7. PUBLIC CONSUMPTION GOODS: THE SAMUELSON-LINDAHL MIXED ECONOMY

APPENDIX 7.A - COST SHARES AND INDIVIDUAL BUDGET CONSTRAINTS UNDER PROPORTIONALITY AND PROGRESSIVITY

In *Section 5* (Subsection *A progressive income taxation*) of this Chapter, we pick up **Stiglitz**'s statement (**2015**, p. 233 bottom) that with a *progressive* income taxation the taxpayer's cost share would rise *more than in proportion* with gross income. We may restate the claim as follows

Let y_1, y_2 *be the gross incomes of a pair of taxpayers, and* s_1, s_2 *be their respective cost shares. Then under proportionality*

7.A1 $$y_2 = \alpha y_1 \Rightarrow s_2 = \alpha s_1, \forall \alpha$$

while under progressivity

7.A2 $$\begin{gathered} \alpha > 1 \Rightarrow s_2 > \alpha s_1 \\ \alpha < 1 \Rightarrow s_2 < \alpha s_1 \end{gathered}$$

The problem with this statement is that under a *progressive tax code* the *derivation of individual cost shares* is not as simple as it is with a proportional one, because, whereas there is a *unique 'type' of proportional tax code*, yielding a unique tax revenue for every GDP level (independent of gross income distribution), the *progressive tax code* (continuous or brackets progressivity) may take *many different shapes*, which implies that *total tax revenues* depend *also* on *gross income distribution*, and *marginal tax revenues* depend on how the tax code is adjusted to increase revenue.

However, in spite of all these special properties of progressivity, the **proof** of the general statement is **tautological**:

Let y_1 and s_1 be the *gross income* and *cost share* of a *poor taxpayer*, and let $y_2 = 2y_1$ and s_2 be the gross income and cost share of a *rich taxpayer*. By eqs (7.30-34) we know that $s_2 = 2s_1$ and *tax proportionality* are by definition *equivalent properties* of the tax code. Therefore, under *any progressive tax code* the cost share of the rich taxpayer must be *more than proportionally higher* than that of the poor one:

$$s_2 > 2s_1$$

because if it were $s_2 = 2s_1$ the tax code would by definition be proportional ■.

This fact may be represented graphically in **F7.4b**. Eq (7.33) tells us that under proportionality each taxpayer's cost share s is equal to

7.A3 $$s = \frac{y}{Q}$$

Therefore, by eq (7.36), which we repeat here for convenience, the **IB**s (individual budget constraints) of all taxpayers intercept the *G*-axis at the same point $\bar{G}$ which is the amount of *G* obtained when all the economy's resources are put into *G*

7.A4 $$\begin{aligned} & c = y - \frac{y}{Q} h(G) \\ & = 0 \to \frac{y}{Q} h(G) = y \\ & \to h(G) = Q \\ & \to G \,\big|\, h(G) = Q \to G = \bar{G} \end{aligned}$$

Now suppose the cost share of the *poor* taxpayer is $s_1 = \frac{y_1}{Q}$, so that by (7.A4) his **IB** intercepts the G-axis at point $\bar{G}$. By the previous general statement, under *progressivity* the cost share of the *rich* taxpayer must be *more than proportionally higher* than that of the poor one:

$$s_2 > 2s_1$$

If so, the shape of the *rich* taxpayer's **IB** would be something like the *dotted blue curve*, *steeper* than the curve corresponding to proportionality at every level of G, and intercepting the G-axis at some point to the *left* of $\bar{G}$.

To see this take the *rich* taxpayer with $y_2 = 2y_1$ and cost share s_2. This yields

7.A5
$$\begin{aligned} c_2 &= 2y_1 - s_2 h(G) \\ &= 0 \rightarrow s_2 h(G) = 2y_1 \\ &\rightarrow h(G) = \frac{2y_1}{s_2} \end{aligned}$$

Proportionality implies $s_2 = 2s_1 = \frac{2y_1}{Q}$, which yields

7.A6
$$\begin{aligned} h(G) &= \frac{2y_1}{s_2} = \frac{2y_1}{2s_1} = \frac{2y_1}{2y_1/Q} = Q \\ &\rightarrow G = \bar{G} \end{aligned}$$

But *progressivity* implies $s_2 > 2s_1 = \frac{2y_1}{Q}$, which yields

7.A7
$$\begin{aligned} h(G) &= \frac{2y_1}{s_2} < \frac{2y_1}{2s_1} = \frac{2y_1}{2y_1/Q} = Q \\ &\rightarrow G < \bar{G} \end{aligned}$$

■