The structure of production

Since the MO96 positive approach to government *doesn't require a public budget constraint*, this is not contained in the eqs below describing the structure of production of the MOE.

The cost of producing *G* is a function $h(G)$ defined as the opportunity cost of *G* in units of *C* (the economy being in a state of full employment of the given input resources). Total output $Q = GDP$ is thus defined as $Q = C + h(G)$, where *C* is the 'commercial' part of output and $h(G)$ the conventional measure of the 'non-commercial' part. The amount of *tax-rent* extracted is τQ, where τ is the *average tax pressure* (tax-rate) on output.

The extraction of tax-rent decreases the economy's productivity while public investment increases it. Indeed, without *G* the economy's productivity would fall to zero (see next Section). Thus the RI interact with the economy in two opposite ways. With one hand they extract *tax-rent*, and in so doing they *decrease* the economy's productivity, with the other they provide, and bear the cost of, the public production goods necessary for the economy to be productive, and in so doing they *increase* the economy's productivity[1].

The structure of production is thus defined by the following equations:

3.1
$$\begin{aligned} Q &= \eta(\tau)X(G) = Q(\tau, G) \\ &= C + h(G) \\ &\rightarrow C = \eta(\tau)X(G) - h(G) = C(\tau, G) \\ &\rightarrow T = \tau Q(\tau, G) = \tau\eta(\tau)X(G),\ \tau \in [0,1] = T(\tau, G) \end{aligned}$$

represented graphically by the curves drawn in **F3.1**.

$X(G)$, drawn in *black*, is **total (full employment=potential) output** as an increasing function of *G*. This function is a purely 'technological' relationship, independent from whether *G* is tax- or deficit-financed, on the benchmark assumption that if it were tax-financed the tax would be non-distortionary (would cause no decrease in productivity),

$h(G)$, drawn in *red*, is the **opportunity cost of** G,

$Q(\tau, G)$, drawn in *black*, is the **production frontier** or **output function**, **PFR** or **OF** for brevity, divided into a 'commercial' part C and a 'non-commercial' part $h(G)$. It is positive in G and negative in τ,

$C(\tau, G)$, drawn in *blue*, is the **consumption frontier**, **CFR** for brevity,

$T(\tau, G)$, drawn in *brown*, is the **tax-rent revenue function**, **TRF** for brevity,

$\eta(\tau)$, is the **macro coefficient of excess tax burden**, **CEB** for brevity. It is a decreasing function of the **average tax pressure** $\tau \in [0,1]$, representing the % of potential output $X(G)$ produced at different levels of τ, the reduction being caused by the **excess tax burden** of the tax system, **EB** for brevity [2].

In a *macro setting* like the MOE all types of tax-generated social losses are put together into a *reduction of potential output* represented by the CEB. We assume the CEBs to have shapes like those depicted in **F3.3**, ranging between the *two polar cases drawn in thick black* in the Figure. At one extreme the CEB shape $\eta(\tau) = 1 - \tau$ can be viewed as representing the case of *maximum tax-*

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[1] *Tax-rent* is the terminology used in the MO96 paper, in line with the authors' 'positive' approach to government, where taxes are interpreted *sic et simpliciter* as the primary form of *public rent*.

[2] In the tax literature the EB is the *social welfare loss* caused by the levying of a tax. In a *micro setting* the simplest picture of the EB is drawn in **F3.2**. In the market of a good/service *X*, a *specific tax* yields 1) a *tax revenue* measured by the *blue area*, which in terms of the concept of *rent-extraction* (see **CHAPTER 1** (Section 1 – *Introduction to rent extraction*) must be viewed as an extra-gain (surplus) to the tax-raising agent (the government), 2) a loss (of surplus) suffered by the taxpayers, measured by the *red area*, and 3) a *social loss*, measured by the *orange area* - equal to the excess of the loss (red) over the extra gain (blue). This social loss is called the *excess burden* of the tax.

distortion (maximum EB), where taxes cause some maximum output reduction (CEB curves convex towards the origin can be ruled out as intuitively unrealistic). At the other extreme the CEB shape $\eta(\tau)=1$ represents the case of *minimum tax-distortion (zero EB)*, where taxes cause no output reduction. *Actual shapes* of the CEB are more likely to be the convex ones *drawn in red* in the Figure, because actual tax systems may vary greatly from the point of view of their 'efficiency', and the more 'efficient' (less distortionary) ones correspond to CEB curves more convex towards the top-right [3].

Using the $T(\tau,G)$ function in eq (1) we now define the BB eq

3.9
$$\begin{aligned} &T(\tau,G)=\tau\eta(\tau)X(G)=h(G) \\ &\rightarrow G_B(\tau),\ \tau\in[0,1] \\ &\rightarrow \tau(G_B),\ G_B\in[0,\bar{G}_B] \end{aligned}$$

where G_B denotes the BB level of G, and $\bar{G}_B$ the maximum such level. This BB condition (4.9) generates an implicit function $G_B(\tau)$, rising in τ up from 0 to $\bar{\tau}$ and then decreasing back to zero when $\tau\rightarrow 1$. This is visualized in the bottom panel **F3.5b**. The G_B level, determined by the intersection between the TRF defined in (3.1) and the cost curve $h(G)$, rises with τ as long as the TRF rotates upwards, reaches its maximum when the TRF stops increasing at $\tau=\bar{\tau}$, and decreases back towards zero as the TRF rotates backwards when τ continues towards unity. This maximum attainable BB level of *G* has a special place in the MOE, and we therefore give it the special notation $\bar{G}_B$.

But in what follows we need to change $G_B(\tau)$ into its *inverse* $\tau(G_B)$, splitting $G_B(\tau)$ into its two parts and taking only the *lower part*, defined over the interval $G\in[0,\bar{G}_B]$, the upper part being of no economic interest.

Production frontier, consumption frontier and tax revenue function under balanced budget. The case of excess burden

The construction of the PFR, CFR and TRF under BB - **PFR$_B$**, **CFR$_B$**, **TRF$_B$** for brevity – is easily obtained by inserting the (inverted) BB function $\tau(G_B)$ into the output function $Q(\bullet)$ [4]. Using (3.1), and writing G_B in place of G to remind that we are working with the BB level of *G*, this yields

3.10
$$\begin{aligned} &Q\left(\tau(G_B),G_B\right)=\widehat{Q}(G_B)=\eta\left(\tau(G_B)\right)X(G_B) \\ &C\left(\tau(G_B),G_B\right)=\widehat{C}(G_B)=\eta\left(\tau(G_B)\right)X(G_B)-h(G_B) \\ &T\left(\tau(G_B),G_B\right)=\widehat{T}(G_B)=\tau(G_B)\eta\left(\tau(G_B)\right)X(G_B)=h(G_B) \end{aligned}$$

Under BB, as G increases also τ must increase by the amount required to keep the budget balanced. This causes a *downward bending* of $\widehat{Q}(G_B)$ and thus also a *shrinking* of $\widehat{C}(G_B)$. Such

[3] The red curves drawn in **F3.3** have *uniform convexity* towards the top-right, with slope -1 at the point of their intersection with the 45° line τ, but this is only a matter of graphic and analytical convenience, with no particular economic meaning (except for the assumptions underlying the construction of **F3.7** in Section 6). With such uniform convexity the *rising blue lines* $\left(-\dfrac{\eta(\tau)}{\eta'(\tau)}\right)$ drawn in **F3.4** (whose construction and use is explained below) would always intersect their corresponding red curves precisely where these intersect the 45° line τ (because that is where $-\eta'(\tau)=1$). No need for this to happen with more general shapes of the red lines, such as those drawn in **F3.3bis**. Notice further that it would be more general to assume the CEBs to depend both on τ *and* G, with $\eta(\tau,G)$, but the simpler option is clearly an acceptable first approximation (MO96, p. 76 footnote 3).

[4] As a matter of notational convention we introduce here the *cap* notation $\widehat{f}(\bullet)$ to indicate a *composite function* in compact form

downward bending and shrinking continue until τ reaches its threshold level $\bar{\tau}$ and G_B reaches its maximum $\bar{G}_B$. At $\bar{G}_B$ the PFR$_B$ and CFR$_B$ fall down vertically onto the G-axis, because when τ reaches $\bar{\tau}$ and G_B reaches $\bar{G}_B$ total output and consumption are *not zero*. In particular total consumption is

$$\widehat{C}(\bar{G}_B) = \widehat{Q}(\bar{G}_B) - h(\bar{G}_B) \neq 0$$

Using points **P1**, **P2**, **P3** we've drawn such 'corner' level of total consumption as the distance $\overline{\mathbf{P2}\text{-}\bar{G}_B} = \overline{\mathbf{P1}\text{-}\mathbf{P3}}$.

As for the $\widehat{T}(G_B)$ curve, it coincides by construction with the $h(G_B)$ curve, up to $\bar{G}_B$. In **F3.5a** these new PFR$_B$, CFR$_B$ and TRF$_B$ are the smaller thick *black* and *blue* curves and the new thick *brown* curve drawn over $h(G)$.