

CHAPTER NOTES

Cassady (1967) provides a panoramic view of real-world auction institutions, past and present, that is both colorful and insightful. Second-price auctions are also referred to as Vickrey auctions. It was commonly believed that the second-price auction was a purely theoretical construct proposed by Vickrey (1961) as a sealed-bid counterpart of the open ascending-price format. Lucking-Reiley (2000) points out, however, that many stamp auctions have been conducted under second-price rules since the nineteenth century. In this context, they originated as a means of allowing bidders who could not be present at the actual, open ascending-price auction, to submit bids by mail.

Many Internet auction websites have adopted what are effectively second-price rules. For instance, at the popular auction site eBay, goods are sold by means of what appears to be an English auction. Bidders can, however, make use of *proxy bidding* wherein they employ a computer program, sometimes called an “elf,” to bid on their behalf. The computer program raises rival bids by the minimum increment as long as it is below some limit set by the bidder. It is easy to see that this is effectively a second-price auction in which the amount bid is the same as the limit set by a bidder. Again, see the paper by Lucking-Reiley (2000).

There have been many excellent surveys of auction theory. These vary in both content and emphasis, reflecting, as does this book, the interests of the authors and the state of theory at the time they were written. We mention some of the prominent ones. Milgrom (1985) gives a cogent account of the theory of symmetric single-object auctions and shows how the theory may be extended to situations in which there are multiple objects but each bidder wants at most only one. McAfee and McMillan (1987a) also concentrate on the symmetric single object case but emphasize many extensions and applications of the theory. Milgrom (1987) attempts to answer the question of when auctions are appropriate and why they are so prevalent. He places auctions in the larger context of general institutions of economic exchange and evaluates their performance in different environments. The survey by Wilson (1992)—again largely concerning single-object auctions—offers a wide range of examples in which equilibrium bidding strategies can be computed in closed form. Technical aspects of the symmetric private values model are carefully treated by Matthews (1995). Klemperer (2003) emphasizes that many aspects of auction theory have interesting applications to other branches of economic theory.

There is now a substantial and rapidly growing literature concerning empirical work on auctions and the development of associated econometric tools. A detailed discussion would take us too far afield, so we only mention a representative sample of the work. The papers by Hendricks, Porter, and Wilson (1994), Hendricks and Paarsch (1995), and Laffont, Ossard, and Vuong (1995) serve as useful introductions to the area.

Auctions have also been the subject of a now large body of work in exper-

PART I

Single-Object Auctions

1.5 WHAT IS AN AUCTION?

A wide variety of selling institutions fall under the rubric of "an auction." There are hybrid Dutch-English auctions in which the price is lowered until there is an interested bidder and then other bidders are allowed to outbid this amount. There are what may be called "deadline" auctions—commonly used by Internet auction sites—in which the person with the highest standing bid before a fixed stopping time—say, noon on Sunday—is declared the winner. There are "candle" auctions, with a random stopping time, in which the person with the highest bid standing before the wick of a candle burns out wins. One may conceive of a third-price auction or an auction in which the winner pays the average of all the other bids. The range of possibilities is rather wide and even more so when sales of multiple objects are considered. Without adopting a rigid view as to what may be called an auction and what may not, we seek to identify some important features that such institutions have in common.

A common aspect of auction-like institutions is that they elicit information, in the form of bids, from potential buyers regarding their willingness to pay, and the outcome—that is, who wins what and pays how much—is determined solely on the basis of the received information. An implication of this is that auctions are *universal* in the sense that they may be used to sell any good. A valuable piece of art and a secondhand car can both be sold by means of an English auction under the same basic set of rules. Alternatively, both can be sold by means of a first-price sealed-bid auction. The auction form does not depend on any details specific to the item at hand.

A second important aspect of auction-like institutions is that they are *anonymous*. By this we mean that the identities of the bidders play no role in determining who wins the object and who pays how much. So if bidder 1 wins with a bid of b_1 and pays some amount p , then keeping all other bids fixed, if some other bidder—say, bidder 2—were to bid b_1 and bidder 1 were to bid b_2 , then bidder 2 would win and pay p also. Every bidder other than 1 and 2—say, bidder 3—is completely unaffected if bidders 1 and 2 exchange their bids in the manner just described.

In later chapters we place auctions in a larger class of institutions, called *mechanisms*. Mechanisms differ from auctions in that they are not necessarily universal or anonymous.

1.6 OUTLINE OF PART I

Part I presents situations where a *single* indivisible object is sold to one of many potential buyers. Chapter 2 introduces the basic theory of auctions with *private values*, beginning with the case where these are symmetrically and independently distributed. It derives equilibrium strategies in first- and second-price auctions and compares their performance. Chapter 3 concerns the benchmark "revenue equivalence principle," in its simplest form. Chapter 4 is then concerned with amendments to the revenue equivalence principle necessitated by

various extensions to the basic model including asymmetries, risk aversion, and budget constraints. Chapter 5 examines the problem of mechanism design with private values, considering both optimal and efficient mechanisms.

Chapter 6 introduces the model of auctions with *interdependent values* and affiliated signals, again deriving equilibrium strategies in the common auction forms. The main goal here is to rank the common auction forms in terms of the expected selling price. Chapter 7 derives the "revenue ranking principle" and explores some of its implications. Chapter 8 again explores some extensions and qualifications to the basic model necessitated by asymmetries among bidders. Chapter 9 considers the problem of allocating efficiently when bidders are asymmetric, focusing on the efficiency properties of the English auction. Chapter 10 studies mechanism design with interdependent values, again considering both optimal and efficient mechanisms.

Finally, Chapter 11 is concerned with collusive behavior among bidders and the formation of bidding cartels. The models here are with private values.

Figure 1.2 shows the organization of Part I, emphasizing the more or less parallel development of the subject matter in the private value and the interdependent value cases.

Part II of the book concerns *multiple-object* auctions. Chapter 12 serves as an introduction to this part.

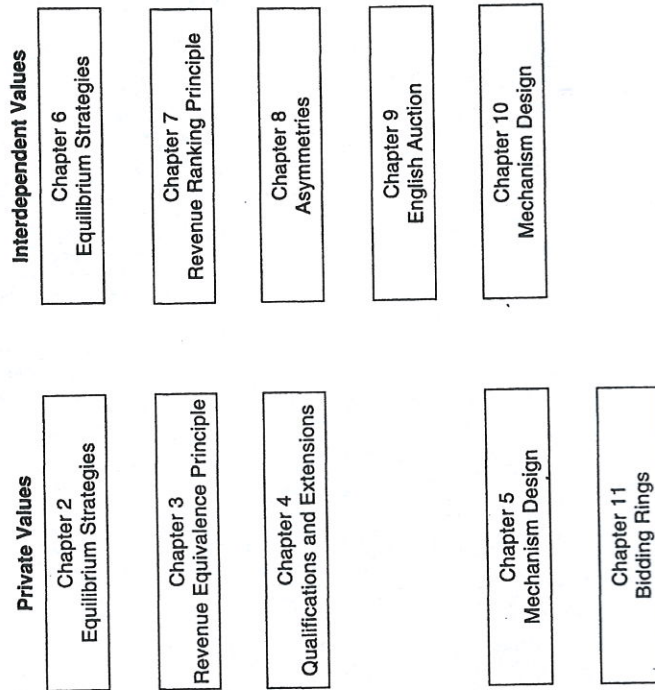


FIGURE 1.2 Outline of Part I.

1.3 EQUIVALENT AUCTIONS

Four auction formats have been outlined here. Two were open auctions—the English and the Dutch—and two were sealed-bid auctions—the first- and second-price formats. These seem very different institutions, and certainly, they differ in the way that they are implemented in the real world. Open auctions require that the bidders collect in the same place, whereas sealed bids may be submitted by mail, so a bidder may observe the behavior of other bidders in one format and not in another. For rational decision makers, however, some of these differences are superficial.

First, observe that the Dutch open descending price auction is strategically equivalent to the first-price sealed-bid auction.² In a first-price sealed-bid auction, a bidder's strategy maps his private information into a bid. Although the Dutch auction is conducted in the open, it offers no useful information to bidders. The only information that is available is that some bidder has agreed to buy at the current price, but that causes the auction to end. Bidding a certain amount in a first-price sealed-bid auction is equivalent to offering to buy at that amount in a Dutch auction, provided the item is still available. For every strategy in a first-price auction there is an equivalent strategy in the Dutch auction and vice versa.

Second, when values are private, the English open ascending auction is also equivalent to the second-price sealed-bid auction, but in a weaker sense than noted earlier. The English auction offers information about when other bidders drop out, and by observing this, it may be possible to infer something about their privately known information. With private values, however, this information is of no use. In an English auction, it clearly cannot be optimal to stay in after the price exceeds the value—which can only cause a loss—or to drop out before the price reaches the value—thus forgoing potential gains. Likewise, in a second-price auction it is best to bid the value (this is discussed in more detail later). Thus, with private values, the optimal strategy in both is to bid up to or stay in until the value is reached.

This equivalence between the English and second-price auctions is *weak* in two senses. First, the two auctions are not strategically equivalent. Second, and more important, the optimal strategies in the two are the same only if values are private. With interdependent values, the information available to others is relevant to a particular bidder's evaluation of the worth of the object. Seeing some other bidder drop out early may bring bad news that may cause a bidder to reduce his own estimate of the object's value. Thus, if values are interdependent, the two auctions need not be equivalent from the perspective of the bidders. Figure 1.1 depicts the equivalences between the open and sealed-bid formats introduced here.

²Two games are strategically equivalent if they have the same normal form except for duplicate strategies. Roughly this means that for every strategy in one game, a player has a strategy in

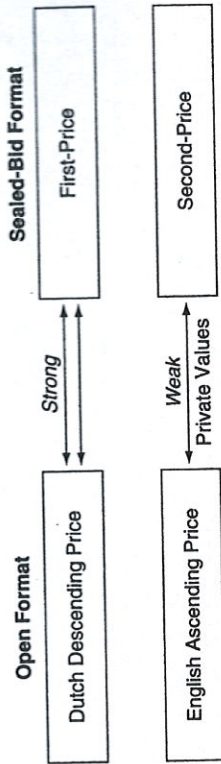


FIGURE 1.1 Equivalence of open and sealed-bid formats.

1.4 REVENUE VERSUS EFFICIENCY

The main questions that guide auction theory involve a comparison of the performance of different auction formats as economic institutions. These are evaluated on two grounds, and the relevance of one or the other criterion depends on the context. From the perspective of the seller, a natural yardstick in comparing different auction forms is the *revenue*, or the expected selling price, that they fetch. From the perspective of society as a whole, however, *efficiency*—that the object end up in the hands of the person who values it the most *ex post*—may be more important. This is especially true when the auction concerns the sale of a publicly held asset to the private sector, so the seller, in this case a government, may want to choose a format that ensures that the object is allocated efficiently, even if the revenue from some other, inefficient format is higher.

But should efficiency be a criterion at all? Why can we not rely on “the market” to reallocate the object efficiently, even if the auction does not do so? After all, if there are unrealized gains from trade, the person who wins the auction can resell the object to someone who attaches a higher value. We will argue that this argument is suspect for many reasons. First, postauction transactions will typically involve a small number of agents, especially in the context of privatization, and so will result in some bargaining about the resale price. Such bargaining is unlikely to result in efficient outcomes, since it will typically take place under conditions of incomplete information. Second, resale may involve significant transaction costs, so it may not take place even when it should. In Chapter 4 we take up the question of whether resale will lead to efficiency more formally. In short, we find that even in the best circumstances—with no transaction costs or bargaining delays—the answer is no. Resale cannot guarantee efficiency, so a policy maker interested in achieving efficiency would do well to choose the auction format carefully.

Of course, revenue and efficiency are not the only criteria that should guide the choice of an auction format. The common auction forms discussed thus far have the virtue of simplicity—the rules of the auction are transparent—and this may be an important practical consideration. Another important factor may be the potential for collusion among bidders. As we will see later, auction formats

In what follows, an auction will be understood to include the process of procurement via competitive bidding. Of course, in this case it is the person bidding lowest who wins the contract.

Why are auctions and competitive bidding so prevalent? Are there situations to which an auction is particularly suited as a selling mechanism as opposed to, say, a fixed, posted price? From the point of view of the bidders, what are good bidding strategies? From the point of view of the sellers, are particular forms of auctions likely to bring greater revenues than others? These and other questions form the subject matter of this book.

1.1 SOME COMMON AUCTION FORMS

The open ascending price or *English* auction is the oldest and perhaps most prevalent auction form. The word *auction* itself is derived from the Latin *augere*, which means "to increase" (or "augment"), via the participle *auctus* ("increasing"). In one variant of the English auction, the sale is conducted by an auctioneer who begins by calling out a low price and raises it, typically in small increments, as long as there are at least two interested bidders. The auction stops when there is only one interested bidder. One way to formally model the underlying game is to postulate that the price rises continuously and each bidder indicates an interest in purchasing at the current price in a manner apparent to all by, say, raising a hand. Once a bidder finds the price to be too high, he signals that he is no longer interested by lowering his hand. The auction ends when only a single bidder is still interested. This bidder wins the object and pays the auctioneer an amount equal to the price at which the second-last bidder dropped out.

The *Dutch* auction is the open descending price counterpart of the English auction. It is not commonly used in practice but is of some conceptual interest. Here, the auctioneer begins by calling out a price high enough so that presumably no bidder is interested in buying the object at that price. This price is gradually lowered until some bidder indicates her interest. The object is then sold to this bidder at the given price.

The sealed-bid *first-price* auction is another common form. Its workings are rather straightforward: Bidders submit bids in sealed envelopes; the person submitting the highest bid wins the object and pays what he bid.

Finally, there is the sealed-bid *second-price* auction. As its name suggests, once again bidders submit bids in sealed envelopes; the person submitting the highest bid wins the object but pays not what he bid but the second-highest bid.

1.2 VALUATIONS

Auctions are used precisely because the seller is unsure about the values that bidders attach to the object being sold—the maximum amount each bidder is willing to pay. If the seller knew the values precisely, he could just offer the object to the bidder with the highest value at or just below what this bidder is willing to pay. The uncertainty regarding values facing both sellers and buyers

If each bidder knows the value of the object to himself at the time of bidding, the situation is called one of privately known values or *private values*. Implicit in this situation is that no bidder knows with certainty the values attached by other bidders and knowledge of other bidders' values would not affect how much the object is worth to a particular bidder. The assumption of private values is most plausible when the value of the object to a bidder is derived from its consumption or use alone. For instance, if bidders assign different values to a painting, a stamp, or a piece of furniture only on the basis of how much utility they would derive from possessing it, perhaps viewing it purely as a consumption good, then the private values assumption is reasonable. On the other hand, if bidders assign values on the basis of how much the object will fetch in the resale market, viewing it as an investment, then the private values assumption is not a good one.

In many situations, how much the object is worth is unknown at the time of the auction to the bidder himself. He may have only an estimate of some sort or some privately known signal—such as an expert's estimate or a test result—that is correlated with the true value. Indeed, other bidders may possess information—say, additional estimates or test results—that if known, would affect the value that a particular bidder attaches to the object. Thus, values are unknown at the time of the auction and may be affected by information available to other bidders. Such a specification is called one of *interdependent values* and is particularly suited for situations in which the object being sold is an asset that can possibly be resold after the auction. A special case of this is a situation in which the value, though unknown at the time of bidding, is the same for all bidders—a situation described as being one of a pure *common value*.¹ A common value model is most appropriate when the value of the object being auctioned is derived from a market price that is unknown at the time of the auction. An archetypal example is the sale of a tract of land with an unknown amount of oil underground. Bidders may have different estimates of the amount of oil, perhaps based on privately conducted tests, but the final value of the land is derived from the future sales of the oil, so this value is, to a first approximation, the same for all bidders.

Note that the term *interdependence* refers only to the structure of values and how these are affected by information held by other bidders. It does not refer to any statistical properties of this information—that is, how the signals observed by the bidders are distributed. Thus, we could have a situation in which values are interdependent so a particular bidder's value depends on a signal observed by another bidder, but at the same time, the signals themselves are statistically independent. Similarly, we could have a situation in which the values are not interdependent so a particular bidder's value depends only on his own signal, but the signals themselves are correlated.

¹Sometimes the term *common values* is itself used to label what we have called *interdependent*

Introduction

In A.D. 193, having killed the Emperor Pertinax, in a bold move the Prætorian Guard proceeded to sell off the entire Roman Empire by means of an auction. The winning bid was a promise of 25,000 sesterces per man to the Guard. The winner, Didius Julianus, was duly declared emperor but lasted for only two months before suffering from what is perhaps the earliest and most extreme instance of the “winner’s curse”: He was beheaded.

Auctions have been used since antiquity for the sale of a variety of objects. Herodotus reports that auctions were used in Babylon as early as 500 B.C. Today, both the range and the value of objects sold by auction have grown to staggering proportions. Art objects and antiques have always been sold at the fall of the auctioneer’s hammer. But now numerous kinds of commodities, ranging from tobacco, fish, and fresh flowers to scrap metal and gold bullion, are sold by means of auctions. Bond issues by public utilities are usually auctioned off to investment banking syndicates. Long-term securities are sold in weekly auctions conducted by the U.S. Treasury to finance the borrowing needs of the government. Perhaps the most important use of auctions has been to facilitate the transfer of assets from public to private hands—a worldwide phenomenon in the past two decades. These have included the sale of industrial enterprises in Eastern Europe and the former Soviet Union, and transportation systems in Britain and Scandinavia. Traditionally, the rights to use natural resources from public property—such as timber rights and offshore oil leases—have been sold by means of auctions. In the modern era, auctions of rights to use the electromagnetic spectrum for communication are also a worldwide phenomenon. Finally, there has been a tremendous growth in both the number of Internet auction websites, where individuals can put up items for sale under common auction rules, and the value of goods sold there.

The process of procurement via competitive bidding is nothing but an auction, except that in this case the bidders compete for the right to sell their products or services. Billions of dollars of government purchases are almost exclusively made in this way, and the practice is widespread, if not endemic, in business.

Auction Theory

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