7. Public consumption goods: the Samuelson-Lindahl mixed economy

1. Social optimality with public goods

A premise on normative versus positive economics

The topic of this Chapter is the efficient resource allocation between private and public uses. It belongs to the field of **normative economics** because the study of *efficient allocations* is a *normative investigation.* The question we want to answer is not: ' how *does* a society *actually* allocate its resources between private and public uses?', which is a *positive investigation*, but 'what are the properties of a *good* allocation between etc.?', or equivalently: 'how *should* a society allocate its resources between etc.?'.

This distinction between the positive and normative approaches sinks its roots in the fundamental **distinction/inseparability** between '**what is**' and '**what ought to be**', which identifies and separates the *soft sciences of society* from the *hard sciences of nature*, and must always be kept in mind by the social scientist if he wants to avoid running into a tangle of confusions and contradictions. It is however worth reminding that this separation between the two branches of knowledge is being in turn currently disputed by the school of thought of **sociobiology**, whose foundations were laid by the distinguished biologist **Edward Osborn Wilson (1998)**, recently deceased (2021), and strongly opposed by his Harvard colleague and equally distinguished biologist **Richard Lewontin** (2005), also recently deceased (2021).

The Samuelson-Lindahl mixed economy

We consider an economy consisting of one **private consumption good** $C = \sum_i c_i$, one **public consumption good** G, and n agents-taxpayers. The technology of this economy is represented by a standard **production/consumption frontier,** CFR**,** between the public consumption good G and the private one C, produced with the *full employment* of given resources (physical capital, labour and technology).

It is useful to carefully specify the assumptions underlying such CFR. The CFR rests on the assumption of an **opportunity cost function** $h(G)$ which specifies the production cost of G in terms of the physical units of C that are sacrificed when some of the given resources are diverted from the production of C to the production of G. The basic technology assumption of this economy is that there is a **fixed total output** $Q = GDP$ equal to the amount of C that *could* be produced if all available resources were put into it.

This is of course an *unrealistic benchmark scenario* because no viable commercial economy can exist without public goods. It is precisely for this reason that, after developing certain concepts and results within the **Samuelson-Lindahl economy**, SLE, of this Chapter we shall extend it into the **McGuire-Olson economy**, MOE, of Chapter 8 (*Public production goods*).

Given Q, if the economy wants to produce some G some resources must be diverted from C into G, according to the cost function $h(G)$. Thus, for whatever output mix (G, C), **total output itself remains conventionally measured in physical units of** C, as the amount of C *actually* produced, plus the amount of C that is *forgone* because some resources are diverted into G. In other words, $Q = GDP$ is the - fixed - level of *potential* C, either *actually produced*, or else *sacrificed* for producing G. The CFR is thus the relationship $C(G)$ obtained by subtracting $h(G)$ from the fixed level Q, according to the following expression

$$\begin{aligned} Q &= C + h(G) \\ &\rightarrow C = Q - h(G) = C(G) \end{aligned} \tag{7.1}$$

The first eq in (7.1) defines Q as the actual amount of C produced plus the amount of C that is forgone for producing G. The second gives C as a function of G, for given Q. The **marginal rate of transformation** of C into G, MRT, i.e. how many additional units of C must be sacrificed for one more unit of G, is the negative of the slope of the CFR, and it is the same thing as the **marginal social (opportunity) cost** of G measured in units of C, MSC. Formally:

$$\begin{aligned} \underset{C(G)}{MRT}(G, C) &= -C'_G = h'(G) = MSC(G) \\ &= -\frac{\Delta C}{\Delta G} \end{aligned} \tag{7.2}$$

(the minus sign is needed because $C'_G < 0$).

In the sequel we shall make frequent reference to passages in **Stiglitz**'s textbook (**2015**). For the reader familiar with that textbook reviewing those passages may serve as an introduction to some of the issues discussed in this Chapter.

Balancing the budget

Here is another assumption of this **static one-period economy** frequently left in the shadow. Since there is no future, there is *neither saving nor investment*, which implies that there is also *no room for a non-balanced public budget*. This economy incorporates by logical necessity a *balanced budget constraint*, BB. It contains the hidden assumption that the *cost* $h(G)$ *of public goods* must by construction be matched by an *equal amount of tax revenues*. It therefore contains implicitly the hidden assumption that taxes carry no **excess burden** = *social loss*, EB.

2. The two agents geometry of the efficiency conditions

Efficient allocations. The standard 'total' diagram

F7.1a shows how to derive graphically the CFR defined by eq (7.1). F7.1b is the *standard Samuelson 'total' diagram* showing the graphical procedure for constructing **Pareto allocations**, PAs, in the case of 2 taxpayers,

A and **B**. First we fix an indifference curve I_A of **A**, then we draw the **residual consumtion frontier**, RCFR, available to **B** by subtracting I_A from the CFR, and finally we find the highest indifference curve max I_B attainable by **B**, i.e. the one tangent to the RCFR. The resulting allocation (G^*, c_A^*, c_B^*) is *by construction* a PA. Since the slope of the RCFR is equal to the difference between the slope of the CFR and the slope of I_A, and is also equal to the slope of I_B, the allocation satisfies the condition

$$MRS_A(G, c_A) + MRS_B(G, c_B) = h'(G) = MSC(G)$$

that the marginal social cost of G must be equal to the **vertical sum** of the individual marginal benefits derived from G itself. The other PAs are obtained in the same way. Starting with an indifference curve I_A tangent to the CFR, with a corresponding zero RCFR, and then moving down to lower indifference curves I_A and higher RCFRs, with corresponding higher indifference curves max I_B, we obtain the *infinite set* of PAs (which of course need not lie along a vertical line)

This graphical procedure, which is the mixed economy counterpart of the standard graphical procedure for constructing PAs in a private economy with 2 inputs (capital and labour), 2 private outputs (**X** and **Y**) and 2 individuals (**A** and **B**), the **so-called** $2 \times 2 \times 2$ **economy**, was introduced by **Paul Samuelson (1954** and **1955)** and has remained to this day the standard illustration of the conditions for the **efficient resource allocation between private and public uses**. Since we shall use this formal and graphical framework also for discussing **Lindahl allocations** (G levels and shares), first introduced by the Swedish economist **Erik Lindahl (1919)**, in honour of this great economist we shall call it the **Samuelson-Lindahl mixed (private and public) economy**, SLE.

In Chapter 8 on *public production goods* we extend this economy into one where public goods G have a *productive role*, and the taxes required to finance them under the BB constraint do cause an EB. The extension, derived from **Martin McGuire** & **Mancur Olson (1996)**, will be denoted as the **McGuire-Olson mixed economy**, MOE.

Efficient allocations. The standard 'per unit' diagram

The same type of information contained in F7.1a&b is contained in the *standard **'per-unit'** diagram* drawn in F7.6, where instead of representing *total* variables we represent their per unit *marginal* counterparts. This latter Figure is discussed below in Section 6.

The role of the 'duality' between **total** and **per-unit** diagrams in theoretical economic reasoning is underlined by **Layard & Walters 1978**, p. 39. This old microeconomics textbook is still an excellent one. In contrast to many others it emphasizes, from cover to cover, the need to keep the *normative* definitions of *efficiency* conditions fully separated from the *positive* ones of *(market) equilibrium* conditions.

3. The formalism of social optimization.

Maximizing additive social welfare functions

The conventional meaning of social optimization is the maximization of some **social welfare function**, SWF. Now, using a social welfare function is, conceptually speaking, a questionable tour de force. It implies assuming the existence and measurability of something called **social welfare**, and this in turn implies assuming the existence and measurability of something called **individual welfare** or **utility**. All this is avoided by restricting the concept of **social optimality** to that of PAs, which are not defined in terms of individual utility and and social welfare, non-observable nor measurable, but in terms of **individual (**and possibly social**) preferences**. Since these are defined in terms of behaviour, they are - in principle - observable and measurable

The formal derivation of the conditions for PAs is obtained by using individual utilities and social welfare only as a **mathematical abstraction** playing a purely **instrumental role**, and then discarding them.

1) First, we assume the existence of **individual utilities** u_i, and define **social welfare** W as a function having individual utilities as its arguments.

2) Second, we formally derive the conditions for the maximization of this SWF.

3) Third, we eliminate social welfare and individual utilities from the equations by rearranging the *First order conditions*, FOC. In the present very simple context this is a trivial exercise, but it is nevertheless useful as an example of how the procedure works *in general*, and of what is gained and what is lost when we want to deal with issues of social optimality without making use of the questionable concept of a scalar welfare measurement.

Define the standard **Additive social welfare**, ASWF

$$W(u_i(\cdot)) = \sum_i u_i(G, c_i) \tag{7.3}$$

Notice that many other types of SWF are used in the literature, for example the more general form $W = W(u_i(\cdot), \cdot)$, but we do not need them here.

Now we maximize (7.3) over $G, (c_i)$ subject to the production constraint (7.1)

$$\max_{G,c_i} \sum_i u_i(G, c_i) \text{ subject to } C + h(G) = Q \tag{7.4}$$

Using the **Lagrangean**

$$L(\cdot) = \sum_i u_i(G, c_i) - \lambda(C + h(G) - Q) \tag{7.5}$$

we obtain the following FOC

$$\begin{aligned} &1 \text{ eq} \ : && \sum_i MU_{iG}(G, c_i) = \lambda h'(G) && (7.6)\\ &\text{n eqs:} && MU_{ic_i}(G, c_i) = \lambda, \forall i && (7.7)\\ &1 \text{ eq} \ : && C + h(G) = Q && (7.8) \end{aligned}$$

These are $2+n$ eqs in the $2+n$ variables $G, (c_i), \lambda$. Under the usual **implicit function conditions for the existence and uniqueness of solutions to a system of equations** they yield a **unique solution** $G^*, (c_i^*), \lambda^*$, where $G^*, (c_i^*)$ is the **socially optimal** allocation, while λ^* has the well-known meaning of *Lagrangean multipliers*, with which we are not concerned here. Not being interested in λ we eliminate it from the FOC by dividing (7.6) by (7.7): eq (7.6) becomes eq (7.9) and the n eqs (7.7) become the $n-1$ eqs (7.10)

$$\text{1 eq} \quad : \quad \sum_i \frac{MU_{iG}(G, c_i)}{MU_{ic_i}(G, c_i)} = \sum_i \underset{c_i(G)}{MRS_i}(G, c_i) = h'(G) \tag{7.9}$$

$$\text{n-1 eqs:} \quad MU_{ic_i}(G, c_i) = MU_{jc_j}(G, c_j), \forall i \neq j \tag{7.10}$$

$$\text{1 eq} \quad : \quad C + h(G) = Q \tag{7.11}$$

These new $1+n$ eqs in the $1+n$ variables $G, (c_i)$ yield the same unique socially optimal solution $G^*, (c_i^*)$ as before, the only change being the disappearance of λ. In this new system **marginal utilities,** MUs, have disappeared from eq (7.6), having been substituted by **marginal rates of substitution**, MRS, whose 'existence' doesn't require the 'existence' of individual utilities, but only that of individual preferences-indifference curves - see below (7.12). However, we still find *MUs* in (7.10).

Marginal utilities and marginal rates of substitution

The economic meaning of the MRS of c into G, denoted $\underset{c(G)}{MRS}(G, c)$ with subscript $c(G)$, appearing in (7.9), and its relation to the ratio of the **marginal utility** of G over that of c, is highlighted by substituting derivatives with **finite differences**

$$\frac{MU_G(\cdot)}{MU_c(\cdot)} \simeq \frac{\frac{\Delta u}{\Delta G}}{\frac{\Delta u}{\Delta c}} = \frac{\Delta u}{\Delta G} \frac{\Delta c}{\Delta u} = \frac{\Delta c}{\Delta G} \simeq \underset{c(G)}{MRS}(G, c) \tag{7.12}$$

This is the amount of c that the individual is **willing to pay** in exchange for a **marginal unit of** G. It is the **marginal willingness to pay**, MWP, for G, which is the same thing as the **individual marginal benefit, MB, derived from** G, measured in **units of** c as the **numeraire**.

From maximum social welfare to Pareto efficiency

In order to remain strictly within the boundaries of the Paretian world we need to eliminate the MUs altogether from the optimality conditions by eliminating eq (7.10), reducing the system to

$$\text{1 eq:} \quad \sum_i \frac{MU_{iG}(G, c_i)}{MU_{ic_i}(G, c_i)} = \sum_i \underset{c_i(G)}{MRS_i}(G, c_i) = h'(G) \tag{7.13}$$

$$\text{1 eq:} \quad C + h(G) = Q \tag{7.14}$$

These are 2 eqs in the $1 + n$ variables $(G, (c_i))$. Under the the **above usual conditions** they yield a *unique* solution only when $n = 1$ (a single agent-taxpayer). When $n > 1$ (2 or more agents-taxpayers) they yield an *infinite set* of solutions. These solutions $G^*, (c_i^*)$ form the **set of all** PAs. The general concept of social welfare optimality expressed by (7.9-11), which includes some kind of distributional requirement, has been reduced to the concept of Pareto allocations expressed by (7.13-14), which does not.

Eqs (7.13-14) **have a central status in this chapter**.

Extension to multiple private and public goods

It is a (relatively) simple matter to extend the above formalism to the case of *many private goods* and *many public goods*, starting with the case of *two private goods*, X and Y, and *two public goods* G and H, and then generalizing. Such generalizations, though formally interesting, do not contain substantial new insights.

It might instead be of some interest, at least in theory, to consider an economy consisting *only of public goods*. The allocations of such an economy could still be evaluated in terms of *efficiency*, but they would *no longer possess distributional properties* because of the non-rival nature of public goods. However we do not pursue this matter further.

Efficiency versus distribution

The change from system (7.9-11) to system (7.12-13) looks innocent enough, but it is not. Eq (7.10) ties down the allocation to the requirement that the distribution of the private good C among the n consumer-taxpayers must be such that the **marginal utility** derived from the individual consumption of C must be **the same for all individuals**. This is clearly a **distributional requirement**, not an efficiency one, and it is precisely this distributional requirement that selects a **single PA** out of the infinitely many. But the trouble with it is that it depends on the fiction that something called individual utilities exists as an *observable/measurable/interpersonally comparable* fact of the real world, and that any given structure of preferences is represented by an infinite set of equivalent scalar *uf*s which are *monotonic transformations* of each other. We can visualize what it means to satisfy the equity eqs (7.10) in the optimization process using again F7.1b. Satisfying the **equity requirement** means selecting, out of the infinite PAs constructed by moving **downwards** along ever lower indifference curves of A and **upwards** along ever higher indifference curves of B, the particular pair (I_A, I_B) ensuring

$$MU_{Ac_A}(G^*, c_A^*) = MU_{Bc_B}(G^*, c_B^*)$$

Of course, with taxpayers with different needs/preferences that particular pair (I_A, I_B) will not yield an egalitarian distribution of private consumption.

Optimal distribution with representative consumer-taxpayers

By way of example we may make the convenient assumption of a **representative consumer-taxpayer**, namely that all consumer-taxpayers have the same identical preference structure. In this way we bypass the problem posed by the existence of infinitely many scalar *ufs* representing the same preference structure. Assuming identical agents we can use the **same utility function** $u(G, c)$ for all of them. If so the unique PA maximizing the ASWF becomes the **egalitarian** one, where total private consumption is distributed **equally** among taxpayers: $C^* = nc^*$, because, under our assumption of a unique solution, *only* such egalitarian distribution would ensure the satisfaction of the equal MUs eqs (7.10). Other, non-egalitarian distributions would still be PAs, but they would no longer satisfy (7.10), and thus no longer maximize $W = \sum_i u_i(G, c_i) = nu(G, c)$ over $G, (c_i)$.

4. Cost shares, individual 'demand' for public goods, tax-prices

Cost shares and individual budget constraints

As already stated above, in this **one-period economy** there is **neither saving nor investment**. The individual taxpayer's gross income is by construction divided between **private consumption** and **taxes**. His **individual budget constraint,** IB, is thus

$$\begin{aligned} c + sh(G) &= y \qquad (7.15) \\ c &= y - sh(G) = c(G) \end{aligned}$$

where $s \geq 0$ is the **individual share in the cost of** G, the share of total public expenditure charged by the government onto the individual taxpayer, and y is his **gross income** (income before taxes). The IB is drawn in F7.2a. It is understood that in the present discussion the cost share s is not the result of some *voluntary or impersonal adjustment process*. It is **established by the government** through its exclusive **power to tax**, and will in general differ between taxpayers, depending on what criteria the government chooses for *distributing the total tax burden* (more on this below).

For simplicity we shall deal with the *individual tax burden* as if it were **fixed directly** in the form of a **personalized cost share**. But in the **real world** taxes are rarely fixed in this way. Actual tax systems consist mostly of taxes defined in terms of **commercial tax bases** (such as **income, expenditures, real and financial assets, physical quantities,** etc.), and **individual cost shares descend indirectly from such tax systems**.

We remind that if s were actually fixed directly as an exogenously given personalized share in the cost of G, this would be a **price taking** condition that would carry **no tax-induced** EB. Since a direct s entails no EB, it could look like a type of **lump-sum taxation**, but stricly it would not be one because in order for a tax to be truly lump-sum the payment must be **disconnected from any economic variable**, while the tax debt determined directly in the form of a cost share varies with G.

The individual 'demand' for public goods as a function of cost share and gross income

Individual demands for G and c as functions of s and y are obtained in the usual way, by maximizing the taxpayer's utility subject to the IB

$$\max_{G,c} u(G,c) \text{ subject to } c + sh(G) = y \tag{7.16}$$

From the Lagrangean

$$L(\cdot) = u(G,c) - \lambda(c + sh(G) - y) \tag{7.17}$$

we obtain the FOC

$$MU_G(G,c) = \lambda sh'(G) \tag{7.18}$$
$$MU_c(G,c) = \lambda \tag{7.19}$$
$$c + sh(G) = y \tag{7.20}$$

Eliminating λ by dividing (7.18) by (7.19) yields

$$\frac{MU_G(\cdot)}{MU_c(\cdot)} = \underset{c(G)}{MRS}(G,c) = sh'(G) = p \tag{7.21}$$
$$c = y - sh(G) \tag{7.22}$$

This is a system of 2 eqs in 2 variables G, c and 2 parameters s, y. It therefore yields the individual demands for G and c as functions of s and y

$$\begin{array}{l} G^D(s,y) \\ c^D(s,y) \end{array} \tag{7.23}$$

or of p and y

$$\begin{array}{l} G^D(p,y) \\ c^D(p,y) \end{array} \tag{7.23bis}$$

The construction is shown graphically in F7.2 (eq (7.23)) and F7.3 (eq (7.23bis)) respectively, where the demand functions $G^D(s,y)$ and $G^D(p,y)$ are drawn in **inverted form**, with s and p on the vertical axis and G^D on the horizontal axis: $s(G)$ and $p(G)$. Notice again that in deriving demand functions the **utility function** $u(G,c)$ plays a **purely instrumental role**. In passing from (7.18-19) to (7.21) utility disappears, being sibstituted by the MRSs. The existence of demand functions doesn't require the existence of individual utilities. It only requires the existence of individual preferences.

On this the brave reader may test his capacity for abstraction by checking **Debreu 1959**.

The term p introduced in (7.21) stands for what is known as the **tax-price**, whose meaning will be explained in detail below.

Eqs (7.21-22) have a central status in this chapter.

The diagrams

If compared with ordinary demand functions for private goods, the demand functions (7.23) are of a special type, because s is a **share in the total cost** of G, not a **price per unit** of G. The graphical counterpart of (7.23) is given in F7.2b. As already mentioned, the **upper panel a** shows the different tangency points on the different budget curves corresponding to different values of the individual tax share s (the budget curves would be straight lines if the cost function $h(G)$ were linear ($h(G) = hG = G$, by the appropriate normalization of the units of measurement of C and G). In the **lower panel b** this relationship is converted into a schedule showing how the **individual's desired level of G changes when s changes, keeping y fixed**.

The tax-price

We consider now the concept of the personalized **tax-price**, a concept having a central place in **Stiglitz's** textbook **(2015)** discussion of the *efficiency conditions for public goods* (p.117) and of *public expenditure and public choice* (pp. 233).

The *tax-price of a taxpayer* is the *marginal increase in his tax liability* caused by a *marginal increase in G*. The definitional relationship between the taxpayer's personal **tax liability** t_i and the personalized **cost share** s_i is given by the following expressions

$$t_i(G) = s_i h(G) \tag{7.24}$$
$$t_i'(G) = s_i h'(G) = p_i \tag{7.25}$$

Notice that the *personalized tax-price* p_i of (7.21-25) combines two natures into one: it is a **price** of G, in the sense that it is what the taxpayer must pay for an additional unit of G, in the form of an increased tax liability, but it is also a **tax** because G is not bought in the market but collectively decided by the government and charged coercively on the taxpayer.

The individual 'demand' for G as a function of the tax-price

In F7.3 we see how to convert graphically the individual 'demand' curve for G as **a function of the cost share** s into its equivalent as **a function of the tax-price** p. It is the graphical counterpart of eq (7.23bis), where on the vertical axis, instead of measuring s we measure $\underset{c(G)}{MRS}(G,c) = sh'(G) = p$, which is the individual MWP or MB according to the identities

$$\underset{c(G)}{MRS}(G,c) = MWP(G) = MB_G(G) = sh'(G) = p$$

In this way the resulting schedule of the individual's **desired level of public goods** as a function of the **tax-price** becomes formally similar to his **ordinary demand for private goods** as a function of their **market price**. The notation $sh'(G) = p$ introduced in (7.21-25) is meant to represent precisely the fact that $sh'(G)$ is the **'price' that the taxpayer must pay for a unit increase in** G, a price taking the form of the **increase in his tax liability** required to cover the additional cost of G.

While the schedule in F7.2b is different from an ordinary demand function because s is not a price, the schedule in F7.3b is similar to the individual's **ordinary demand for private goods**. However the similarity is **purely formal**. The schedule is not a demand in the market sense, of the different amounts of G that the individual would purchase in the market at different values of the tax-price p. Public goods are not sold in the market, nor are they chosen individually. They are provided jointly to everybody in the same amount through a **collective-political decision process**, and financed by the coercive payment of taxes. G is a quantitative indicator of the amount/quality of public goods and services provided, i.e. of the degree to which public non-rival shared interests are being satisfied by the government, which is the same for everybody. What may, and indeed generally does, differ is the marginal (and total) evaluation of those shared interests by different taxpayers. Because of this non-trivial special property of the relationship in question, to call it the **individual 'demand' for** G, is **misleading**. It is more appropriate to call it the **individual's desired level of** G, for which he - as a tax-payer - would **vote** - as a voter - if he were asked to, and if he **knew his cost share and tax-price**. It seems equally misleading to call the **vertical sum** of such individual 'demands' the **'collective' demand for** G (**Stiglitz 2015**, p. 117-9).

It is for this reason that when talking of G we put the word 'demand' in inverted commas.

Measuring expenditure in physical units of G

The above construction is slightly more technical than the **Stiglitz** one (**2015**, pp. 120-1) because we use a general **nonlinear** cost function $h(G)$ with increasing MSC. By making it linear and normalizing the *cost of public goods*, measured in units of forgone private consumption, *to their physical amount*

$$h(G) = h \times G = G(h = 1$$

(which in the present context seems a perfectly acceptable simplification) the construction becomes the same of **Stiglitz**. In F7.3 the IBs become *straight* *lines* of unitary slope, the MSC becomes constant at $h = 1$, and s becomes the same as p (the rising lines in F7.3b become horizontal).

However, such normalization eliminates the possibility of simulating the effects of changes in the production cost of G.

5. Tax systems, tax-prices, balanced budget

The concept of the tax-price is strictly related to the BB constraint. The BB means that the government always keeps **total tax revenues** $T(G)$ **equal to total public expenditures** $h(G)$, either by choice, or by constitutional obligation, and therefore always changes tax revenues by the same amount by which it changes expenditures. Formally, and using (7.24-5), we have

$$T(G) = \sum_i t_i(G) = \left(\sum_i s_i\right) h(G) = h(G) \tag{7.26}$$

$$T'(G) = \sum_i t'_i(G) = \left(\sum_i s_i\right) h'(G) = h'(G) \tag{7.27}$$

with

$$\sum_i s_i = 1 \tag{7.28}$$

implied by the BB condition restated in terms of cost shares.

Graphically the IBs of the individual taxpayer defined by (7.15) shown in F7.2a are defined in terms of the individual cost shares s_i, but as already noticed cost shares are in general the *result of the structure of the tax system*, and of the particular way in which taxes are changed when G changes. Following **Stiglitz (2015,** pp. 233-4**)**, we consider how the IBs and the *tax-prices* look like in two simple cases: 1) a **uniform cost share**, where the BB is satisfied by requiring every taxpayers to pay **the same cost share** $1/n$, and 2) a **proportional income tax system**, where all taxpayers pay the same (proportional) tax rate τ on their **gross income**, required to cover expenditures. The two types of IBs are shown in F7.4.

A uniform cost share

Given n taxpayers, the government decides to cover the cost of G by charging taxpayers directly with a *uniform cost share* $s = 1/n$ (which of course implies $\sum_i s_i = 1$). Substituting s with $1/n$ we obtain the following expressions for the individual *tax debt*, IB and *tax-price*, respectively

$$\begin{aligned} t_i(G) &= \frac{1}{n}h(G) \\ c_i &= y_i - \frac{1}{n}h(G) \\ t'_i(G) &= \frac{1}{n}h'(G) = p_i \end{aligned} \tag{7.29}$$

These IBs are shown in F7.4a. Moving from poor to richer taxpayers the IB **shifts upwards** in a **parallel way**. And the intercept of the IB with the G-axis is

$$\begin{aligned} c &= y - \frac{1}{n}h(G) \\ &= 0 \rightarrow \frac{h(G)}{n} = y \\ &\rightarrow h(G) = ny \\ &\rightarrow G \mid h(G) = ny \end{aligned} \tag{7.30}$$

As the individual gross income y rises the slope of the IB (the tax price $\frac{1}{n}h'(G)$) remains the same at all values of G, while the IB's *intercept* with the G-axis shifts rightwards. All taxpayers, rich and poor, pay *the same tax price*, which changes with G, but not with gross income. Notice that a tax system defined in this way is equivalent to **fixing cost shares directly**, so that these would indeed have the full nature not of a tax but of a *price*. Notice further that - at least formally - the individual cost shares s_i fixed directly *need not be all equal across taxpayers*.

A proportional income tax system

Here the stylized tax system itself is entirely determined by **a single parameter**: the uniform **proportional income tax rate** τ that must be levied on all gross incomes in order to cover the budget. The individual tax debt is

$$t_i = \tau y_i = s_i h(G) \tag{7.31}$$

In the SLE the sum of individual gross incomes is by construction equal to the fixed total output: $\sum_i y_i = Q$. This yields the tax rate τ that ensures a balanced government budget

$$\sum_i t_i = \tau \sum_i y_i = \tau Q = h(G) \rightarrow \tau = \frac{h(G)}{Q} \tag{7.32}$$

trivially equal to the ratio of public expeditures over Q. Then we substitute this τ into the individual tax debt (7.31) obtaining the cost share s_i charged to the i-th taxpayer

$$t_i = \frac{h(G)}{Q} y_i = \frac{y_i}{Q} h(G) = s_i h(G) \rightarrow s_i = \frac{y_i}{Q} \tag{7.33}$$

equal the share of the individual's gross income y into Q.

Finally we substitute this cost share into the taxpayer's IB (7.15) and tax-price (7.25)

$$c_i = y_i - \frac{y_i}{Q} h(G) \tag{7.34}$$

$$t_i'(G) = \frac{y_i}{Q} h'(G) = p_i \tag{7.35}$$

In the new IB **the (proportional) tax rate disappears**. It is no longer a parameter because it changes with $h(G)$ (which changes when G and/or h change). The IBs generated by a *proportional income tax* are shown in F7.4b. Moving from poor to richer taxpayers the IB *rotates upwards*, pivoting on the *fixed intercept* $\bar{G} \mid h(G) = Q$ on the G-axis

$$\begin{aligned} c &= y - \frac{y}{Q} h(G) \\ &= 0 \rightarrow \frac{y}{Q} h(G) = y \\ &\rightarrow h(G) = Q \\ &\rightarrow G \mid h(G) = Q \rightarrow G = \bar{G} \end{aligned} \tag{7.36}$$

Richer taxpayers pay higher tax prices than poorer ones, because s_i is equal to the share of the individual's gross income into $Q = GDP$. Notice that, unlike in the previous case, this time **cost shares are not fixed directly**. They are derived **from the operation of an ordinary tax system under the BB constraint**, and would therefore entail a more or less large amount of **tax-induced** EB associated with that tax system (*income taxation* in the particular case).

A progressive income taxation

Deriving individual cost shares under a **progressive income tax system** is not as simple as under proportionality, because they depend on
1) the particular type of progressive tax code,
2) the gross income distribution, and
3) how the progressive tax code is adjusted to keep the budget balanced.

There is however **one general proposition that can be proved**. Under **proportionality** we see by (7.35) that the individual cost share is equal to the share of the individual gross income into GDP, so that if gross income doubles the corresponding cost share must also double, as shown in F7.4b. But under **progressivity** if gross income doubles the corresponding cost share must **more than double**, because **if it didn't the income tax would be proportional**! In the Figure when gross income y rises under progressivity the slope of the IB along verticals in G must icrease *more than proportionally* (**Stiglitz 2015**, p. 233 bottom), which means that the IB of the rich taxpayer would take a shape like the *dotted blue line*. A formal 'proof' of this proposition is given in APPENDIX 7.A

Notice further that according to standard results of the *economics of taxation*, the EB caused by a *progressive* tax system tends to be *greater* than that of a *proportional* one.

Introducing average income.

Using **Stiglitz**'s notation **(2015,** p. 233**)** $n\bar{y} = Q$ (with $\bar{y}$ average gross income) the taxpayer's tax debt (7.34) becomes

$$t_i(G) = \frac{y_i}{n\bar{y}} h(G) \tag{7.37}$$

The expression highlights that under proportional taxation the taxpayer's cost share is higher, equal or lower than $1/n$ depending on whether his gross income y_i is higher, equal or lower than average income $\bar{y}$. Formally this is correct because this model doesn't allow for the distortionary effects of income taxation. But non lump-sum taxes cause efficiency losses, while direct fixing of cost shares doesn't. Thus, in comparing a uniform tax system $\frac{1}{n}h(G)$ with a proportional income tax system τy we must consider that the two are different even for the average income earner. In both cases he would pay $1/n$ of $h(G)$, but the mechanism generating the tax price would be different. In the former case his tax debt is a direct charge and carries no distortion. In the latter it is generated by the increase in the proportional income tax rate τ required to keep the budget balanced and carries the distortion associated with τ.

Resuming the point made in the previous Section 4 (§ *Measuring expenditure in units of* G) we see that if we put $h(G) = G$ the above expressions for cost shares and tax prices become the same as in Stiglitz (2015, pp. 117 ff.), with G in place of $h(G)$ and $s_i = p_i$ everywhere.

6. The Lindahl existence theorems. Theorem I (LET I)

The theorems of welfare economics

We now introduce the concept of a **Lindahl equilibrium**, whose peculiar meaning is highlighted by the **Lindahl existence theorems,** LET**.** These are theorems which, in spite of their formal simplicity, embody a *concept of non-trivial social significance* concerning the intrinsic *qualitative difference* between the *private* and the *public* parts of an economy. Conceptually, the LET are obtained by **extending the classical Theorems of welfare economics,** TWE**, to a mixed economy**.

The TWE are classical theoretical results in economics, whose **non-trivial 'cultural' message** is the powerful 'ideological' **link** they establish between the **positive** (equilibrium) and the **normative** (efficiency) approaches in economics. They establish a link between *how the economy* ***actually*** *allocates its resources* (*positive* economics), and *how the economy* ***should*** *allocate them* (*normative* economics), asserting that under *appropriate conditions* the *two types of allocations* ***coincide***. Such link has clearly a powerful ideological connotation. Now, some of the best minds in the social sciences, in science and in philosophy have been, and are, engaged in explaining to the public at large how **theoretically restrictive** and **empirically quasi-meaningless** are the assumptions required for the theorems' *validity*, but so far this highly commendable *Kulturkampf* doesn't seem to have achieved the desired weakening of that message, at least in the **western cultural milieu** where the message was born and developed.

Granted that it is extremely important to warn all young students of economics about the powerful ideological weight of the message, the special point we want to make here is that that weight should be scaled down not only because of the restrictiveness of the theorems' assumptions, but also because of the fact that these are constructed with reference to an **exclusively private/commercial economy** of only rival interests and markets.

The TWEs come in two versions:

TWE I: every **competitve equilibrium,** CE (*equilibrium* is a **positive** concept), combined with an *initial distribution of endowments*, is efficient, i.e. a PA (*efficiency* is a **normative** concept).

TWE II: every PA (a **normative** concept) can be achieved by a CE (a **positive** concept) combined with the appropriate *initial endowments*.

For reference, and for a concise view of what these theorems are about, we give in F7.5 the **graphic proof** of the two theorems in the simplest **pure exchange economy** of **2 goods** and **2 subjects**:

TWE I. Start at the *initial endowment* point 1. Then a *competitve equilibrium* leads to point 4, which is a PA.

TWE II. Start at an arbitrary PA, say point 6. Then *any initial endowment* lying on the budget line going through point 6, such as points 5, or 7, or any other, would under *competitive equilibrium* lead to the original PA of point 6. ■

Extending the theorems to a **general intertemporal production commercial economy** with many individuals (consumers-workers) and firms (producers) requires a set of **further restrictive assumptions** about the structure of **preferences, technology** and **markets** (see **Arrow & Hahn 2004**, **Debreu 1959**, **Varian 2020**, Chapter on *Production*), but the ideological message remains unaffected.

(12.22) aggiungere nuova appendice (v. file *Externalities*).

Correspondingly, also the LETs come in two versions:

LET I (corresponding to TWE I): in a **mixed private-public economy**, for every given initial gross income distribution (y_{i0}) there exists a vector of personalized cost shares (s_i) ensuring a Lindahl **voting unanimity/equilibrium** (a **positive** concept), which is also a PA with an **efficient** level of G (a **normative** concept)

LET II (corresponding to TWE II): in a mixed economy for every PA with an **efficient** level of G (a **normative** concept) there exists a vector of gross incomes (y_i) and personalized cost shares (s_i) which converts it into a Lindahl **voting unanimity/equilibrium** (a **positive** concept)

LET I extends the TWE I from an *exclusively commercial economy* to a *mixed economy*. The two types of economies exhibit a fundamental difference, which we insist in emphasizing because of its *social importance*. In the ideal scenario of an **exclusively private/commercial economy** only a **market-ensuring public/political authority** is required. **The market works automatically and impersonally**. Individual agents are under no coercive constraints of any kind, except those imposed by a **Nozick minimal state**, NMS (see Chapter 6 on *Nash-Lindahl*, Section 7 on *cooperation failure*). They act in a state of fully free-voluntary exchange transactions. In the **mixed economy** this social condition of full freedom vanishes. The concept of a *mixed economy* is intrinsically associated with the presence of **an active coercive public/political authority**. There can be **no mixed economy without an active public/political authority**, reaching well beyond the limited role of the NMS. The reasons - briefly reviewed below in *Section 6* - are that in the the mixed economy there is an **intrinsic non-removable possibility/incentive to free-ride**. No way has so far been found - anywhere - to remove this possibility/incentive without establishing a public/political authority with the power of making **cooperation/collective action** for the satisfaction of **public interests** into a **publicly sanctioned obligation**. The LET I as the counterpart of the TWE I is further 'evidence' that the **existence of an active government** is an intrinsic requirement of a mixed economy.

We start with any *arbitrary, predetermined exogenously given output and gross income distribution*

$$(y_{i0}),\ \sum\nolimits_i y_{i0} = Q \tag{7.38}$$

under the assumption that this is generated by the market and that the government takes it as a given 'parameter' of the system. The government enters the picture as the agent responsible for its 'constitutionally' unique job: the provision of G and the coercive covering of its cost through its exclusive *power to tax*. Assuming the government to be a *benevolent* one, we suppose that what it wants to do is to determine a PA , and to distribute its cost among taxpayers - under the BB requirement - in such a way as to obtain their *unanimous consent* on that efficient G level (a perfect application of the benefit principle, BP). Assuming further - for convenience - that this benevolent government has the necessary information on the taxpayers' incomes and preferences, we ask: can it do that? In other words: does there exist an $n-$vector (s_i^*) of personalized cost shares in the production cost $h(G)$ s.t. 1) their sum adds to unity, and 2) it ensures a *Lindahl unanimity equilibrium* at a PA?

It is easy to show that such an $n-$vector (s_i^*) does indeed exist.

Proof of LET I

Inserting the individual gross incomes y_{i0} of (7.38) into the *individual demand eqs* (7.21-2)

$$\begin{aligned} \underset{c(G)}{MRS}(G, c_i) &= s_i h'(G) = p_i && (7.21) \\ c_i &= y_{i0} - s_i h(G) && (7.22) \end{aligned}$$

yields the *individual demand functions* as functions of s_i only

$$G_i^D(s_i, y_{i0}) \tag{7.39}$$
$$c_i^D(s_i, y_{i0}) \tag{7.40}$$

Then form the *system*

$$\begin{aligned} \sum\nolimits_i s_i &= 1 & (7.41) \\ G_i^D(s_i, y_{i0}) &= G_j^D(s_j, y_{j0}),\ \forall i \neq j & (7.42) \end{aligned}$$

and ask: does there exist a (unique) n-vector (s_i^*) such that the resulting allocation $(G^*, (c_i^*))$

$$\left\{ \begin{array}{c} G_i^D(s_i^*, y_{i0}) = G^* \\ c_i^D(s_i^*, y_{i0}) = c_i^* \end{array} \right\} \forall i \tag{7.43}$$

satisfies the PA conditions?

$$\begin{aligned} \sum\nolimits_i \underset{c_i(G)}{MRS_i}(G^*, c_i^*) &= h'(G^*) & (7.12) \\ C^* + h(G^*) &= Q & (7.13) \end{aligned}$$

Now:

1) System (7.41-2) has n eqs in the n variables (s_i) (notice that the number of eqs (7.42) is not n but $n-1$), and under the usual conditions for the **existence/uniqueness of solutions of a system of equations** (for the **implicit function theorem** see **Chiang 2005** Chapters 4,5,8 - in particular pp. 194-204) it yields a (unique) unanimity-ensuring solution n-vector (s_i^*) of individual cost shares.

2) Substituting (s_i^*), $(G^*, (c_i^*))$ into the individual demand eqs (7.21-2) we see that they must necessarily satisfy the PA conditions (7.12-3): since $\sum_i s_i^* = 1$ by (7.41-2) and $\sum_i y_{i0} = Q$ by assumption, eqs

$$\underset{c(G)}{MRS}(G^*, c_i^*) = s_i^* h'(G^*) = p_i^* \tag{7.21}$$

imply (7.12)

$$\sum\nolimits_i \underset{c_i(G)}{MRS_i}(G^*, c_i^*) = \sum\nolimits_i s_i^* h'(G^*) = \left(\sum\nolimits_i s_i^*\right) h'(G^*) = h'(G^*) \tag{7.12}$$

and eqs

$$c_i^* + s_i^* h(G^*) = y_{i0} \tag{7.22}$$

imply (7.13)

$$\begin{aligned} \sum\nolimits_i c_i^* + \sum\nolimits_i s_i^* h(G^*) &= \sum\nolimits_i y_{i0} & (7.13) \\ &\rightarrow C^* + h(G^*) = Q \end{aligned}$$

which completes the proof ■.

Extension to multiple private and public goods

As anticipated in *Section 1*, the logic and results of LET I hold without qualitative changes when extended to *multiple private goods and public goods*, but of course the extension can only be made via mathematical formalism.

Diagrammatic illustration of LET I. The 'per unit' diagram

The preceding Subsection on LET I presents the existence, voting unanimity and efficiency properties of Lindahl shares in *purely formal terms*. Such stylized concepts must be appreciated in their capacity to highlight fundamental aspects of the public economy, some of which have just been anticipated. We summarize them here by converting the formal analysis into the *standard 'per unit' diagram* drawn in F7.6. We assume two taxpayers A, B, a given total output and gross income distribution $Q = y_{A0} + y_{B0}$, and a given (opportunity) cost function $h(G)$, with marginal social cost function $MSC(G) = h'(G)$. Then we draw the demand schedules of the two taxpayers using the procedure illustrated in F7.3. We label them $MB_A(G)$ and $MB_B(G)$ respectively. The schedule representing the **marginal social benefit**, MSB, or **marginal social willingness to pay**, MSWP, is obtained by **vertical summation** of the individual marginal benefit schedules

$$MSB_G(G) = MB_A(G) + MB_B(G) \tag{7.44}$$

This aggregate schedule is sometimes called the **collective demand** for G (as in **Stiglitz 2015**, p. 119). But we repeat that it is a very different thing from the *aggregate (collective) market demand* for private goods. The aggregate market demand for a private good is obtained by **horizontal summation** of individual demands, and it does represent the total amount of a good demanded in the market at different prices. Instead, as we made clear in *Section 4*, the individual 'demands' for a public good are not demands in the market sense. They represent the amounts of the good that individual taxpayers 'would like to have', and would vote for, when faced with different cost shares/tax-prices, and it is more appropriate to call them simply the individual MB, or MWP, schedules for additional units of G. Their **vertical summation** is of course even less of a 'demand' than the individual ones, not only because the individual components are not ordinary demands, but also because **their aggregation is vertical and not horizontal**. To call it a collective demand is doubly misleading, and it is all the more advisable to call it simply by its proper name, the MSB or MSWP schedule for additional units of G. The concept of the marginal - or total - social willingness to pay for a public good is essential for the understanding of the social meaning of the concept of **public non-rival (shared) interests**.

We now identify the efficient amount G^* at the intersection between the marginal social benefit schedule $MSB(G) = \sum_i MB_i(G)$ and the marginal social cost schedule $MSC(G)$

If the government knew the taxpayers' true preferences, and if it wanted to achieve a *unanimously accepted* PA, subject to the exogenously given total output and gross income distribution $Q = y_{A0} + y_{B0}$, and the exogenously given marginal social cost $MSC(G)$ of providing G, then it would choose the solution vector (G^*, s_A^*, s_B^*). With these individual cost shares

$$\begin{aligned} \underset{c(G)}{MRS}(G^*, c_i^*) &= s_i^* h'(G^*) = p_i^*, \\ i &= A, B \end{aligned} \tag{7.21}$$

both taxpayers want the government to provide exactly the efficient amount G^*. As is apparent in the Figure, the government could choose a **different cost share vector,** say 1/2, possibly because the exogenously given gross income distribution is $y_{A0} = y_{B0} = \frac{1}{2}Q$, and on **equity grounds** it wants the two taxpayers to pay the same cost share. In this case **there would no longer be unanimity**. A would want $G_A > G^*$, and B would want $G_B < G^*$, and whatever amount G the government chooses (G_A, G_B, G^*, or any other), the $n-1$ eqs (7.42) would no longer be satisfied. If the government knew the true preferences of A and B it could still choose the PA amount G^*, but if it wants to stick to the equity-based equal cost shares, it would have to drop the pretence of receiving a unanimous indication from the floor, because even if the chosen G level were efficient, one taxpayer would like the government to increase it and the other would like the government to decrease it.

Notice that if we started off with either one or more of the following:

1) a different exogenous total output,

2) a different gross income distribution of the given output,

3) a different cost function $h(G)$ implying a different $MSC(G)$ schedule, then the $MB_i(G)$ schedules, the PA amount G^*, and the cost shares s_A^*, s_B^* would in general all be different from the previous ones (F7.2 provides an intuitive understanding of these dependencies).

The diagrammatic **per unit** analysis of F7.6 can be repeated using the **total** diagram of F7.1b. Starting from a predetermined income distribution $Q = y_{A0} + y_{B0}$ on the vertical axis, we find the unanimity cost shares (s_A^*, s_B^*) ensuring the same desired G by both taxpayers.

7. Theorem II (LET II)

In parallel to LET I, LET II extends the TWE II from an *exclusively commercial economy* to a *mixed one*.

We start from an arbitrary PA $(G^*, (c_i^*))$ satisfying conditions (7.12-3), and then ask: does there exist a (unique) $2n$-vector (s_i^*, y_i^*) of *personalized cost shares* s_i^* in the *production cost* $h(G^*)$ and *individual gross incomes* y_i^* such that

1) they transform such PA into a Lindahl unanimity equilibrium

$$\left\{ \begin{array}{c} G_i^D(s_i^*, y_i^*) = G^* \\ c_i^D(s_i^*, y_i^*) = c_i^* \end{array} \right\} \forall i \tag{7.45}$$

and

2) the sum of personalized cost shares s_i^* totals 1, and the sum of individual gross incomes y_i^* totals the given $Q = GDP$?

$$\begin{aligned} \sum\nolimits_i s_i^* &= 1 \\ \sum\nolimits_i y_i^* &= Q \end{aligned} \tag{7.46}$$

It is again easy to show that such (unique) $2n$-vector (s_i^*, y_i^*) does indeed exist.

Proof of LET II

The proof consists simply in *inverting the procedure for deriving the demand functions* (7.23): instead of going from (s_i, y_i) to (G, c_i) we go from (G, c_i) to (s_i, y_i). Taking an arbitrary PA $(G^*, (c_i^*))$ from eqs (7.12-3)

$$\sum\nolimits_i \underset{c_i(G)}{MRS_i}(G^*, c_i^*) = h'(G^*) \tag{7.12}$$

$$C^* + h(G^*) = Q \tag{7.13}$$

we insert it into the $2n$ individual demand eqs (7.21-2),

$$\underset{c(G)}{MRS}(G^*, c_i^*) = s_i h'(G^*) = p \tag{7.21}$$

$$c_i^* = y_i - s_i h(G^*) \tag{7.22}$$

obtaining the following system of $2n$ eqs in $2n$ variables

$$\begin{aligned} G_i^D(s_i, y_i) &= G^* \\ c_i^D(s_i, y_i) &= c_i^* \end{aligned}$$

which, under the usual implicit function assumptions, yields a (unique) solution $2n$-vector (s_i^*, y_i^*) satisfying (7.45).

Notice that this inverted procedure can be graphically verified in F7.2a: first we take some point (G_0, c_0), and then we find the pair (s, y) such that the tangency between the IB and the indifference curve is exactly at that point.

As for (7.46), we see that (7.21+12) imply

$$\begin{aligned} \sum\nolimits_i \underset{c_i(G)}{MRS_i}(G^*, c_i^*) &= \sum\nolimits_i s_i^* h'(G^*) = \left(\sum\nolimits_i s_i^*\right) h'(G^*) = h'(G^*) \\ &\rightarrow \sum\nolimits_i s_i^* = 1 \end{aligned} \tag{7.47}$$

and (7.22+13) imply

$$\begin{aligned}\sum_i c_i^* + \sum_i s_i^* h(G^*) &= \sum_i y_i^* \\ &= C^* + \left(\sum_i s_i^*\right) h(G^*) \\ &= C^* + h(G^*) \\ &= Q\end{aligned} \tag{7.48}$$

which completes the proof. ■

Extension to multiple private and public goods

As with LET I, the logic and results of LET II hold without qualitative changes when extended to *multiple private and public goods*.

Diagrammatic illustration of LET II. The 'total' diagram

The unanimity equilibrium of LET II defined by eqs (7.45-8) can be shown graphically using the *standard Samuelson 'total' diagram* drawn in F7.1b, where, starting from a PA $(\mathbf{P}^*, \mathbf{P}_A^*, \mathbf{P}_B^*)$ we draw the IBs of the two taxpayers **A** and **B** obtained by selecting the two pairs (s_A^*, y_A^*) and (s_B^*, y_B^*) which convert that allocation into a Lindahl equilibrium.

Relation between LET I and LET II

Notice the *substantial difference* between the **existence problems** addressed by LET II and LET I.

In LET II we start off with a PA and then want to find out whether any such allocation did map into a $2n$ **vector** $((s_i^*), (y_i^*))$ **of cost shares** and **gross incomes** leading to it under **unanimity**.

In LET I we let **one half of the** $2n$ **vector** - the half consisting of the **gross income distribution** - to be **exogenously supplied by the market**. We then consider the remaining half - the half consisting of the **cost shares** - and want to find out whether there exist some n **vector** of such cost shares leading to a PA under **unanimity**.

The special economic meaning of the $2n$ **vector** of **cost shares** and **gross incomes** lies in the fact that it is simultaneously associated with a PA and the **voting unanimity** property of the collective decision process leading to the choice of G. In other words, given a PA in a mixed economy, the Lindahl $2n$ vector of eqs (7.45-6) converts that allocation into a **Lindahl unanimity equilibrium**. The concept of a *Lindahl unanimity equilibrium* is the counterpart - for a mixed economy - of the concept of a **competitive equilibrium** for a pure commercial economy. With reference to the public part of an economy it **substitutes the concept of competitive equilibrium with that of voting unanimity**. There is however an **important aspect of this transition from a commercial to a mixed economy**, which in the LET II version of the theorem remains somehow left in the background. Whereas a commercial competitive equilibrium can - at least in principle - arise out of impersonal market transactions (demand and supply) among a sufficiently large number of individual traders (where 'sufficiently large' means that traders must act as **price takers**), the same doesn't hold for a Lindahl equilibrium. The vector of individual cost shares yielding voting unanimity will **never arise spontaneously out of some free negotiating process among the voters**. Such vector does exist, but it works only if it is perceived by the voters as **imposed by an outside authority**. In other words, as already pointed out in stating the LET I, it works only if the voters regard themselves as *price-takers* with respect to it. **The 'impersonal authority' of the competitive market must be replaced by the 'subjective authority' of a political organization endowed with coercive power**. Moreover the political organization would need to know the taxpayers' true preferences on G if it wants to achieve both **Pareto efficiency** and **unanimity**.

In the **more operational** LET I **version of the theorem** this aspect of the transition is instead *immediately apparent* because we start off by **explicitly assuming an active government** constitutionally responsible for providing G and covering its cost with taxes.

8. The distribution of the tax burden. The benefit and ability to pay principles

F7.6 is also useful for discussing the fundamental difference between the **benefit principle,** henceforth BP, and the **ability to pay principle,** henceforth APP, in financing public expenditure. For the sake of simplicity let us keep the assumption of a **benevolent government** with a reasonably good knowledge of the taxpayers' true **preferences** for G, and of their income, expenditure, assets and liabilities, and other market indicators of their *wealth*, and therefore of their **ability to pay**.

We first recall a preliminary distinction already discussed in previous sections, between two different ways of charging for covering public expenditures. Tax-payers could be charged by fixing *directly* their individual **cost shares**, globally or separately for different public goods or groups of them, or *indirectly* by applying **specific or ad valorem taxes** on their **market transactions and/or assets**. The *first charging modality* resembles the pricing mechanism, and as such would carry *no* EB. The second modality can be converted into the quasi-price mechanism of the tax-price only through a rather roundabout transformation (see Section 5, Subsection *A proportional income tax system*, eqs (7.31-36)) whose 'pricing' dimension may not be so easily perceived by the taxpayer, while at the same time it does cause the well known distortions associated with non lump-sum taxation. The first modality looks in principle more attractive but it is difficult to imagine how it could be implemented in practice except in very circumscribed situations. As a consequence it is the second modality that serves almost everywhere as the general financing mechanism.

The qualitative distinction between the BP and the APP is *conceptually* independent from the previous one between *direct* and *indirect* cost shares. In short, if we look at F7.6 we see that charging - either directly, or indirectly by taxing market variables - on the basis of the BP means, for any given level of G, distributing the tax (cost) burden among **A** and **B** so as to bring their respective marginal costs as close as possible to their respective marginal benefits (respective marginal willingness to pay), *independently* of their respective wealth. On the other hand, charging according to the APP means, for any given level of G, distributing the tax (cost) burden among **A** and **B** in a more or less close proportion to their respective market wealth, independently of their respective marginal benefits. Specifically, if the government wants the cost share of G charged onto a taxpayer to be based more on some principle of **distributional equity** than on his individual preference for G, then it will link it to his ability to pay, which is clearly related to his market wealth.

Indeed, the concept of *distributional equity* may actually be split into two distinct 'views', one empasizing **equality**, the other **fairness** (imparzialità, equità). The APP is more closely associated with the **equality/egalitarian perspective**, but the BP may claim a certain **fairness value**, because even from a distributional point of view it seems fair that what people pay for public goods should bear *some relation* to their perceived benefit. The Figure provides an intuitive background to the thesis that a widespread reliance on the BP is more likely to produce some kind of social consensus on the politically chosen level of G than a widespread reliance on the APP.

In our stylized economy, if the government's priority on its expenditure policies were *electoral consensus*, then it would try to bring the distribution of the tax burden closer to the BP. If its priority were instead *distributional equality*, then it would bring that distribution closer to some notion of the APP. In either case it would choose the combination of charging modalities - direct charge or tax-base taxation - most appropriate to the pursuit of its priorities.

Public choices on G and its financing are made by definition either by *authority*, or by some kind of *democratic majority voting* procedure. In the latter case the adoption of the BP might be expected to produce less voting dispersion than the adoption of the APP. It must however be kept in mind that while the BP may be practicable in particular instances, adopting it as a general rule faces *informational difficulties* which are much greater than with the APP. In particular, assessing the (individual and social) willingness to pay for the satisfaction of non-rival public interests faces the **intrinsic obstacle of the free riding incentive to preference underreporting**, embedded in the very nature of non-rival interests. As a consequence, discrepancies between actual and estimated preferences may expose even a benevolent democratic government honestly trying to implement the BP to significant voting dispersion and electoral instability. By way of example, in F7.6 if the government wrongly estimated the two taxpayers to have the same preferences for G (say, the dotted line) it would then wrongly expect to obtain their agreement by providing G^* and charging each with 1/2 of the cost.

9. Lindahl equilibria, free riding, and public efficiency: a summary

We now try to put into a compact synthesis the complex conceptual background of the analysis developed in this Chapter.

In short, the $LETs$ *show that in principle* ***efficient unanimity resource allocations*** *between private and public uses with coercive personalized cost shares* ***do exist****. But they say nothing about a remaining* ***open question****. While the* $TWEs$ *convergence of efficiency and equilibrium in the commercial economy are (under highly restrictive conditions) the result of* ***impersonal market mechanisms****, the* $LETs$ *allocations and cost shares need the emergence of an* ***active political authority endowed with coercive power****, charged with the task of implementing them. This raises the* ***open question****: has the* ***finding*** *and* ***implementing*** *of such optimal allocations on the part of a political authority any chance of being* ***practically possible****?*

What follows is a reflection on this open question, and an attempt to provide tentative answers, keeping a link with what we regard as the best pertinent literature.

The analysis in the Chapter shows that the concepts and logic of **economic efficiency** can be extended from the world of **rival interests (private goods)** to the world of **non-rival (shared) interests (public goods)** without loss of generality. It also shows that the concept of an **efficient equilibrium** under competitive conditions in the market can be carried over into the non-market by converting it into the concept of an **efficient unanimity-based equilibrium under coercive cooperation**, implemented by a **benevolent government** endowed with a plausible knowledge of the taxpayers' preferences. The qualifying properties of such *Lindahl equilibrium* are thus two:

1) substitution of *free competition* among *power-free* individual agents with *government-implemented coercive cooperation* through the imposition of individual cost shares by means of the *power to tax*,

2) the government must be *benevolent*, and must have a *plausible knowledge* of the citizens' public preferences.

These are also the two properties that make the *public cooperative* Lindahl equilibrium into a substantially different concept from the *private competitive* market equilibrium. Whereas in the domain of rival interests, under idealized (and restrictive) conditions, the coordination of individual actions in the form of **free competition** operates under **effciency oriented incentives**, in the domain of non-rival public interests this is no longer true:

1) the coordination mode for the satisfaction of such non-rival interests is **not competiton** but **cooperation**.

2) a **voluntary, power-free cooperation** among agents entails **incentives which do not point towards efficiency**. Under purely voluntary cooperation the **intrinsic possibility of free riding** points towards the **underprovision** of public interests and, when the community is sufficiently large, towards no provision at all, because the single subject doesn't, and can't, know the true preferences of all others (see Chapter 6 on *Nash-Lindahl*). As a consequence

voluntary cooperation must be substituted by government-implemented coercive cooperation.

3) if we want government-implemented coercive cooperation to be oriented towards public efficiency, two more highly demanding requirements are needed. On the one hand the government must be a *benevolent* one, *not a rent-seeker*. On the other it must possess a *reasonably plausible kowledge* of the taxpayers' public preferences. But here again, 1) *first*, in the real world governments are *far from benevolent*, or in any case their level of benevolence is quite low, 2) *second*, even if they were benevolent and wanted to act on the basis of the true public preferences of taxpayers, the intrinsic possibility of free riding leads taxpayers to **underreport their true public preferences**, not only to each other, **but also to the government**, and this would in itself be a **source of underprovision**.

In short, we may **summarize the core of the problem** as follows. The satisfaction of public non-rival interests cannot be entrusted to the operation of the twin engines of *profit* and *competition* because by definition such interests cannot form the object of market transactions. It must therefore be entrusted not to competition but to cooperation. But *free cooperation* in this area faces **unsurmountable obstacles consisting in the intrinsic possibility of a - perfectly legitimate - free riding embedded in the very nature of non-rival interests**, and must therefore be substituted by **government enforced cooperation**. But since the satisfaction of non-rival interests is by definition open to the possibility of free riding, individuals have an incentive either to underreport, or to not report at all, their true public preferences, both *to each other*, when left to cooperate *voluntarily*, and *to the government*, when the latter steps in by *enforcing cooperation* through the use of its political coercive power to tax. The particular *behavioural and informational failures* arising in the context of *government-implemented coercive cooperation* have been much investigated by *theoretical economists*, *game theorists*, *experimental economists* and *political scientists*, with the aim of devising mechanisms capable to redress them. **Stiglitz**'s textbook **(2015,** pp. 249-65, in particular the *Appendix: new preference-revelation mechanisms*, pp. 262-5**)** contains a brief discussion and criticism of such attempts, developed in detail in the specialized literature.

For a more detailed presentation, and references to the original literature, see, among others, **Varian 1992** pp. 426-9, **Varian 2020** pp. 733-8, **Cornes & Sandler 1996**, pp. 198-239, **Mas-Colell et al. 1995** p. 373-4, 876-82).

But in his view (which I personally share) they are little more than sophisticated analytical exercises, with **scant theoretical and practical relevance**, and it is not surprising that none has so far been successfully implemented in the real economy, except in 'classroom' experiments. Indeed, in our opinion they will never be, because in the domain of public non-rival interests the absence of economic incentives to free productive coordination and true preference revelation (either to one another or to an outside authority) is embedded in the very nature of human social behaviour, and can only be compensated *from within*, through a **social-cultural revolution** which at present appears to be extremely unlikely (**Sawicky** 2020), because it requires:

1) the adoption of a **higher political-economic culture** on the part of the political élites (**Judt** 2009, **Castellucci & Gorini** 2014 pp. 8-13, **Gorini** 2023, **Mazzucato** 2018, 2021, **Mazzucato & Collington** 2023, **Stiglitz** 2019),

2) the stregthening and diffusion among a large majority of ordinary people of a new **moral-civic consciousness**, which is equivalent, in Mancur Olson's terminology, to an extraordinary improvement and diffusion of the citizens' endowment of **public human capital** (**Olson 1996** pp. 15-6),

3) bringing back into economics a **secular culture of the state** (**Gorini 2018** pp. 342-8).

As for the topic of **free versus coercive collective action in the public economy**, we have given here only some hints. A more 'analytical' treatment is developed in Chapter 6 on *Nash-Lindahl*.

See INDEX for list of
Sections
Appendices
Figures
Symbols and notations
References
[12/2021]