game.

Notice that under normal institutional arrangements, essentially laws and judiciary, binding agreements, i.e. **court-enforceable contracts**, are in principle possible, though the degree to which they may be actually binding is quite variable, because it depends on the efficiency of the judicial system. However, such agreements become *less and less feasible as the number of potential partners increases*: no binding contracts can be signed among thousands of people.

Richiamo al seminario 29.10.21 di **Bruno Chiarini** e relativa discussione (Castellucci, Coromaldi, D'Amato, Pallante, Pecorella). Sono stati sollevati, tra gli altri, i seguenti tre punti: 1) Natura dell'analisi strategica: non basta imparare la **forma** dell'interazione strategica (problem solving), bisogna anche interrogarsi sulle **ragioni** dei comportamenti strategici. 2) Relazione/distinzione tra **comportamenti strategici** e **comportamenti morali**. 3) Per un'analisi strategica dei fatti sociali bisogna conoscere la teoria dei giochi, almeno un poco, e la teoria dei giochi non si conosce veramente fino a quando non si capisce che **i giochi non richiedono l'esistenza di soggetti giocatori** (giochi evolutivi, the selfish gene, ecc.).

3 GENERALIZATION TO ARBITRARY NUMBERS OF CHOCES AND PLAYERS

The game-theoretic concepts reviewed above within the simplest format 2×2 are all perfectly general. They can be extended *pari passu* to games where

1) the 2 players have each any number of choices, and
2) any number of players have each any number of choices.

Table 4.A-D replicates the information collected in F4.1, but in the more general situation in which both **Ro** and **Co** have many choices (say, **Ro** 15 and **Co** 20). The Table with its captions is selfexplanatory. The scenario of 2 players having each *many choices* provides a better intuition than the 2×2 case of the different strategy types, in particular it gives a better view of what is meant by *Dominant strategies*, *unique and multiple Nash strategies*, *no Nash strategies*, and **best reply functions**, BRF. For simplicity, **no pay-offs** are inserted in the Table, which therefore doesn't provide information on which of the represented strategies are PS.

Scenarios with more than 2 players cannot be easily represented visually, but attempts to do so with 3 players are left as an exercise to the reader.

4 LOGICAL RELATIONSHIP BETWEEN STATEGY TYPES

F4.2 is a graphic device for highlighting the logical relations between the various types of games and (pure) strategies. For simplicity we confine our argument to **pure one-stage games**, and to games having a **finite number of strategies**.

It is easy to prove that, given any such set of games, **every game in the set must necessarily possess some** PS.

Proof. Take any game in the set, and select an arbitrary strategy $\mathbf{S}_0$ out of its finite number of strategies. Suppose no other strategy benefits some players without damaging the others. Then $\mathbf{S}_0$ is a PS. Suppose instead that there is some other strategy $\mathbf{S}_1$ where some players gain and no player looses. Then $\mathbf{S}_0$ is not a PS, and we move on to this next one. If this is a PS, then there we are. If it is not, then we move to the next *Pareto superior* one $\mathbf{S}_2$. If the number of strategies is finite and we continue this process we must necessarily reach a strategy $\mathbf{S}_n$ with no Pareto improving alternatives, because the *order* 'Pareto superior' is *transitive* ■.

In the Figure the orange frame is this universal set U of one-stage games with finite strategies. Its orange colour means that all games in the set must contain one or more PSs, as just proved. The subset in *light green* consists of all games in U possessing *multiple* NSs. It may of course be empty. Another subset, in *dark green*, consists of all games in U possessing a *unique* NS. A further subset, contained in the dark green one and coloured *in blue*, consists of all games in U with a DS. A DS is also a NS, and if it exists it is unique. This means that if a DS exists, there cannot be other DSs. Therefore the blue area must lie inside the dark green one. Of course, any of these subsets *may* *be empty*, but if they are not, all of their games must in any case case contain some PSs, which may or may not coincide with the equilibrium strategies.