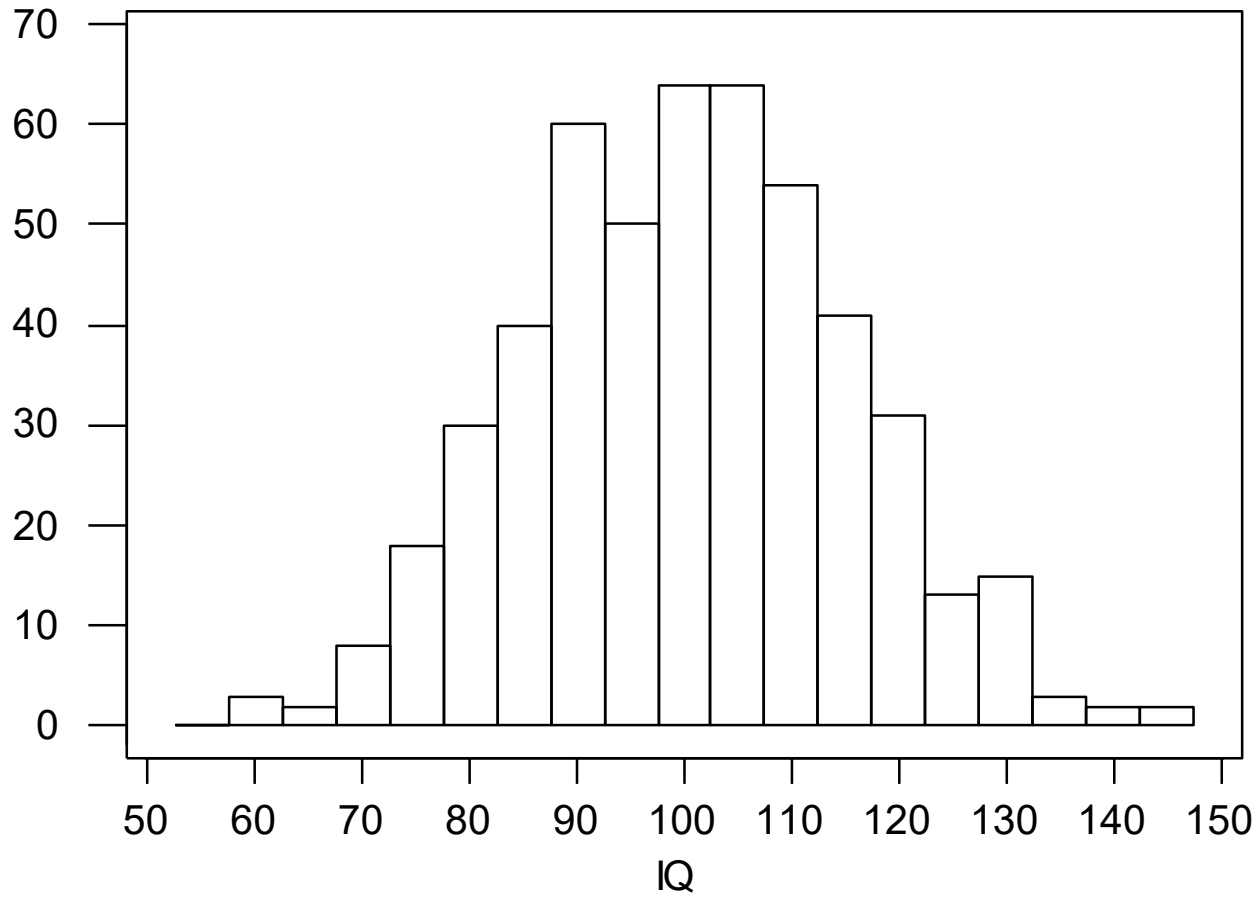


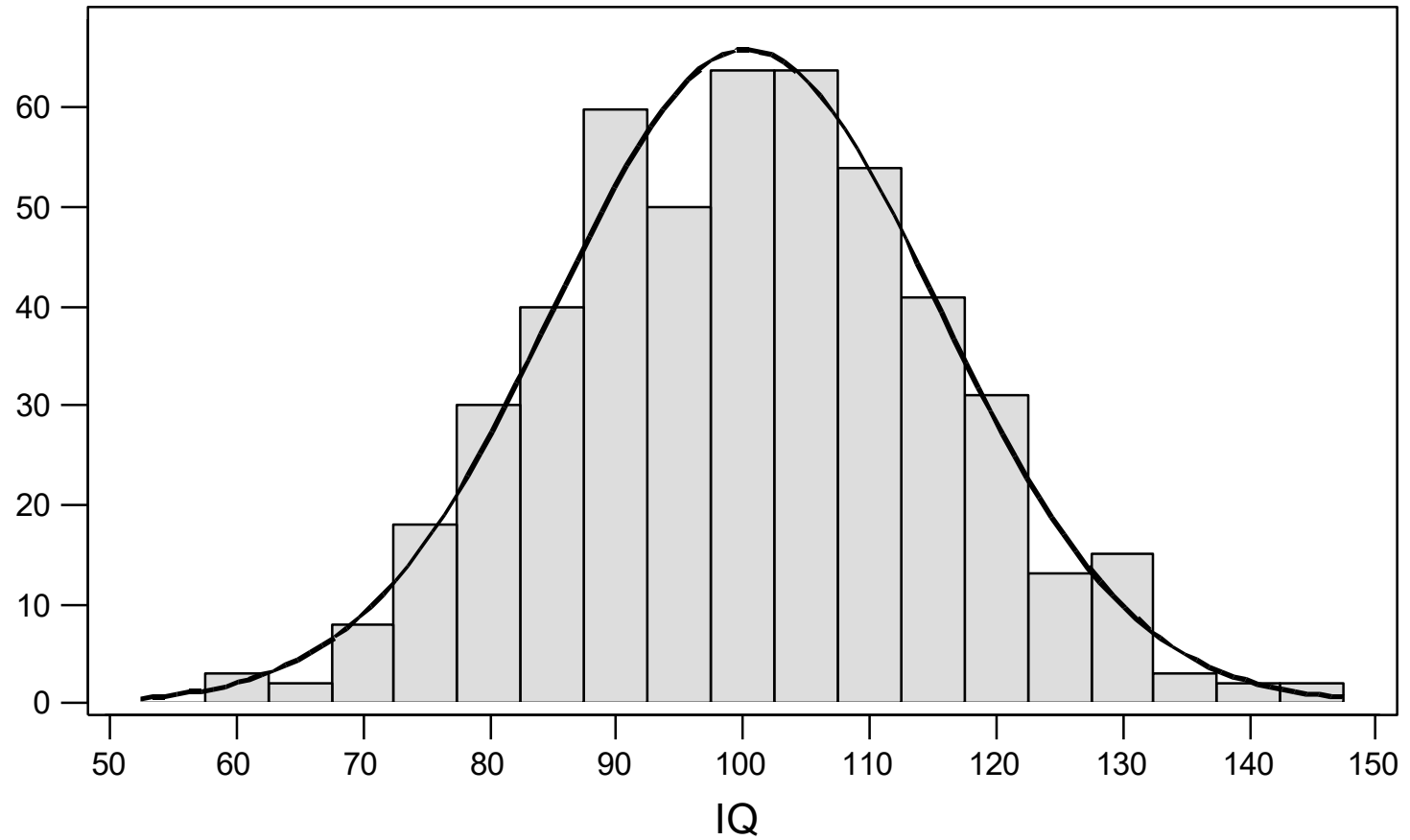
Variabili aleatorie Continue

Maura Mezzetti

Histogram of IQs of 500 randomly selected people

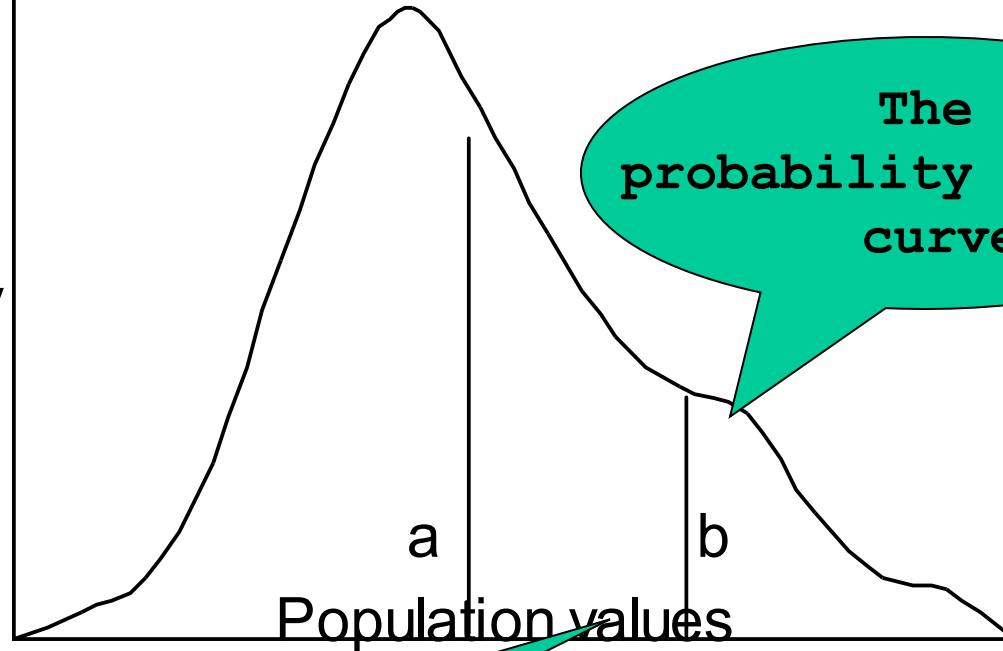


Histogram of IQ data, with 'Model' Curve



Hypothetical Distribution

Relative
frequency

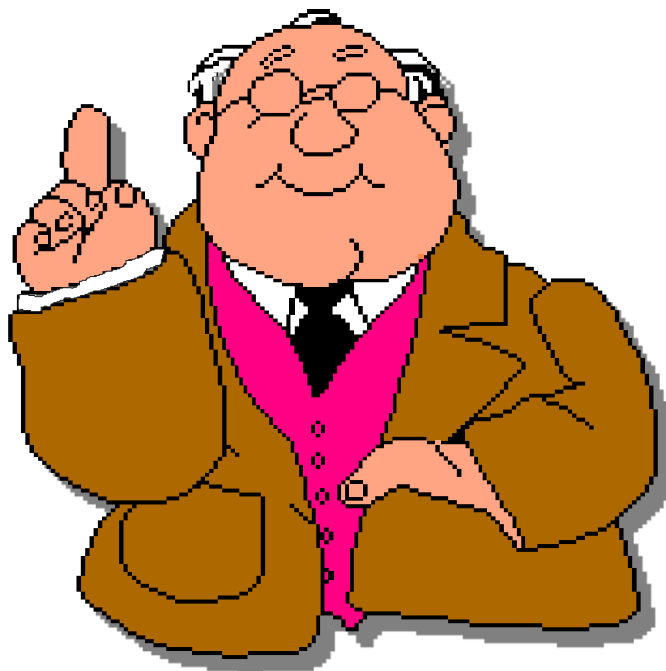


The
probability density
curve

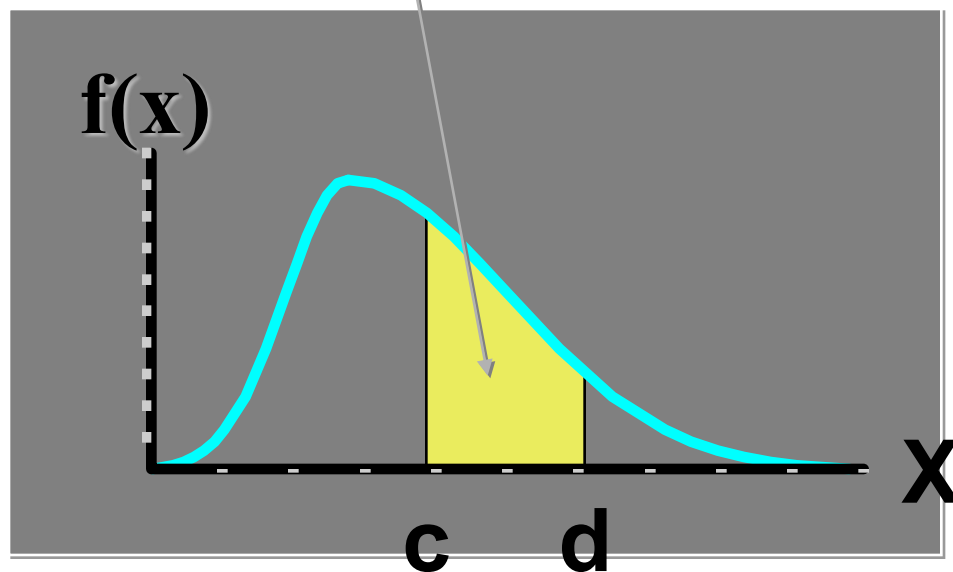
This area equals
 $P(a < X \leq b)$

Variabile aleatoria continua

**La probabilità è
l'area sotto la curva!**



$$P(c < X \leq d) = \int_c^d f(x) dx$$



Distribuzione di Probabilità continua

- Non si può parlare di probabilità di assumere un valore particolare
- $P(X=a)=0$
- Si parla di probabilità di assumere un valore dentro un intervallo
- La probabilità che la x assuma un valore tra x_1 e x_2 è definita come l'area sotto la curva

- $f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

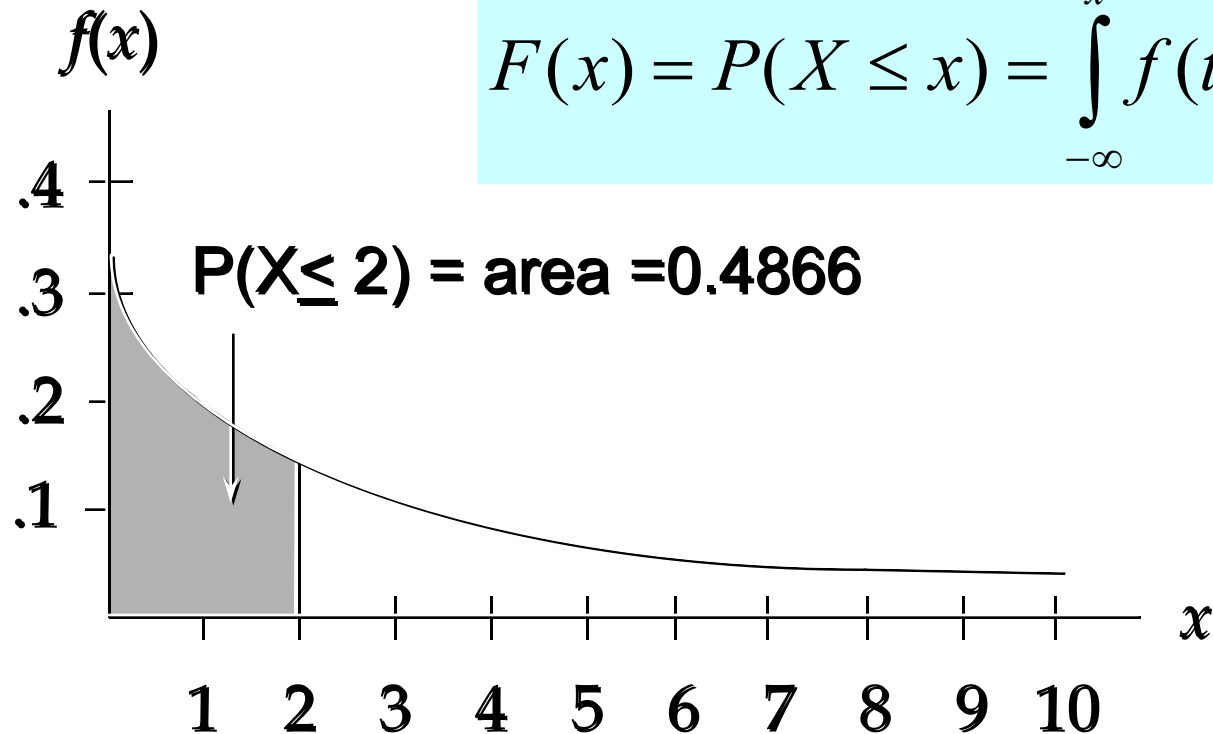
$$\int_a^b f(x) dx = P(a < X \leq b) = P(a < X < b) = P(a \leq X \leq b)$$

-

for each $a < b$

Funzione di distribuzione cumulativa

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$



- F è non decrescente

se $x_1 < x_2$ allora $F(x_1) \leq F(x_2)$

- $F(-\infty)=0$ e $F(\infty)=1$
- $P(X > x) = 1 - F(x)$
- $P(x_1 < X \leq x_2) = F(x_2) - F(x_1)$

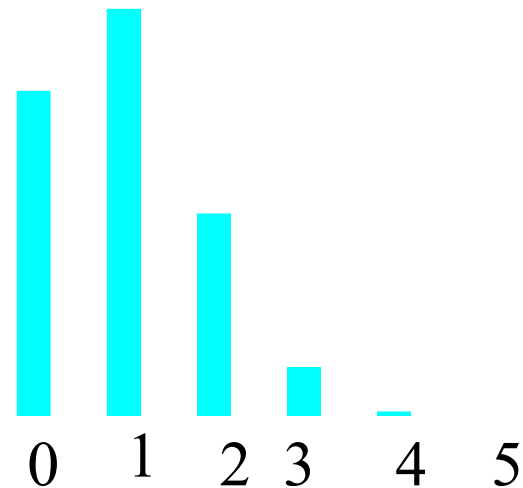
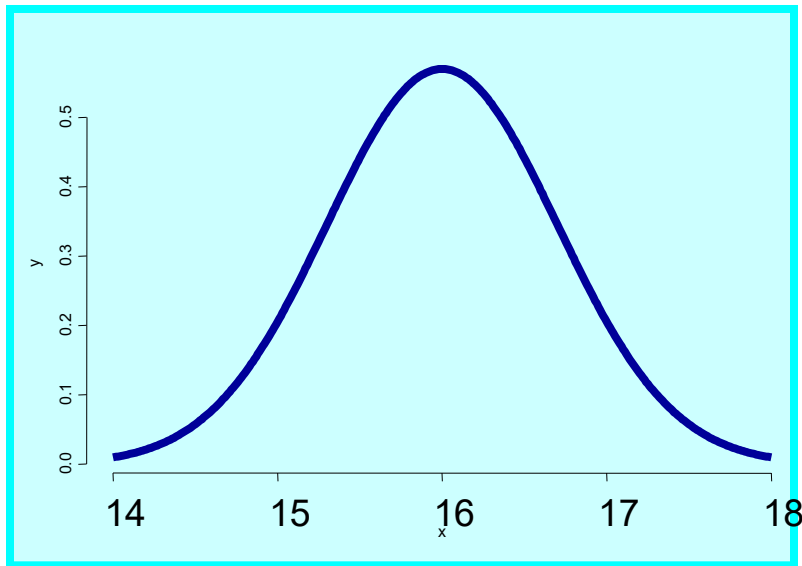
Continua vs Discreta

$$f(x)dx = P(x < X \leq x + Dx)$$

$$p(x) = P(X = x)$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\sum_x p(x) = 1$$



Continua vs Discreta

- Funzione di ripartizione

$$F(x_0) = \int_{-\infty}^{x_0} f(t) dt$$

$$P(a < X \leq b) = \int_a^b f(t) dt$$

$$F(x_0) = \sum_{x \leq x_0} p(x)$$

$$P(a < X \leq b) = \sum_{a < x \leq b} p(x)$$

- Valore atteso

$$E(X) = \int_{-\infty}^{\infty} t f(t) dt$$

$$E(X) = \sum_x x p(x)$$

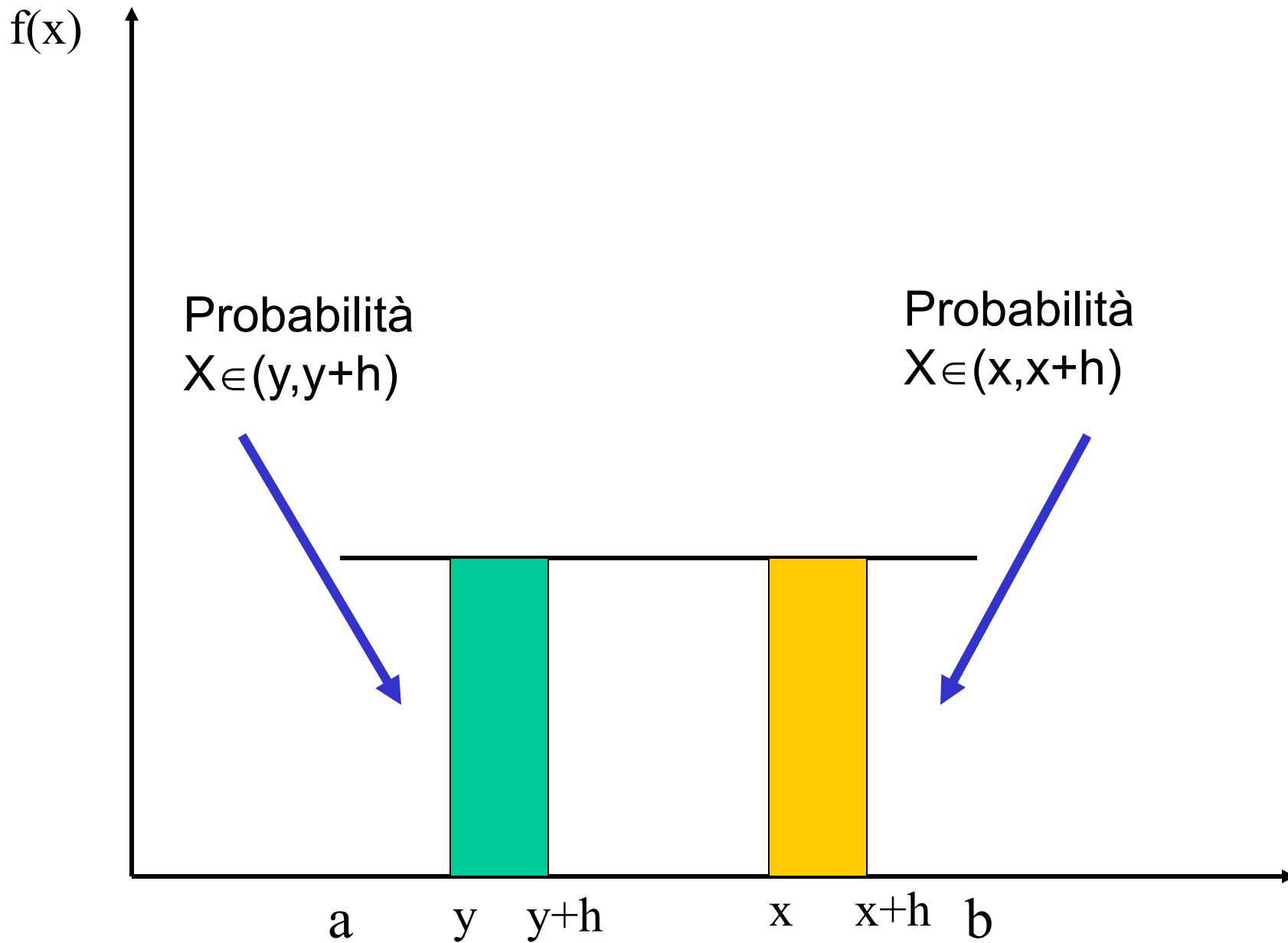
- Varianza

$$Var(X) = \int_{-\infty}^{\infty} (t - E(X))^2 f(t) dt$$

$$Var(X) = \sum_x (x - E(X))^2 p(x)$$

La distribuzione Uniforme

Una variabile aleatoria continua è distribuita in modo Uniforme se la probabilità che X appartenga ad un intervallo è proporzionale alla lunghezza stessa dell'intervallo



Una variabile aleatoria X è distribuita uniformemente tra a e b se la funzione di densità ha la seguente espressione

$$f(x) = \frac{1}{b-a} \quad x \in [a, b]$$
$$0 \quad \textit{elsewhere}$$

- Valore atteso di X

$$E(X) = (a + b)/2$$

- Varianza di X

$$\text{Var}(X) = (b - a)^2/12$$

dove

a = valore minimo

b = valore massimo

Esempio: Slater's Buffet

I clienti di un ristorante pagano per la quantità di insalata che prendono. La quantità di insalata è distribuita in maniera Uniforme tra 200 grammi e 800 grammi.

Densità

$$\begin{aligned} f(x) &= 1/600 \quad \text{per } 200 \leq x \leq 800 \\ &= 0 \quad \text{elsewhere} \end{aligned}$$

x = peso del piatto di insalata

Example: Slater's Buffet

qual è il valore atteso?

Expected Value of X

$$\begin{aligned} E(X) &= (a + b)/2 \\ &= (200 + 800)/2 \\ &= 500 \end{aligned}$$

Variance of X

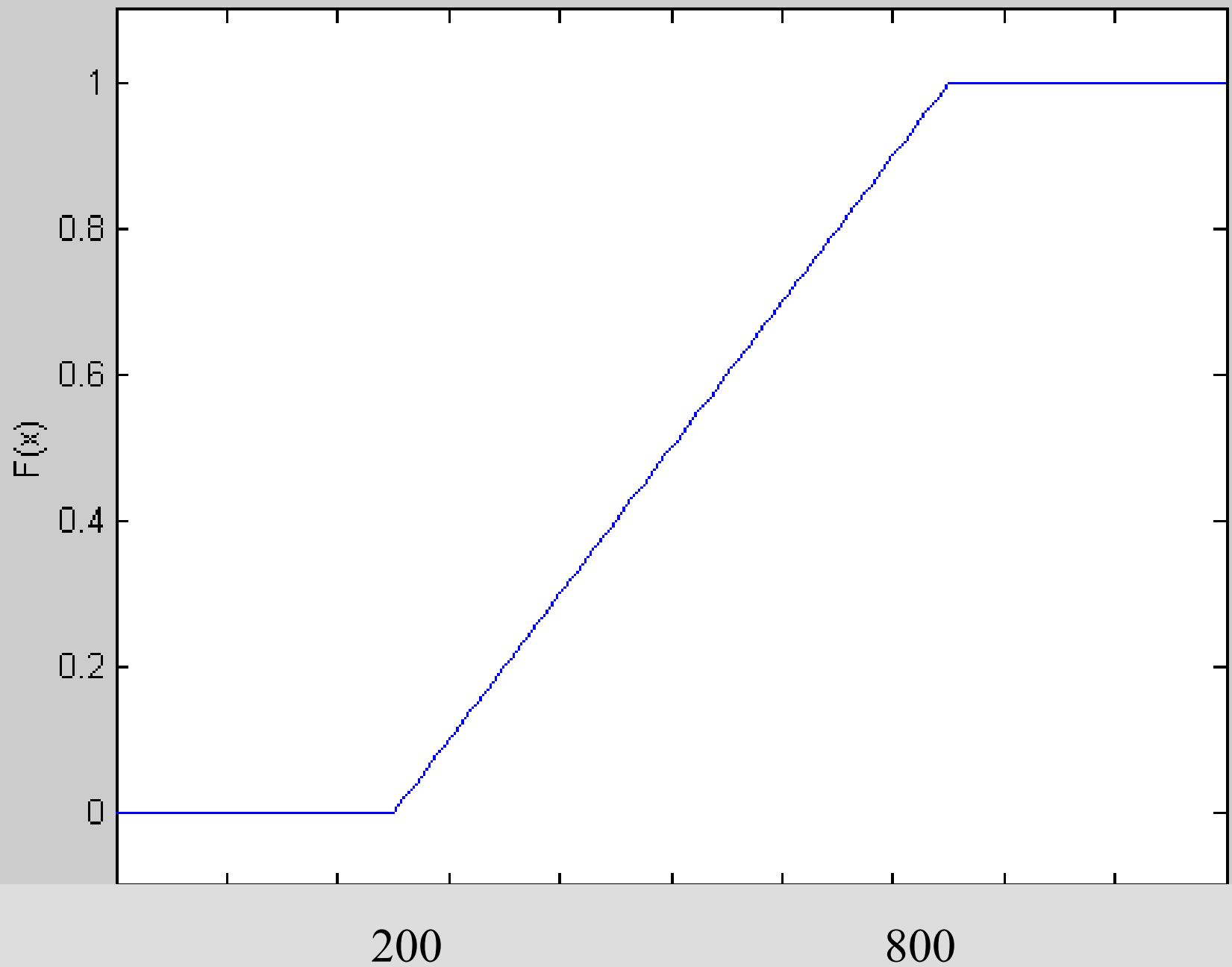
$$\begin{aligned} \text{Var}(X) &= (800 - 200)^2/12 \\ &= (600)^2/12 \\ &= 30000 \end{aligned}$$

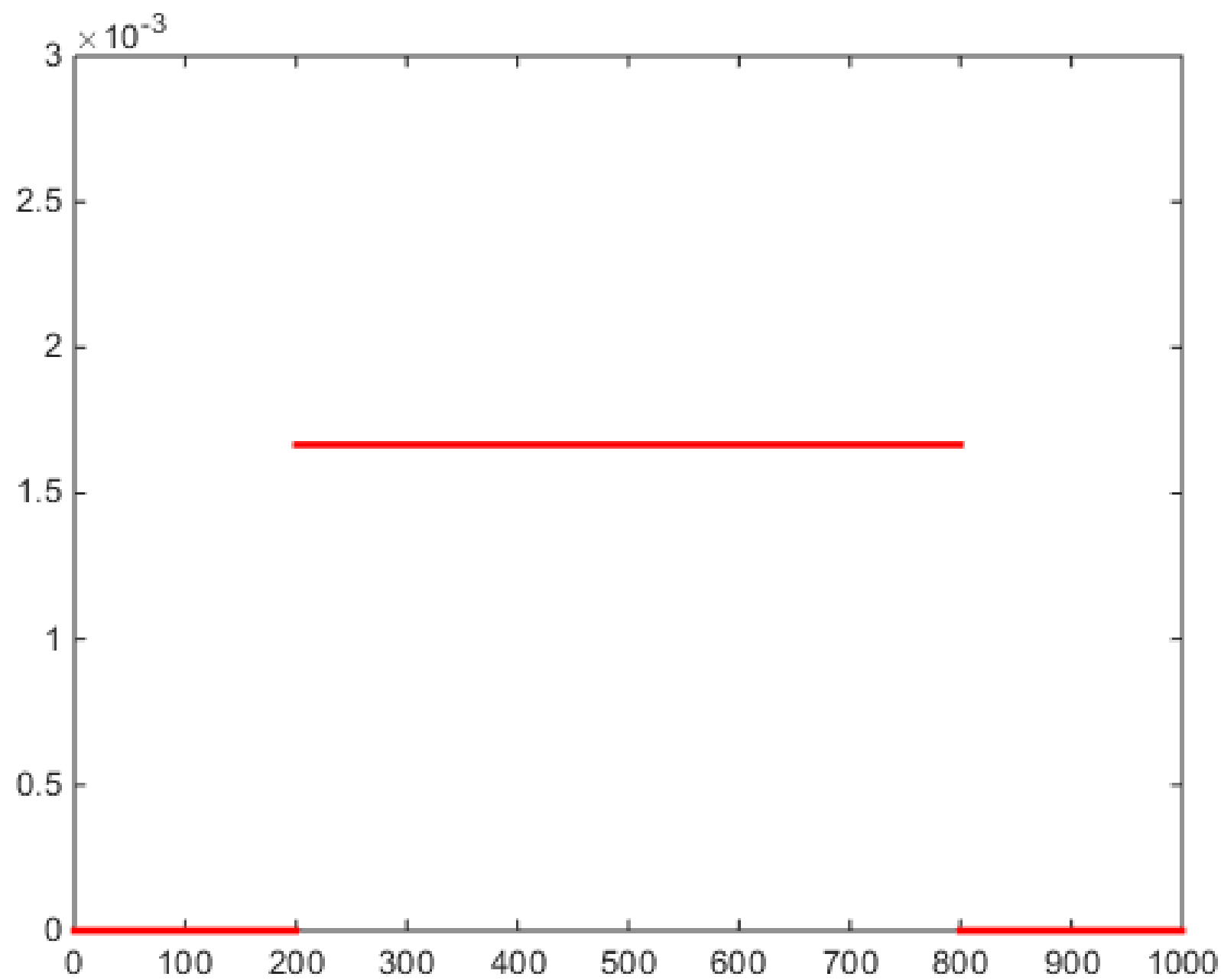
Probabilità un cliente prende meno di
300 grammi

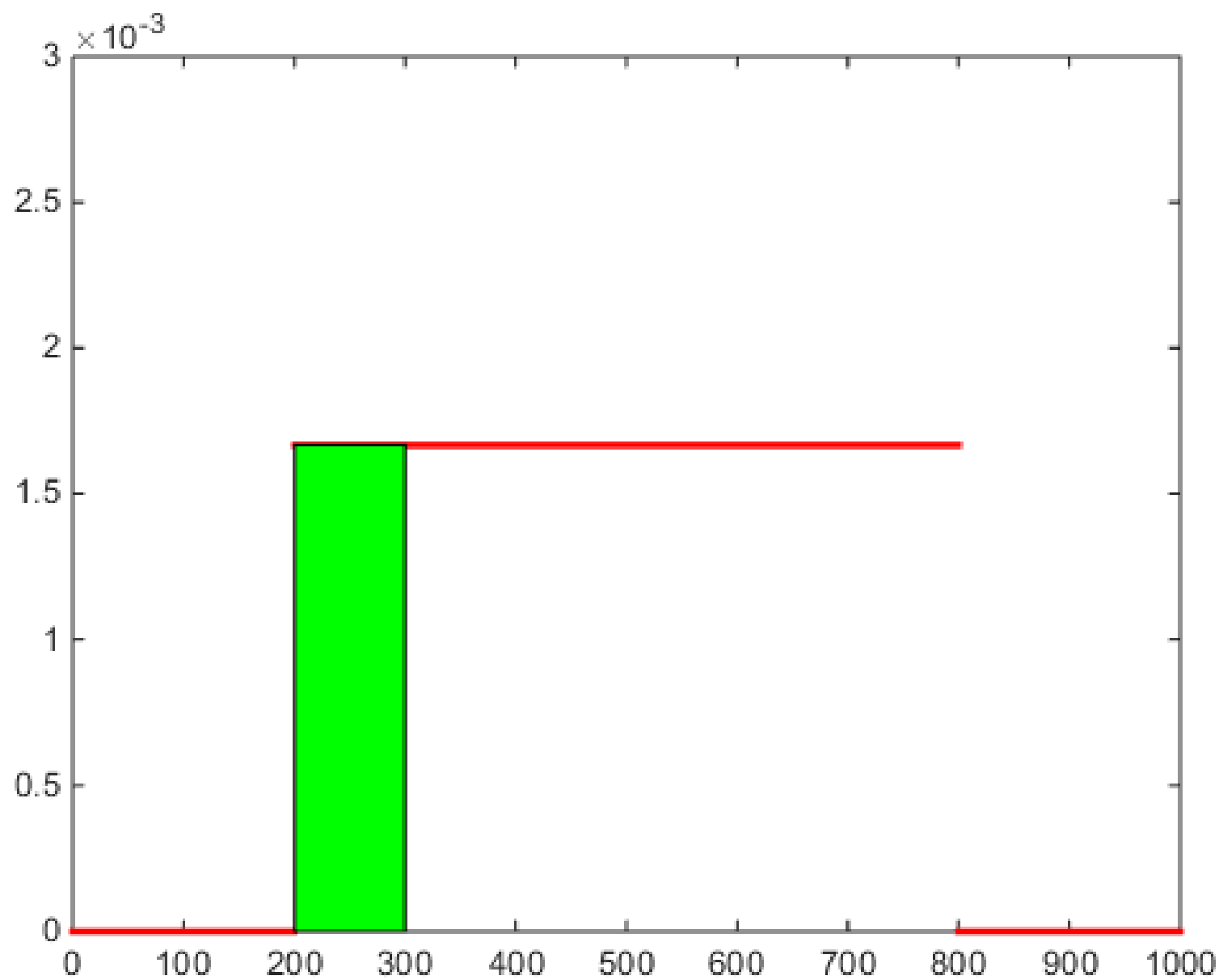
$$P(X \leq 300) = \int_{200}^{300} \frac{1}{600} dx = \left. \frac{x}{600} \right|_{200}^{300} = \frac{100}{600} = \frac{1}{6}$$

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

cumulative distribution function







Esercizio

Il tempo di attesa in posta si distribuisce uniformemente tra 5 e 45 minuti.

Qual è la probabilità che un cliente scelto a caso aspetterà più di 30 minuti?

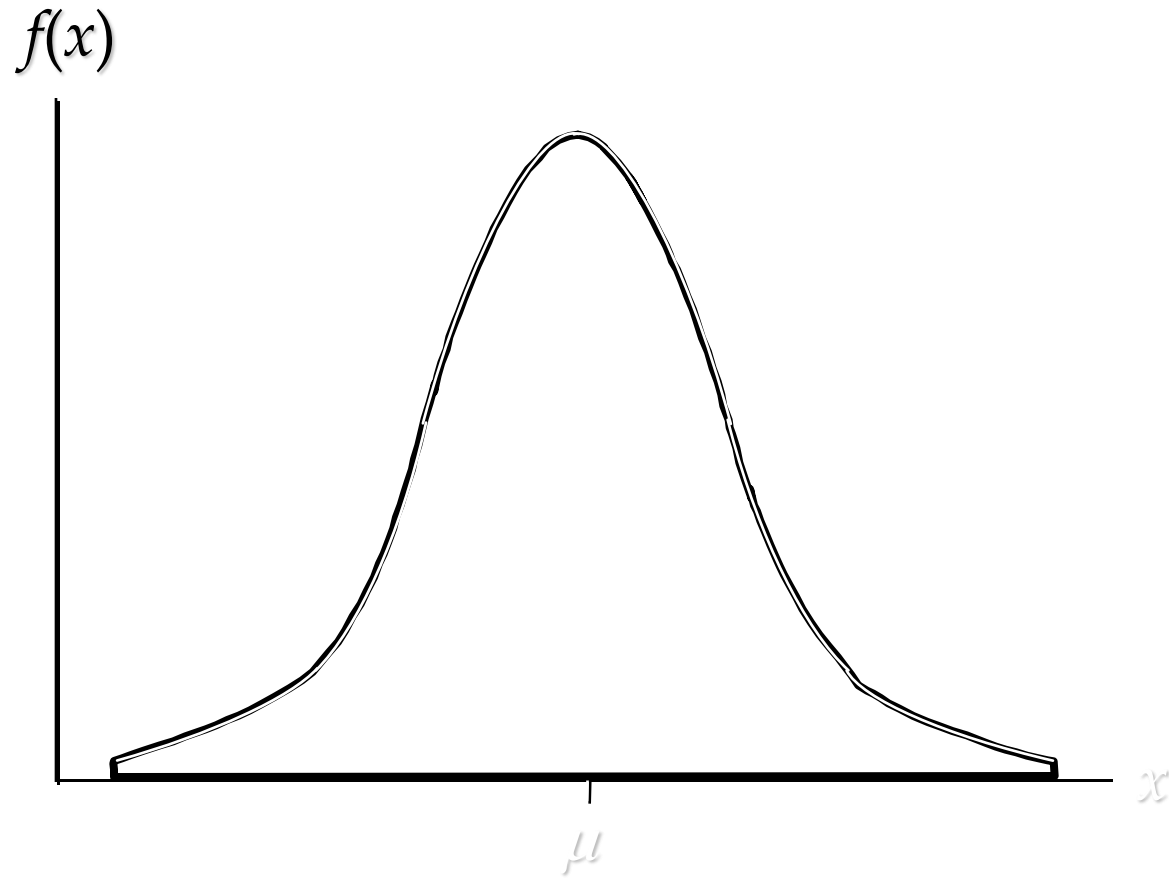
0.375

Qual è il tempo previsto di attesa?

25 min

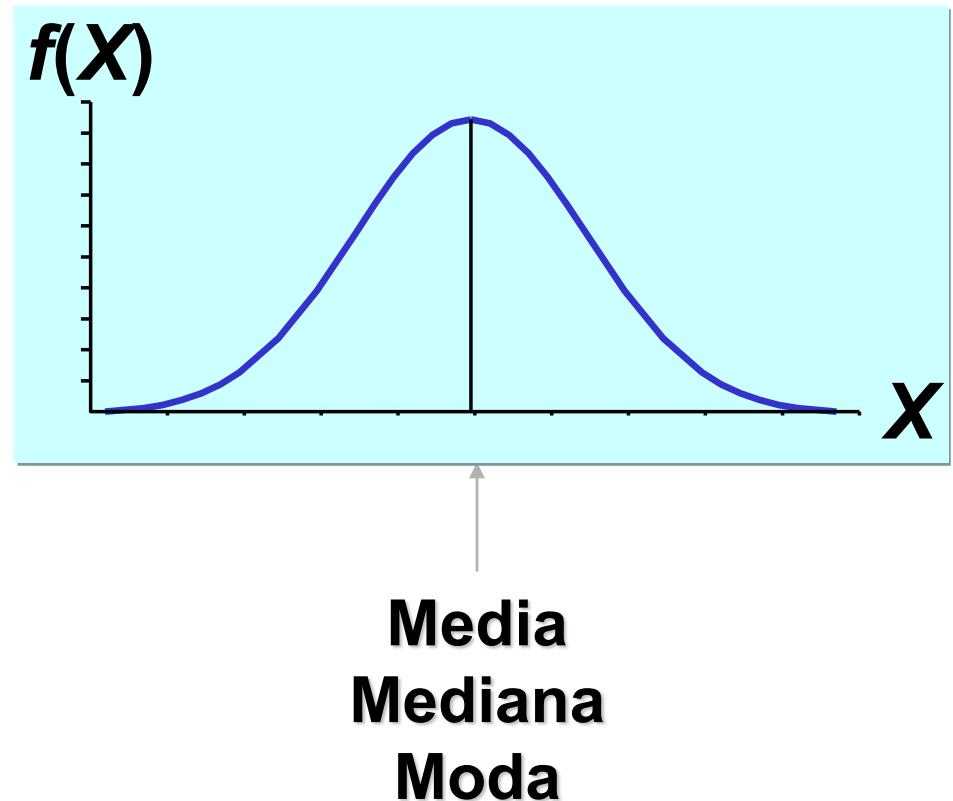


La distribuzione Normale: grafico della distribuzione



La Distribuzione Normale

- 1. Forma “campanulare” e Simmetrica
- 2. Determinata da due parametri, media μ , e deviazione standard σ
- 3. Media, Mediana, Moda sono tutti uguali
- 4. $(Q_3 - Q_1) = 1.33 \sigma$
- 5. Va da $-\infty$ a $+\infty$



Funzione di Densità

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\left(\frac{1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2}$$

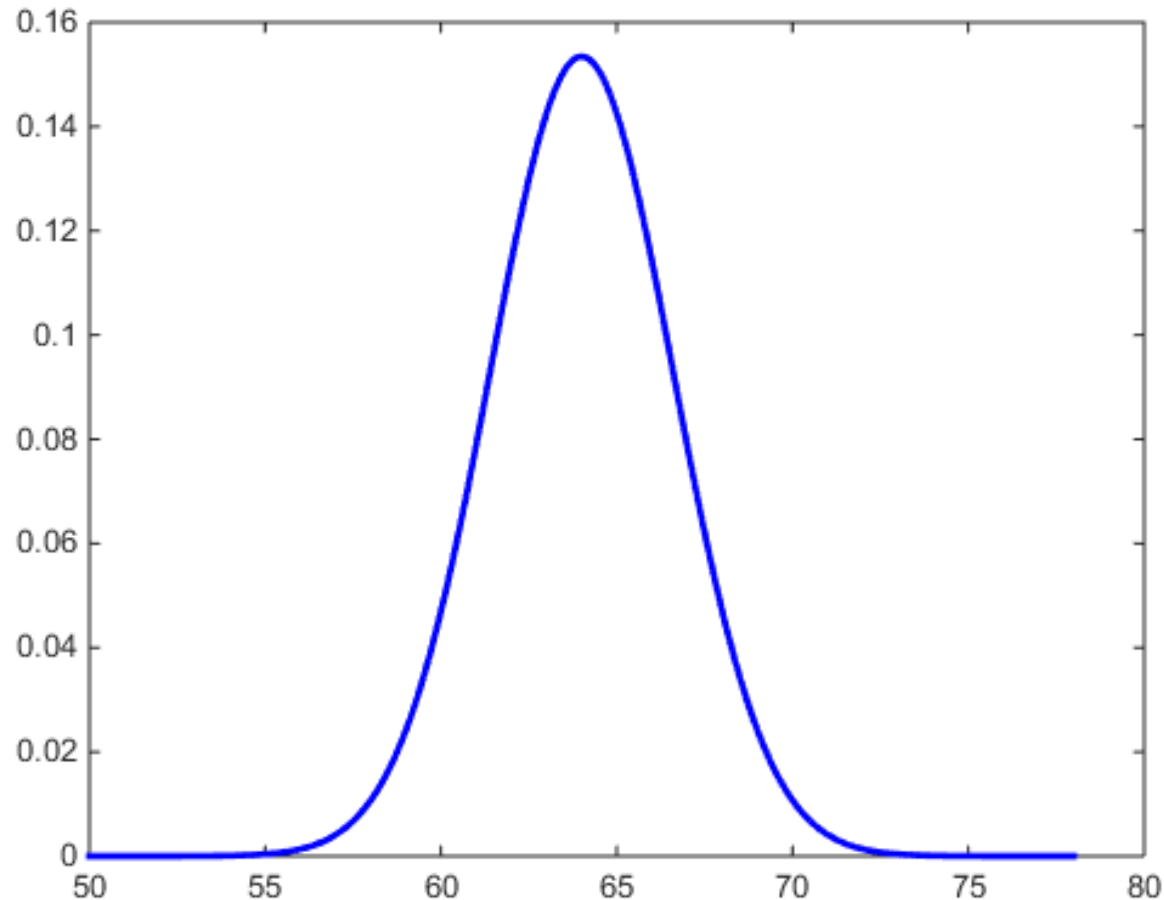
$f(x)$ = funzione di densità

σ = Deviazione Standard

π = 3.14159; e = 2.71828

x = Valore della Variabile aleatoria
($-\infty < x < \infty$)

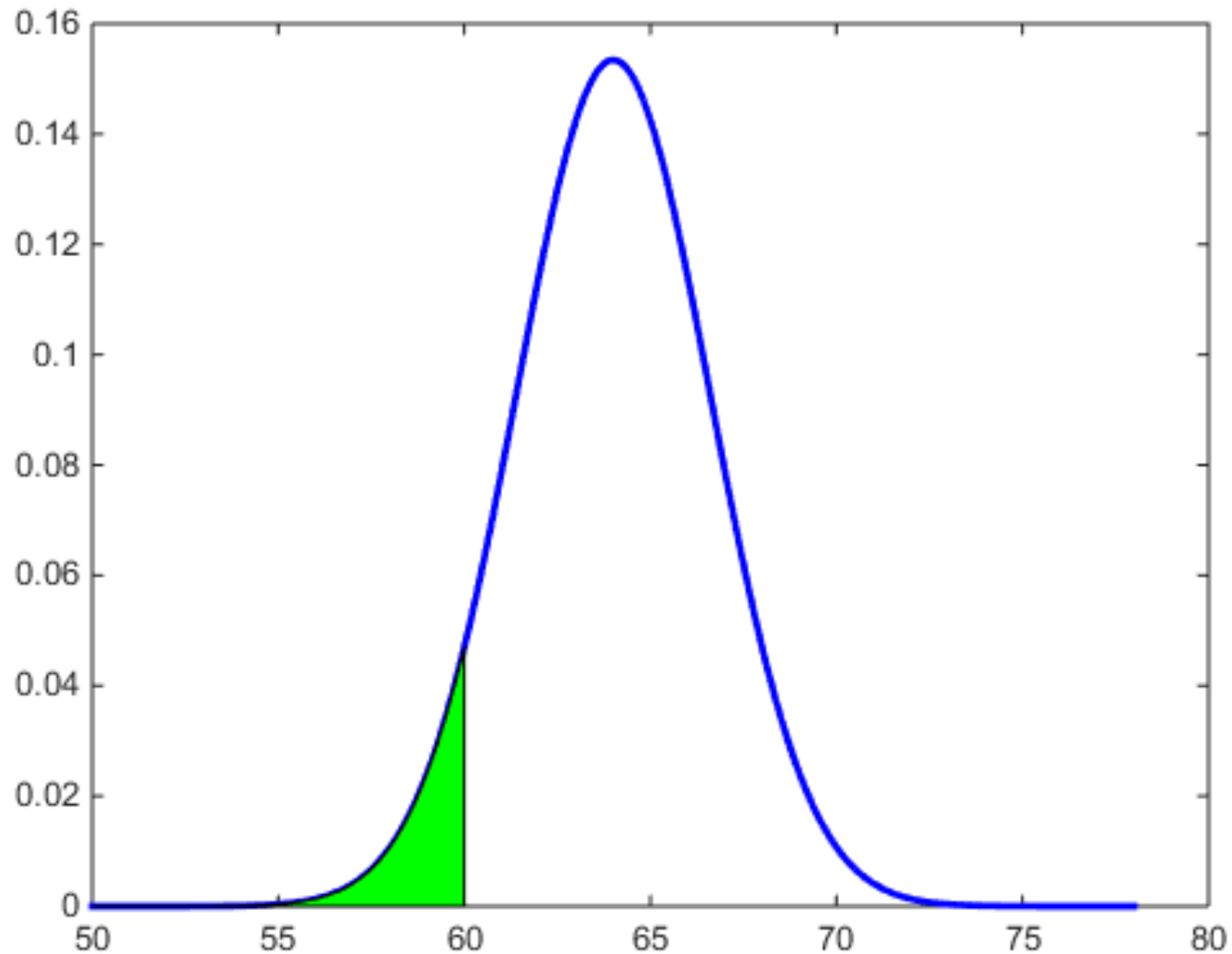
μ = Media



Il risultato al test di ingresso alla Facoltà di Economia si distribuisce come una Gaussiana con $\mu=64$ $\sigma=2.6$

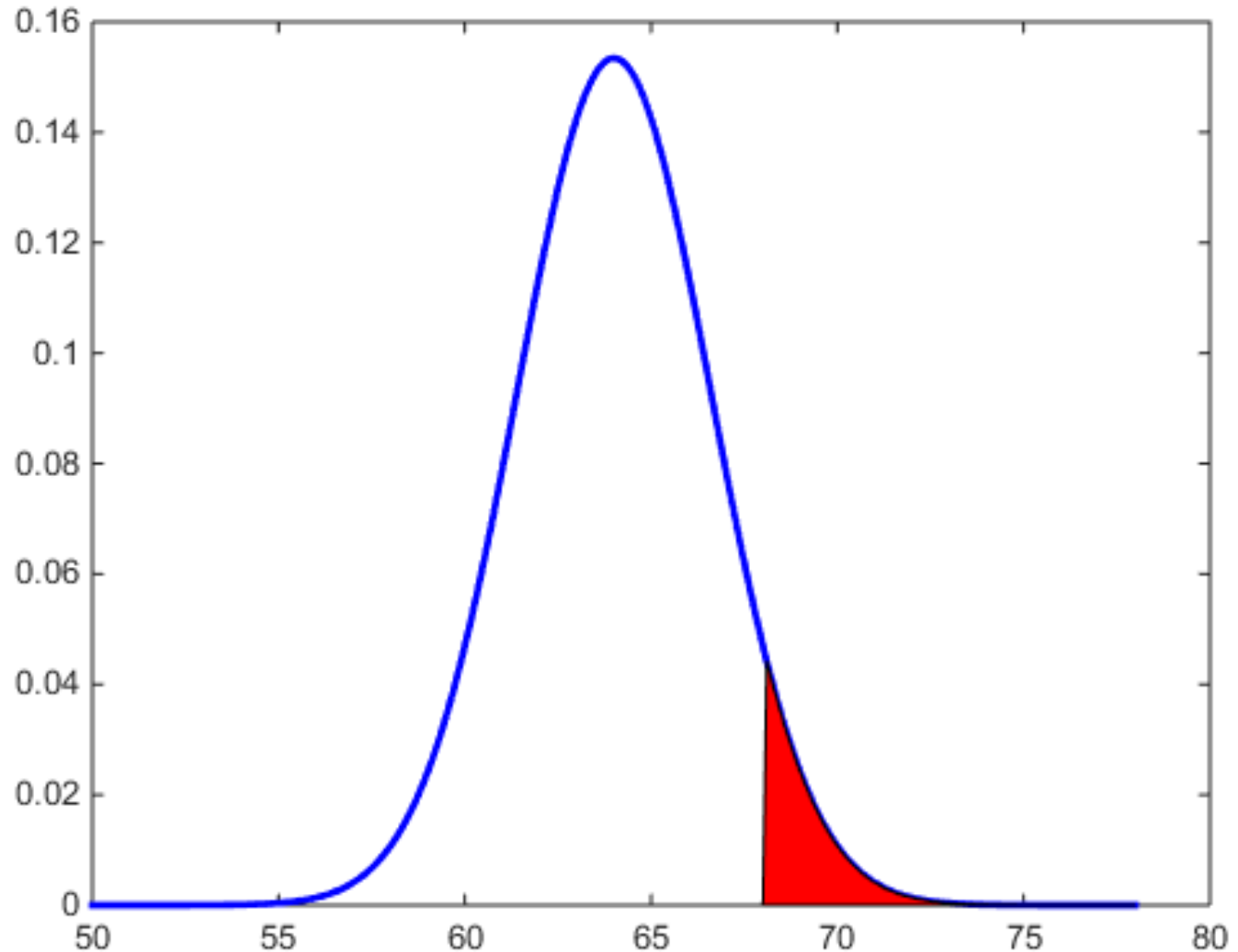
L'area sotto la curva rappresenta la probabilità

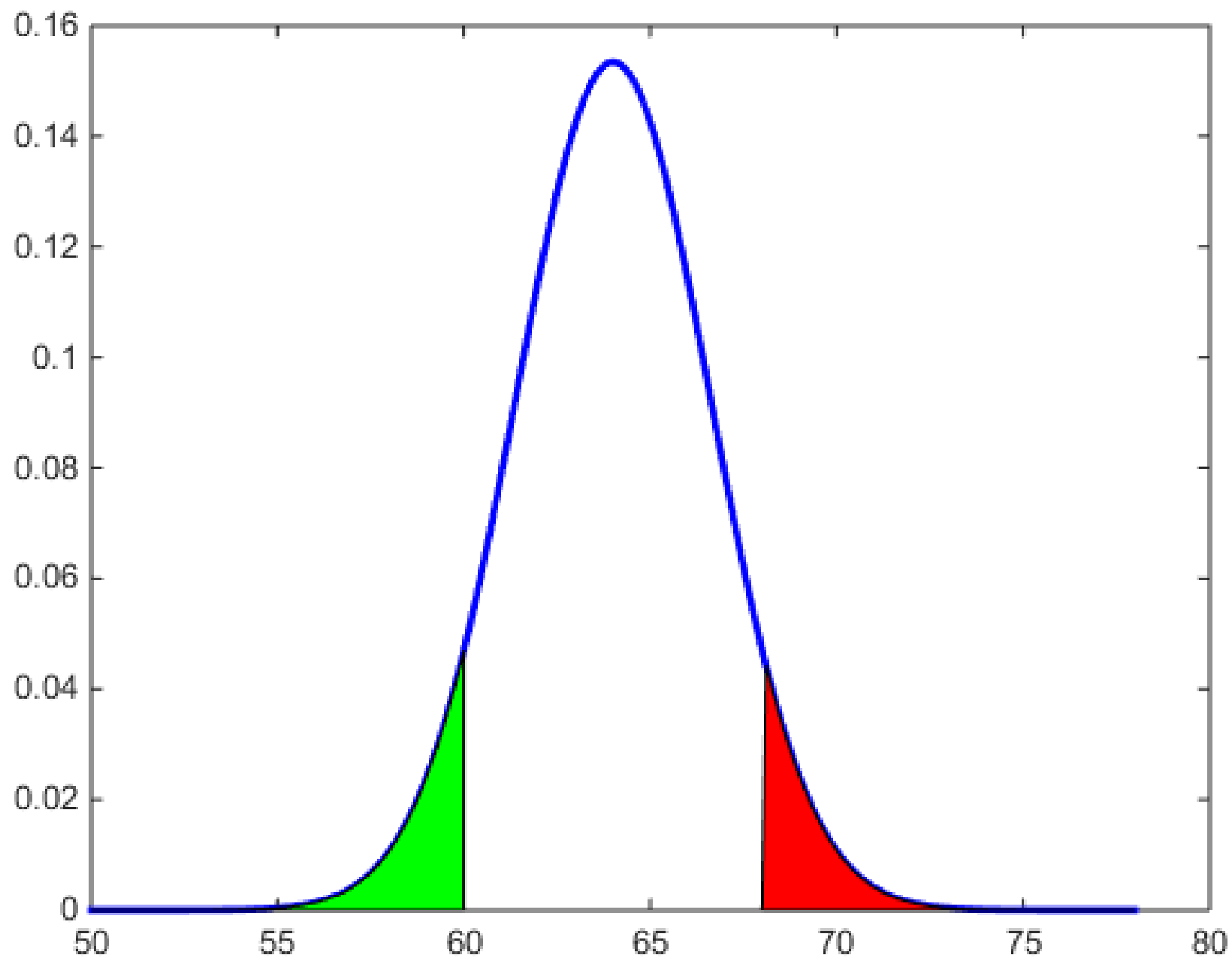
Qual è la probabilità che uno studente preso a caso abbia un risultato inferiore a 60



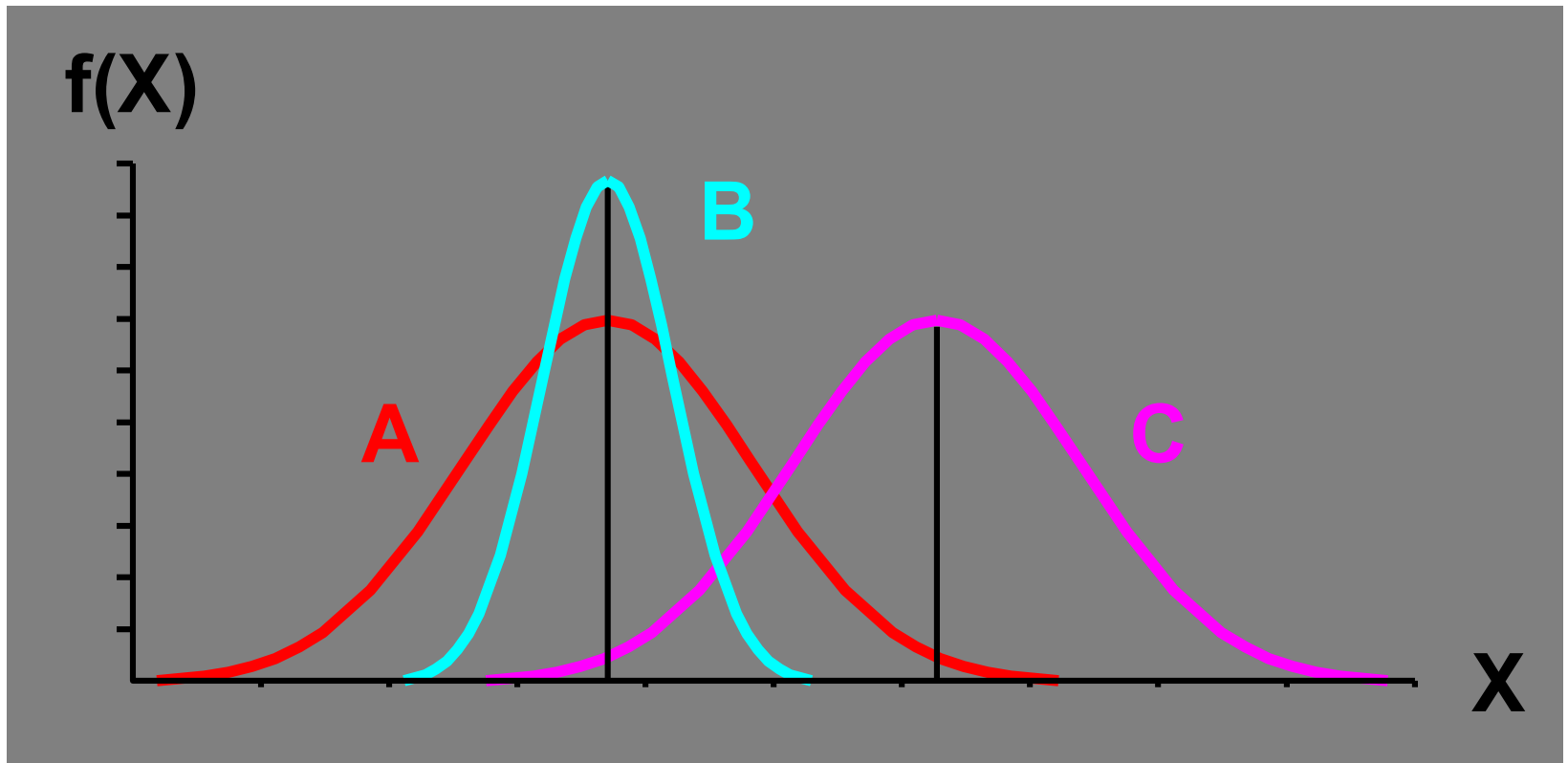
L'area sotto la curva rappresenta la probabilità

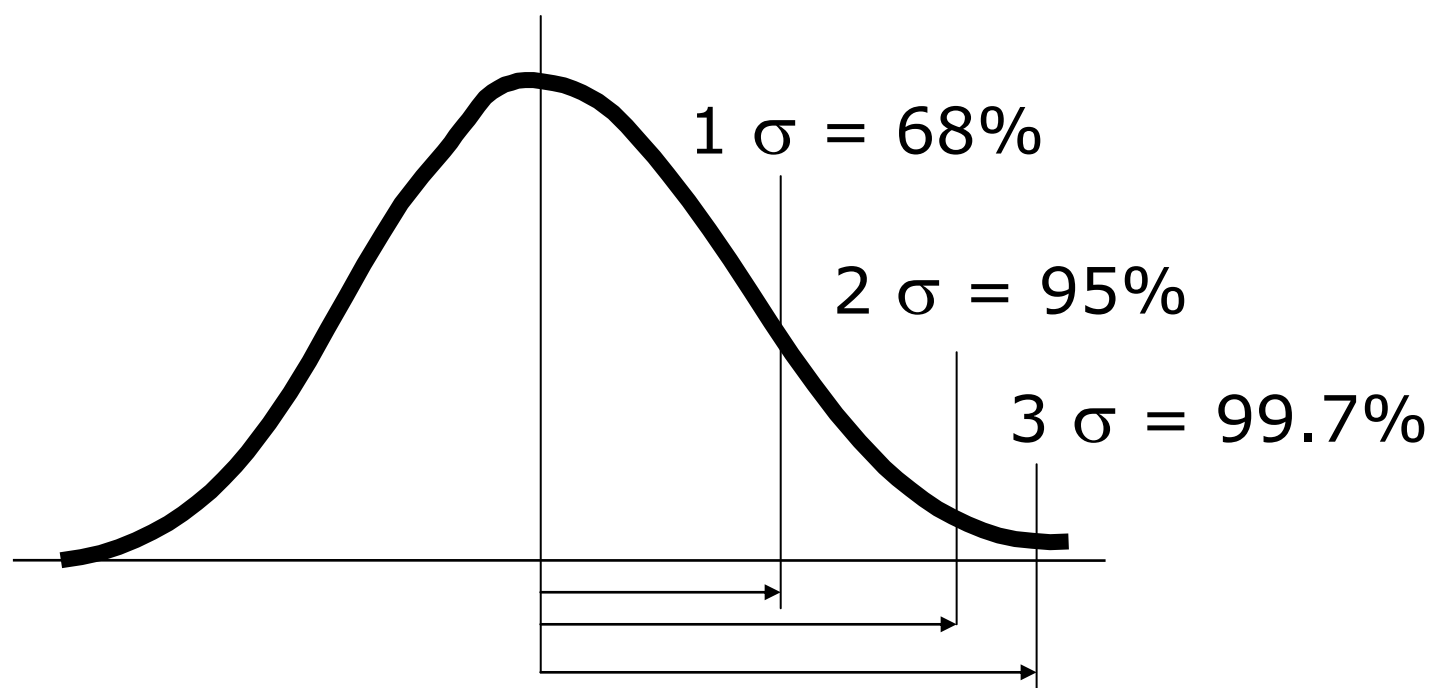
Qual è la probabilità che uno studente preso a casa abbia un risultato superiore a 68





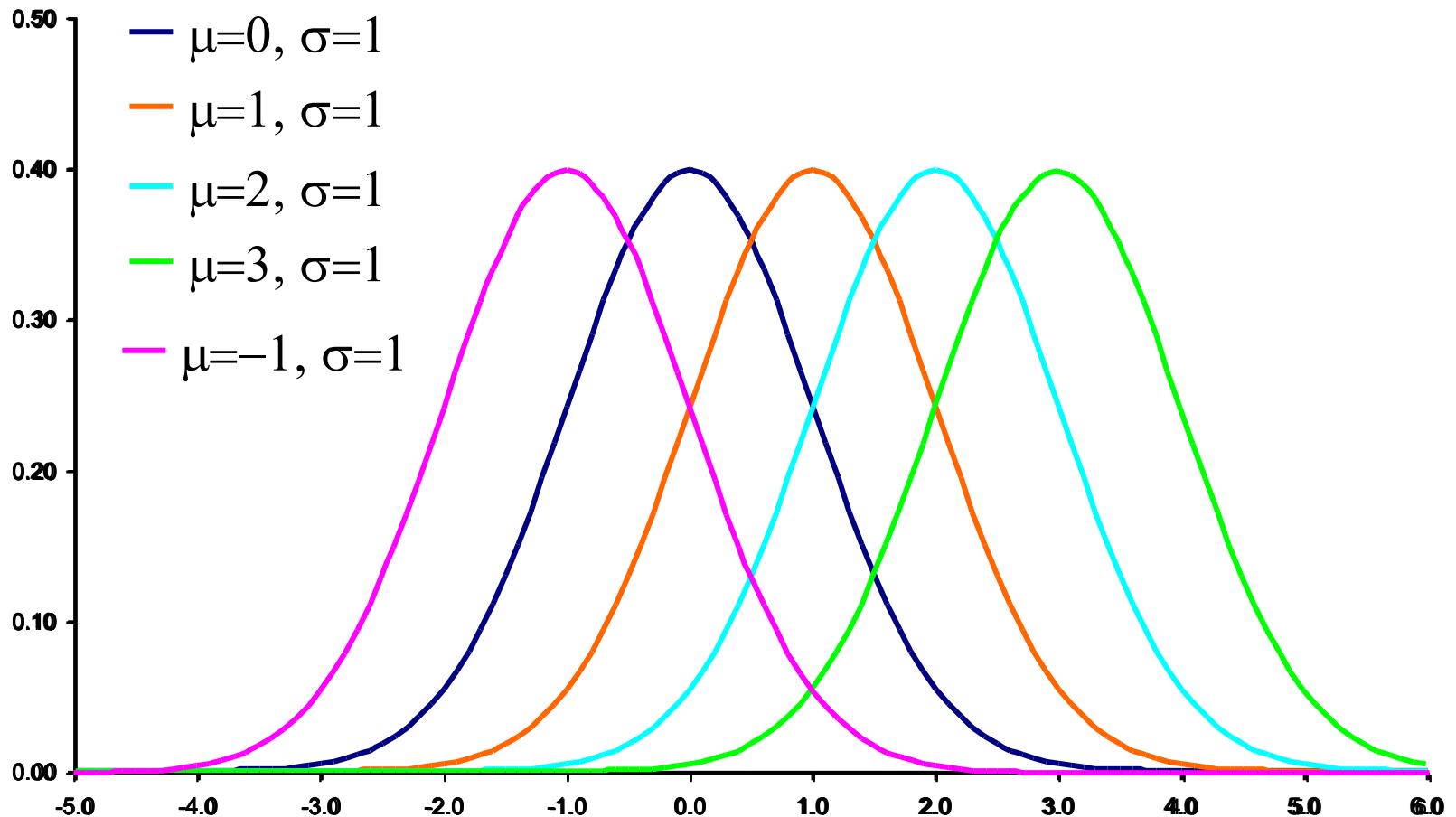
Effetto al variare dei parametri (μ & σ)



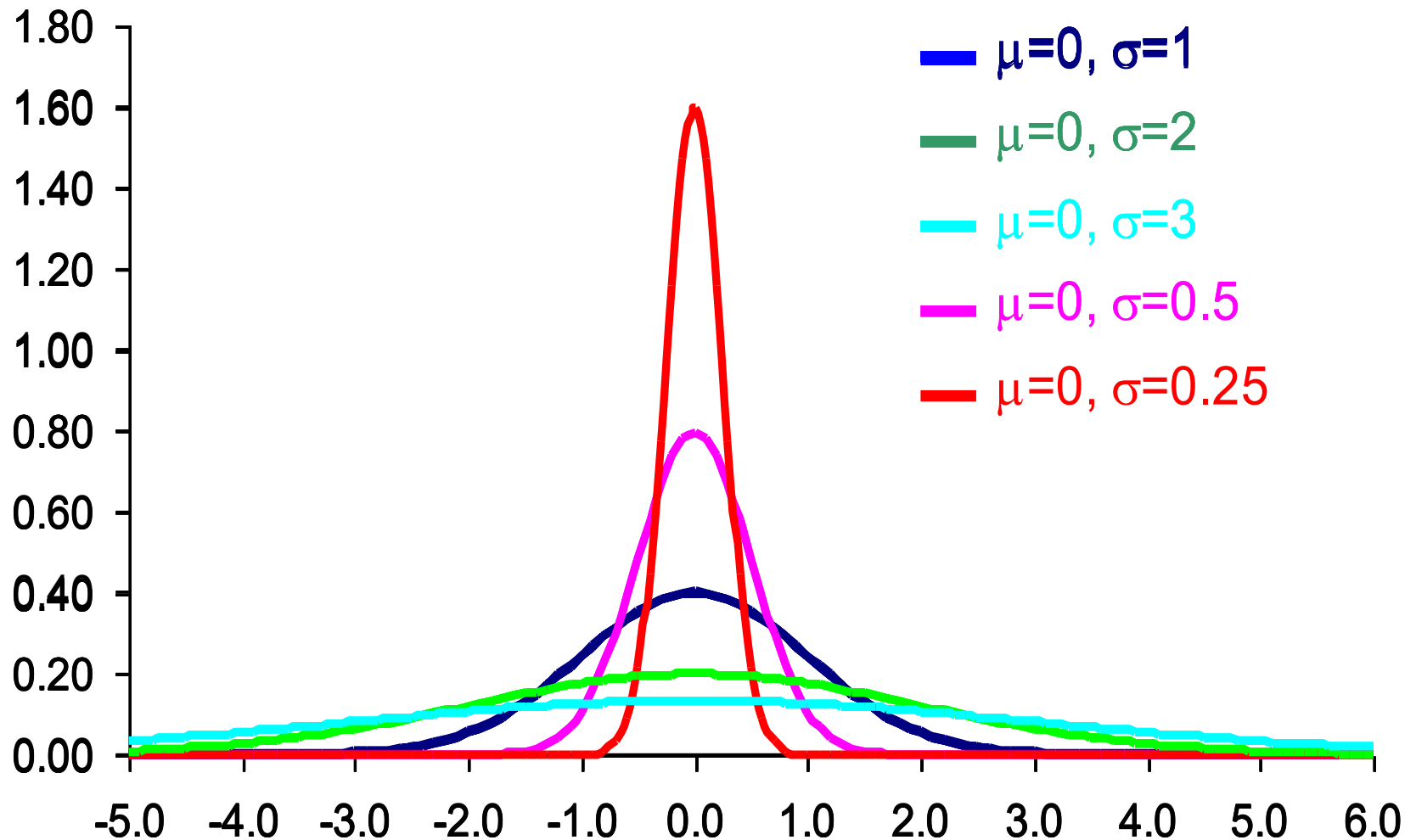


- The graph is symmetric about the mean
- The total area under the curve equals 100%
- Mean to $1 \sigma = \pm 68\%$
- Mean to $2 \sigma = \pm 95\%$
- Mean to $3 \sigma = \pm 99.7\%$

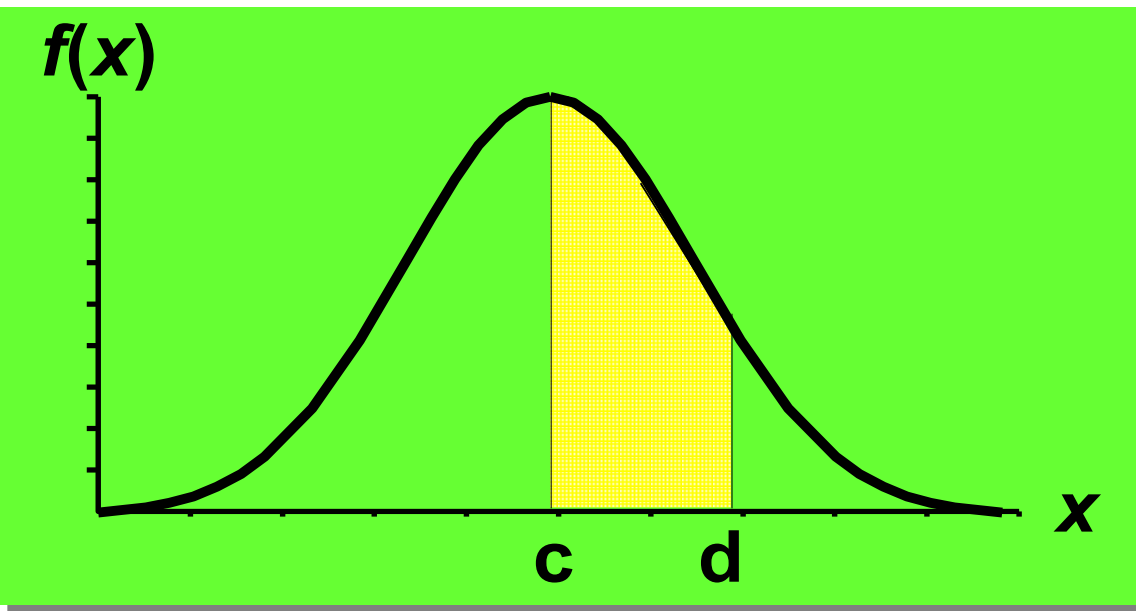
Normal Distributions: $\sigma=1$



Normal Distributions: $\mu=0$

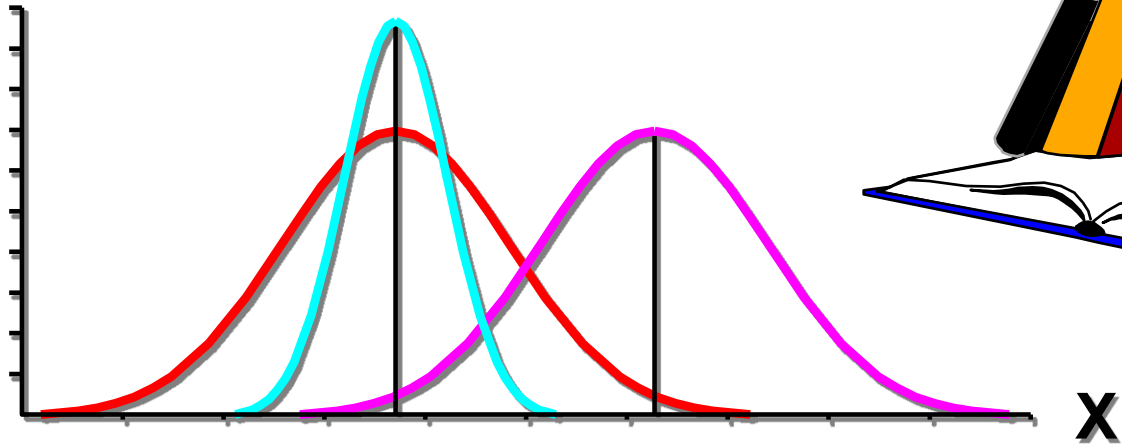


$$P(c < X \leq d) = \int_c^d f(x) dx \quad ?$$



Infinito numero of Tavole

$f(X)$



Standard Normal

$$Z = \frac{X - \mu}{\sigma}$$

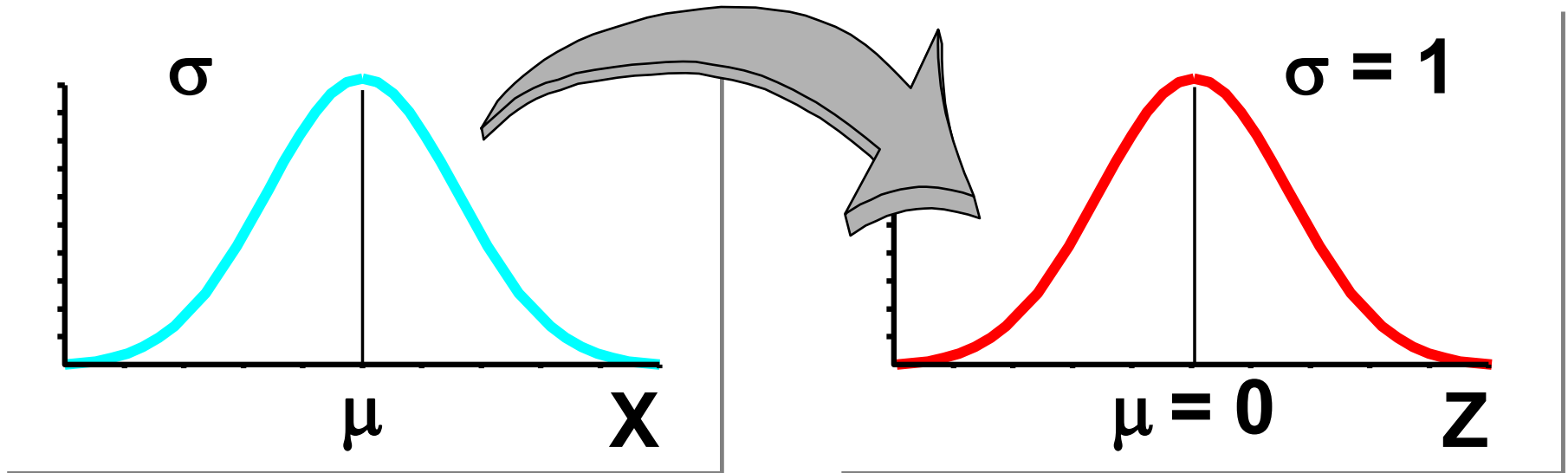
z	Seconda cifra decimale di z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

Standardize the Normal Distribution

$$Z = \frac{X - \mu}{\sigma}$$

**Normal
Distribution**

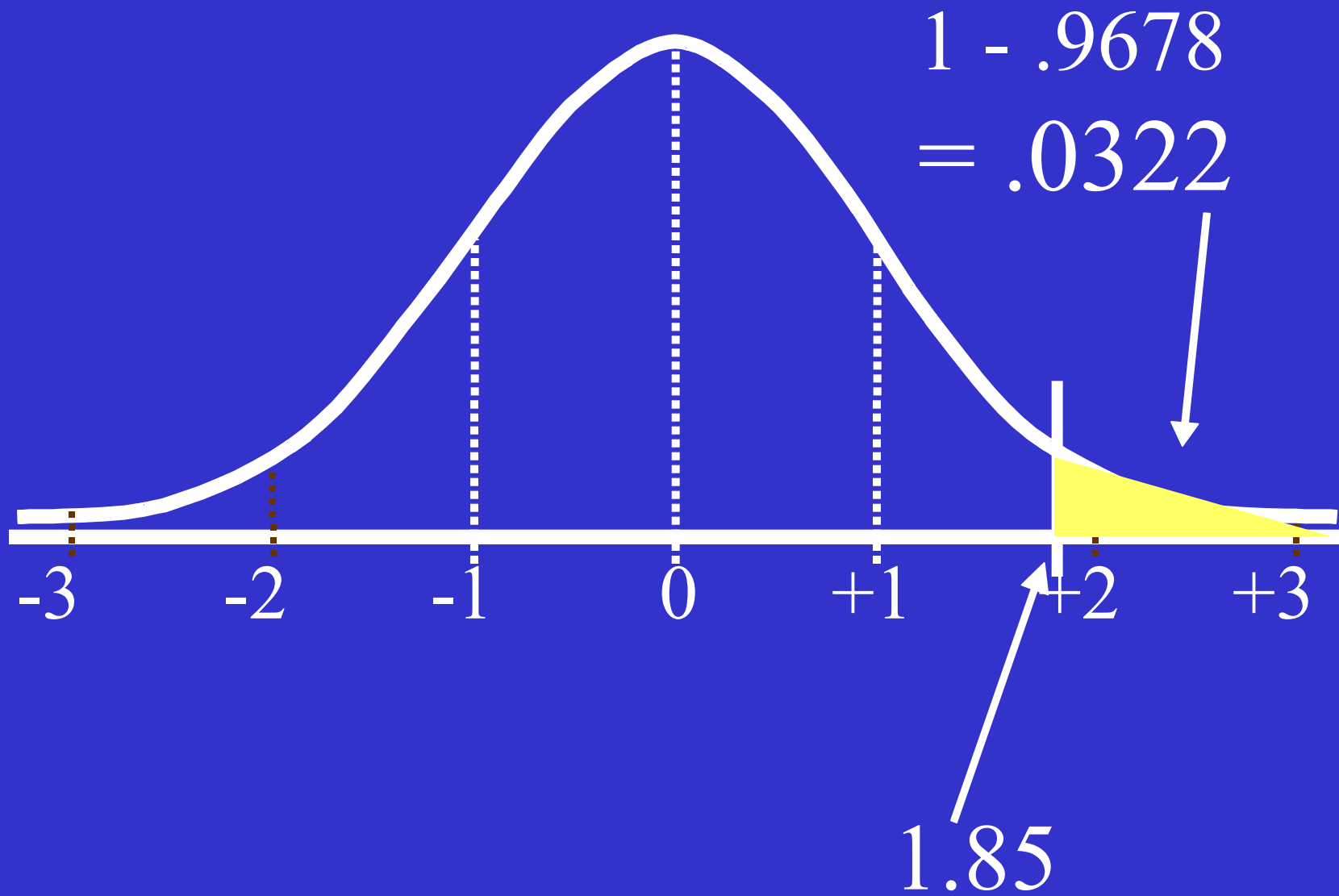
**Standardized
Normal Distribution**



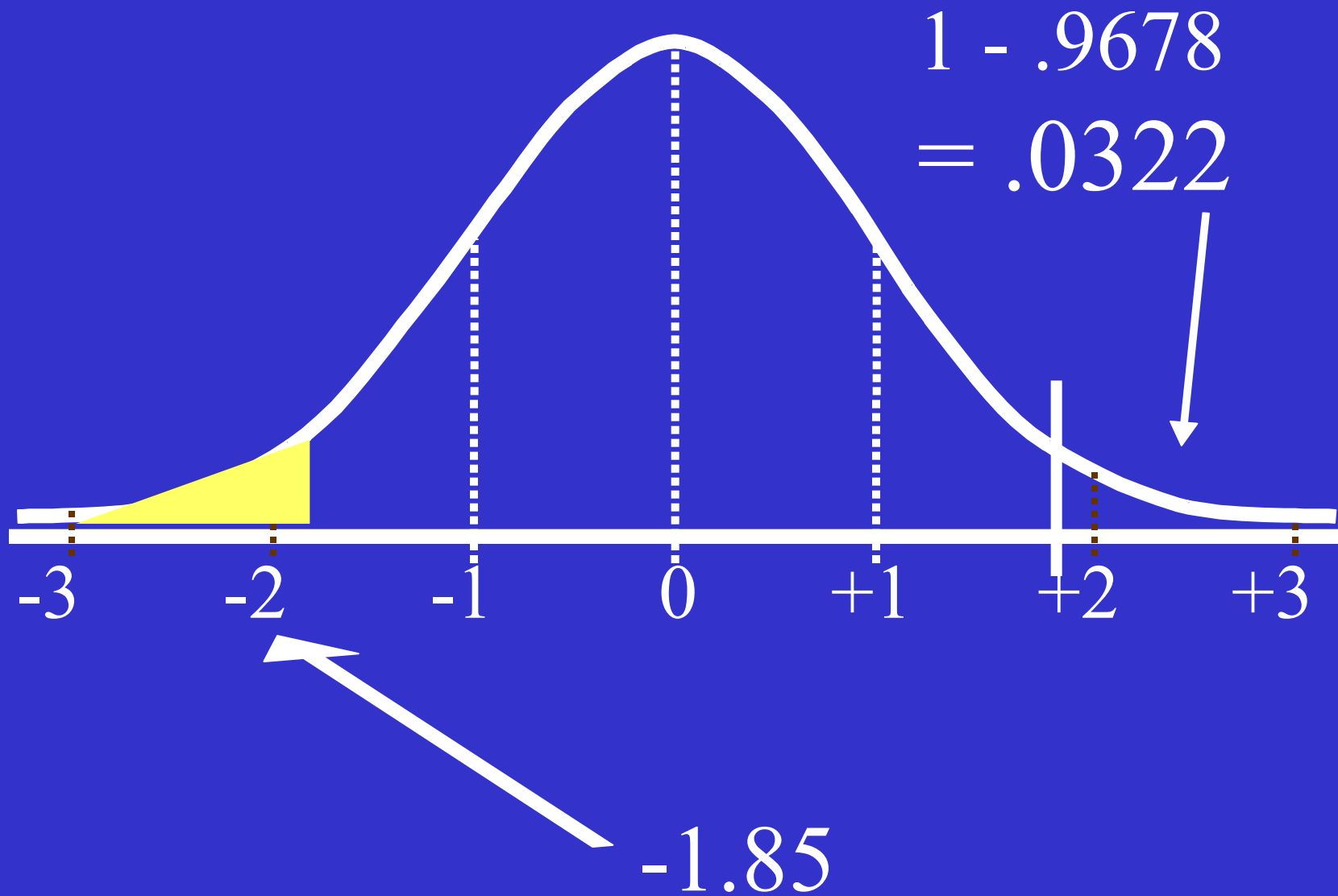
One table!

z	Seconda cifra decimale di z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
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1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
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1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
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2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

Find $P(Z > 1.85)$

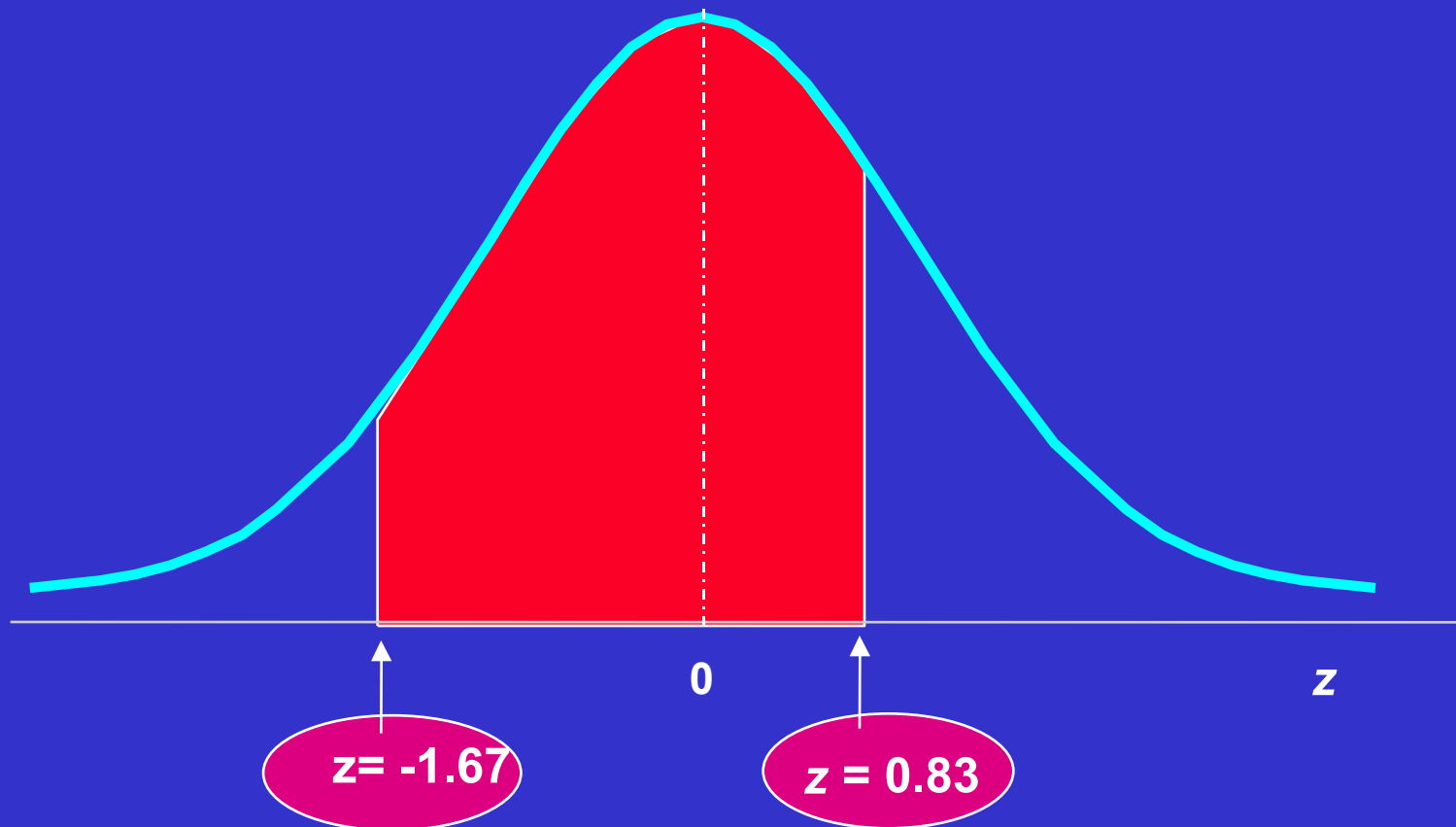


Find $P(Z < -1.85)$

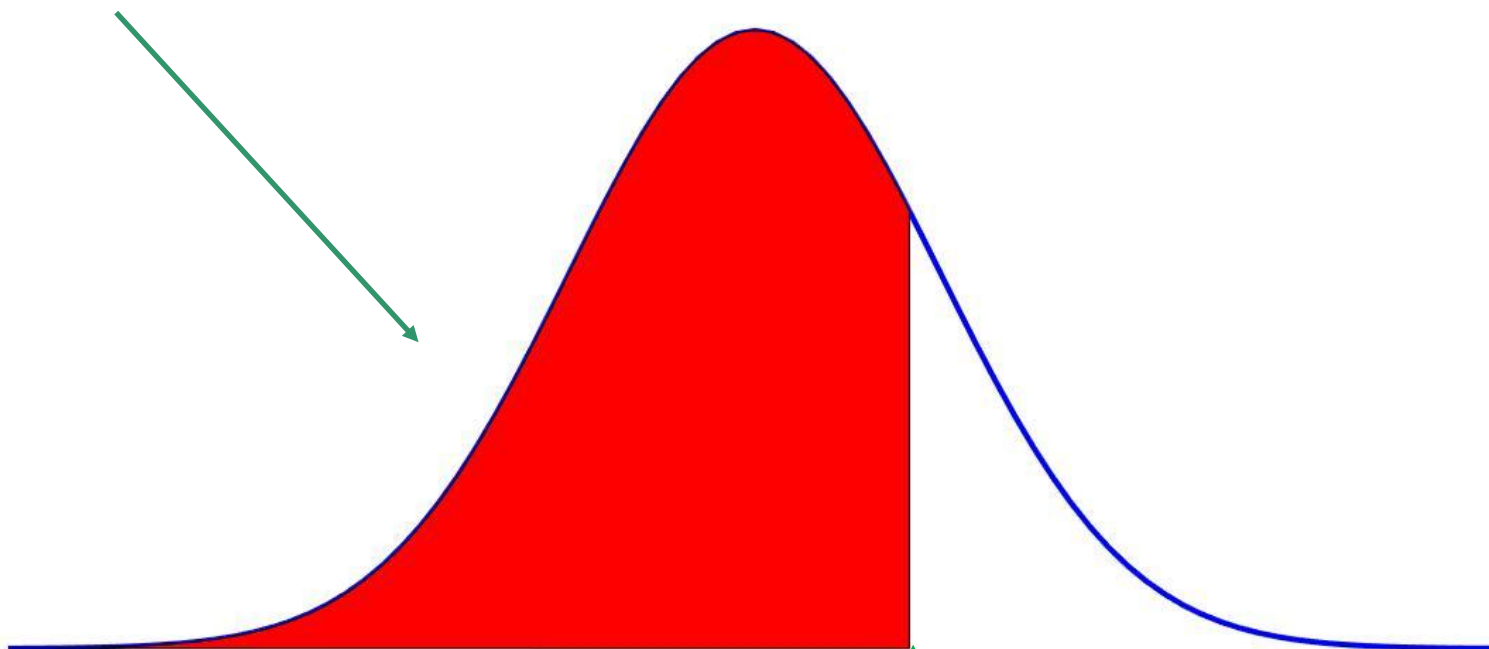


z	Seconda cifra decimale di z									
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0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
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0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

Trovare l'area sotto la curva
(Probabilità $-1.67 < Z \leq 0.83$)



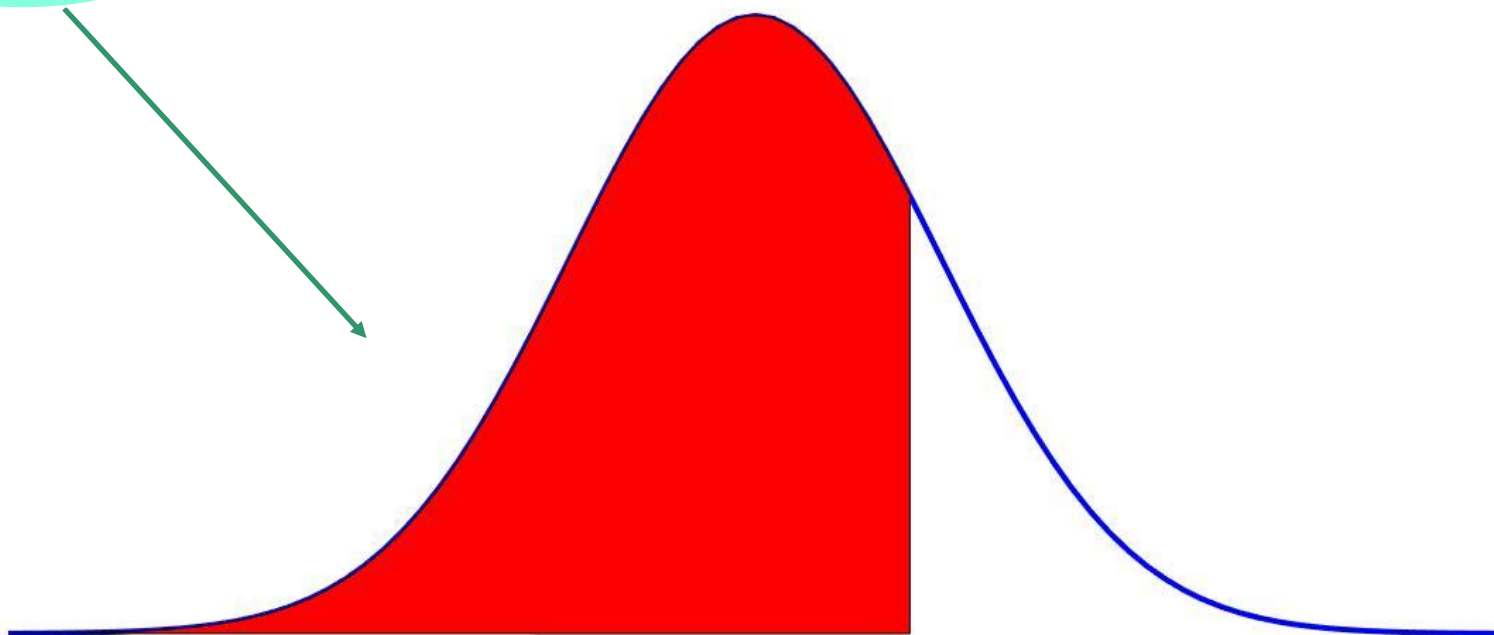
?



0.83

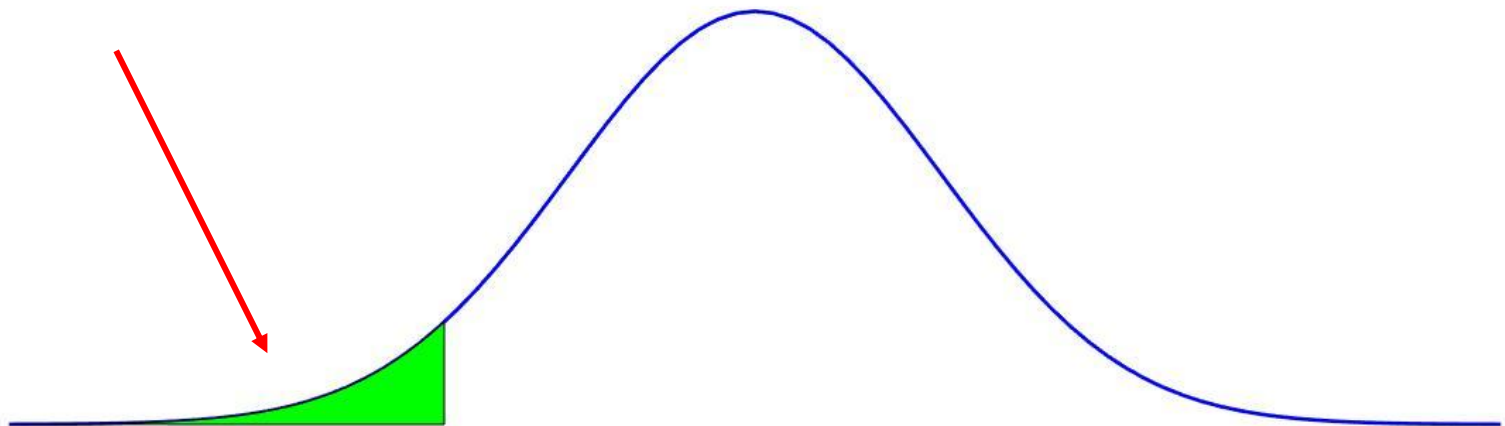
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
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1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
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2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

0.7967



0.83

???

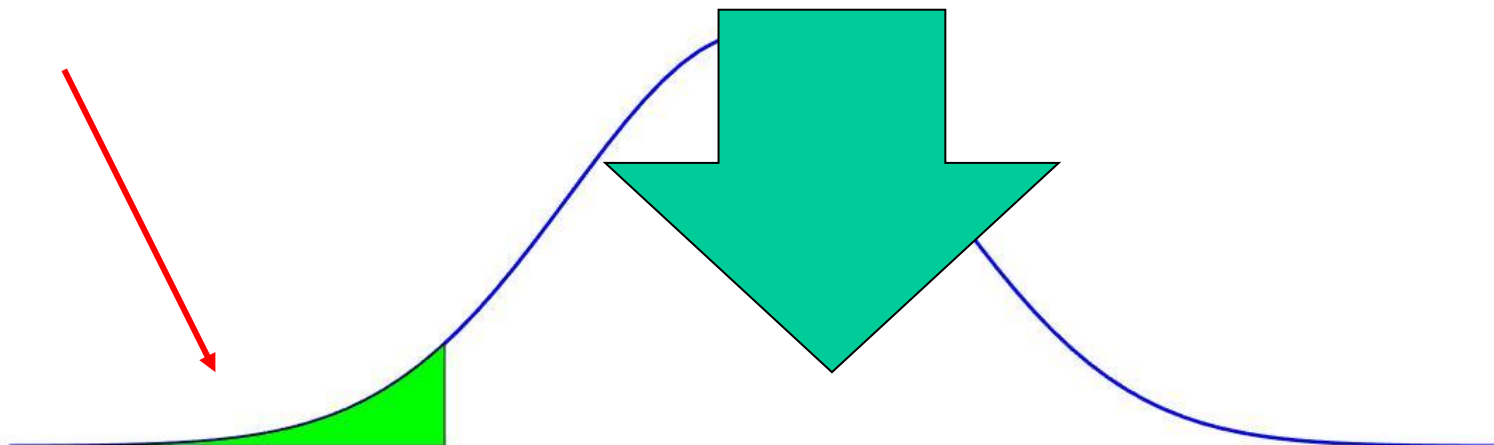


-1.67

z	Seconda cifra decimale di z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
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0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
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1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

???

0.9525

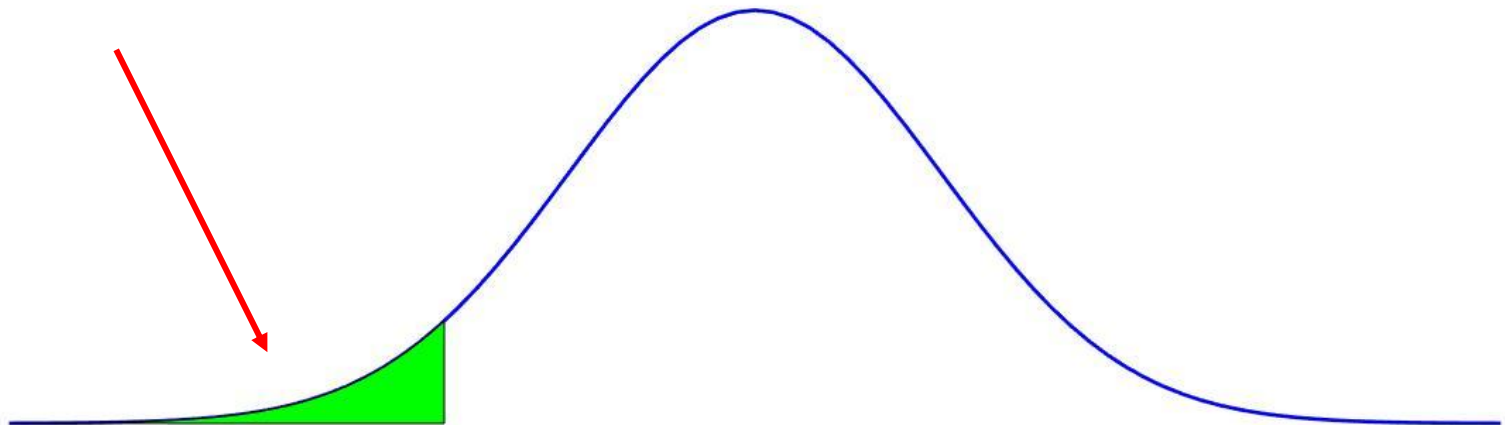


-1.67

1.67

0.0475

$1 - 0.9525$



-1.67

Uso tavola gaussiane

Sia $X \sim N(0.3, s=0.08)$ Trovare:

$$P(X > 0.4)$$

$$P(X < 0.35)$$

$$P(0.28 < X < 0.41)$$

$$P(X < 0.27)$$

$$P(X > 0.19)$$

$$P(X < 0.41)$$

$$P(X < 0.39)$$

Uso tavola gaussiane

a) 0.1056 b) 0.7324 c) 0.5515 d) 0.3557
e) 0.9147 f) 0.9147 g) 0.8888

Calcolare le seguenti probabilità
Utilizzando le tavole della Gaussiana standard:

$$P(0.00 < Z \leq 1.25)$$

ANSWER: 0.3944

$$P(-1.96 < Z \leq 1.96)$$

ANSWER: 0.9500

$$P(1. < Z \leq 1.25)$$

ANSWER: 0.0531

$$P(-1.00 < Z \leq -0.50)$$

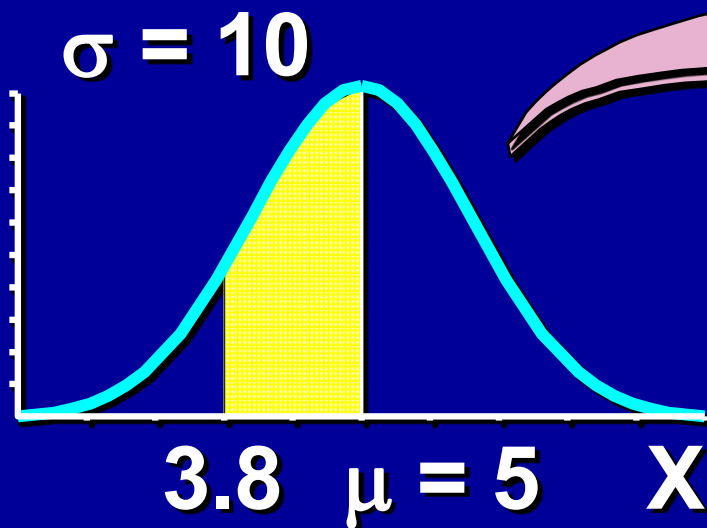
ANSWER: 0.1498

z	Seconda cifra decimale di z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

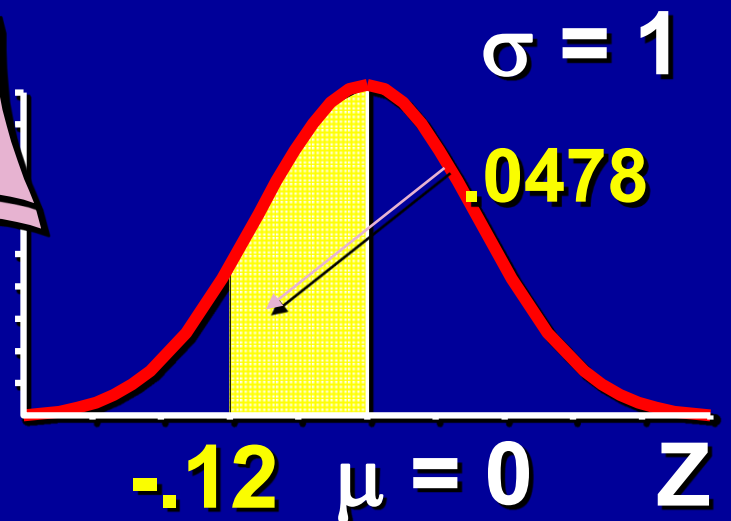
Esempio $X \sim N(5, \sigma=10)$
 $P(3.8 < X \leq 5)$

$$Z = \frac{X - \mu}{\sigma} \quad z = \frac{3.8 - 5}{10} = -.12$$

Normal
Distribution



Standardized
Normal Distribution



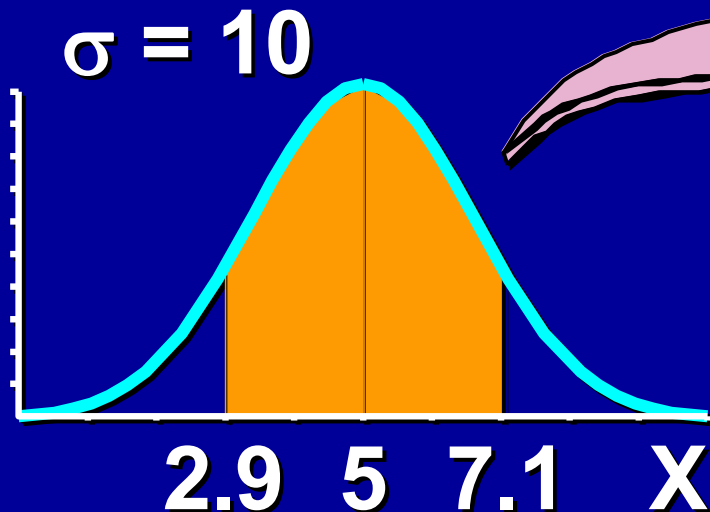
Example

$P(2.9 < X \leq 7.1)$

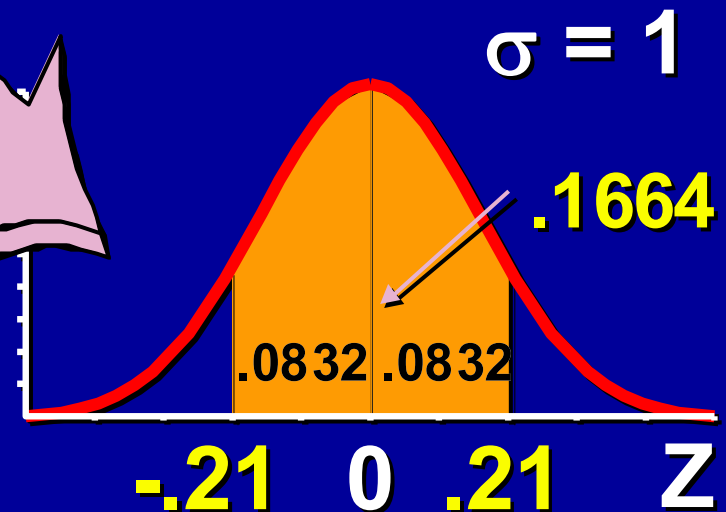
$$Z = \frac{X - \mu}{\sigma} \quad z = \frac{2.9 - 5}{10} = -.21$$

$$Z = \frac{X - \mu}{\sigma} \quad z = \frac{7.1 - 5}{10} = .21$$

Normal
Distribution



Standardized
Normal Distribution



Example

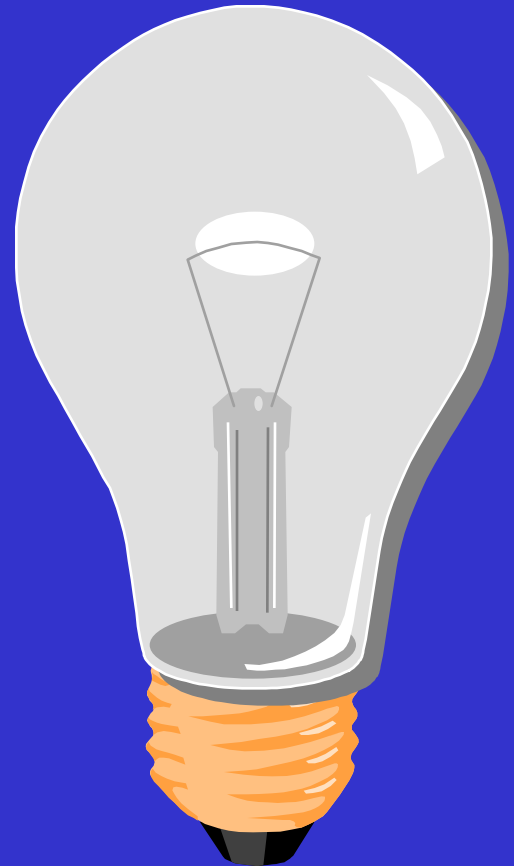
You work in Quality Control for GE.

Light bulb life has a **normal distribution** with

$\mu = 2000$ hours & $\sigma = 200$ hours.

What's the probability that a bulb will last

- A. between 2000 & 2400 hours?
- B. less than 1470 hours?

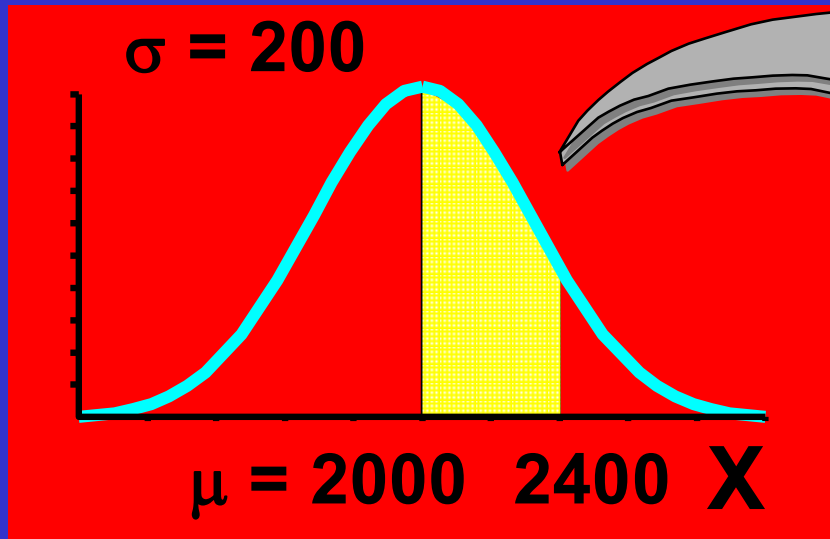


Solution

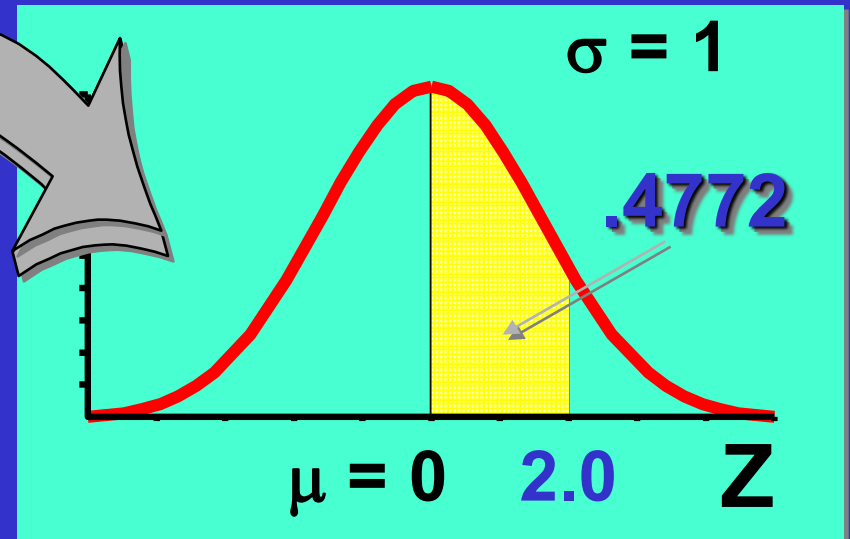
$$P(2000 \leq X \leq 2400)$$

$$Z = \frac{X - \mu}{\sigma} \quad z = \frac{2400 - 2000}{200} = 2.0$$

**Normal
Distribution**



**Standardized
Normal Distribution**

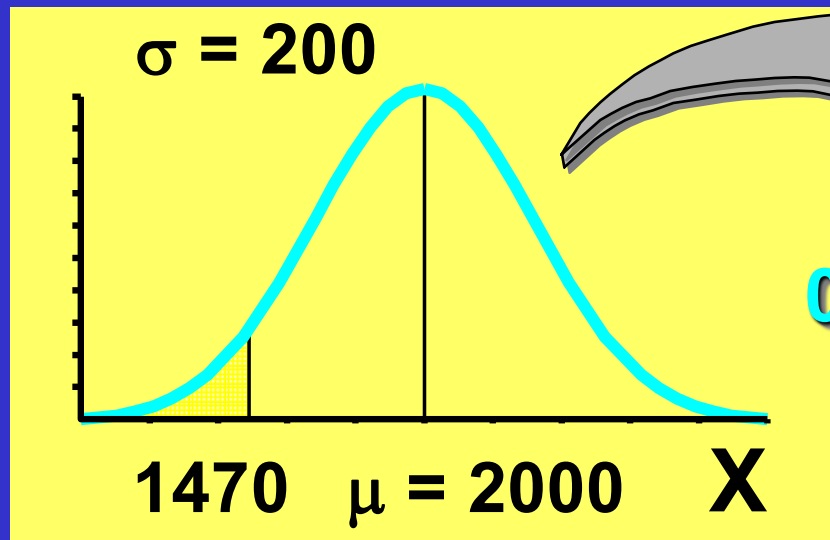


Solution

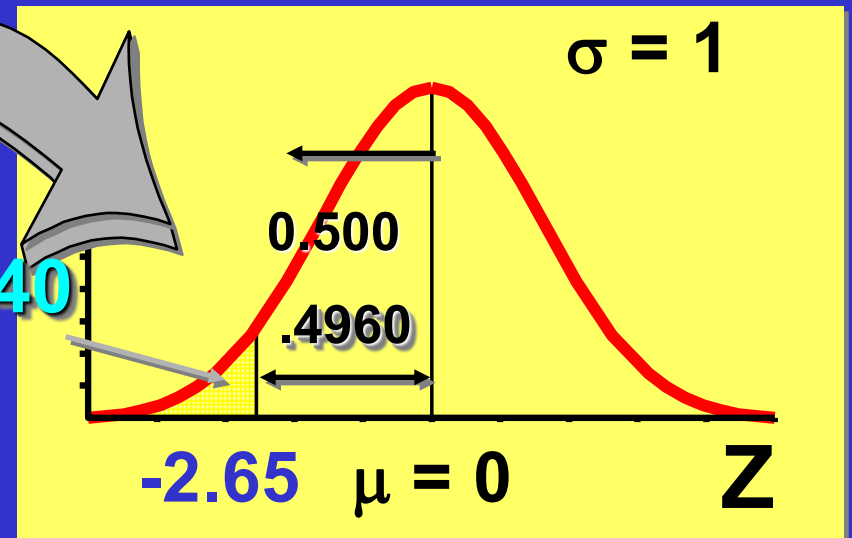
$P(X \leq 1470)$

$$Z = \frac{X - \mu}{\sigma} = \frac{1470 - 2000}{200} = -2.65$$

Normal
Distribution



Standardized
Normal Distribution



0.0040

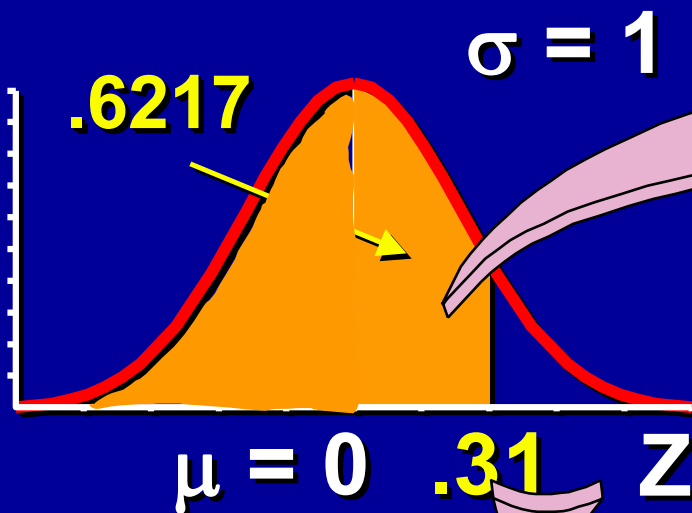
Schema per il calcolo della probabilità per variabili casuali normali

1. Disegnare la distribuzione normale, indicare entrambi i valori x e μ ; colora l'area cercata
2. Convertire i valori per la variabile casuale normale standard, z .
3. Utilizzare la Tabella per trovare l'area appropriata, che è la probabilità che cercate.

Finding Z Values for Known Probabilities

What is z given
 $P(Z \leq z) = 0.6217$?

Standardized Normal
Probability Table (Portion)

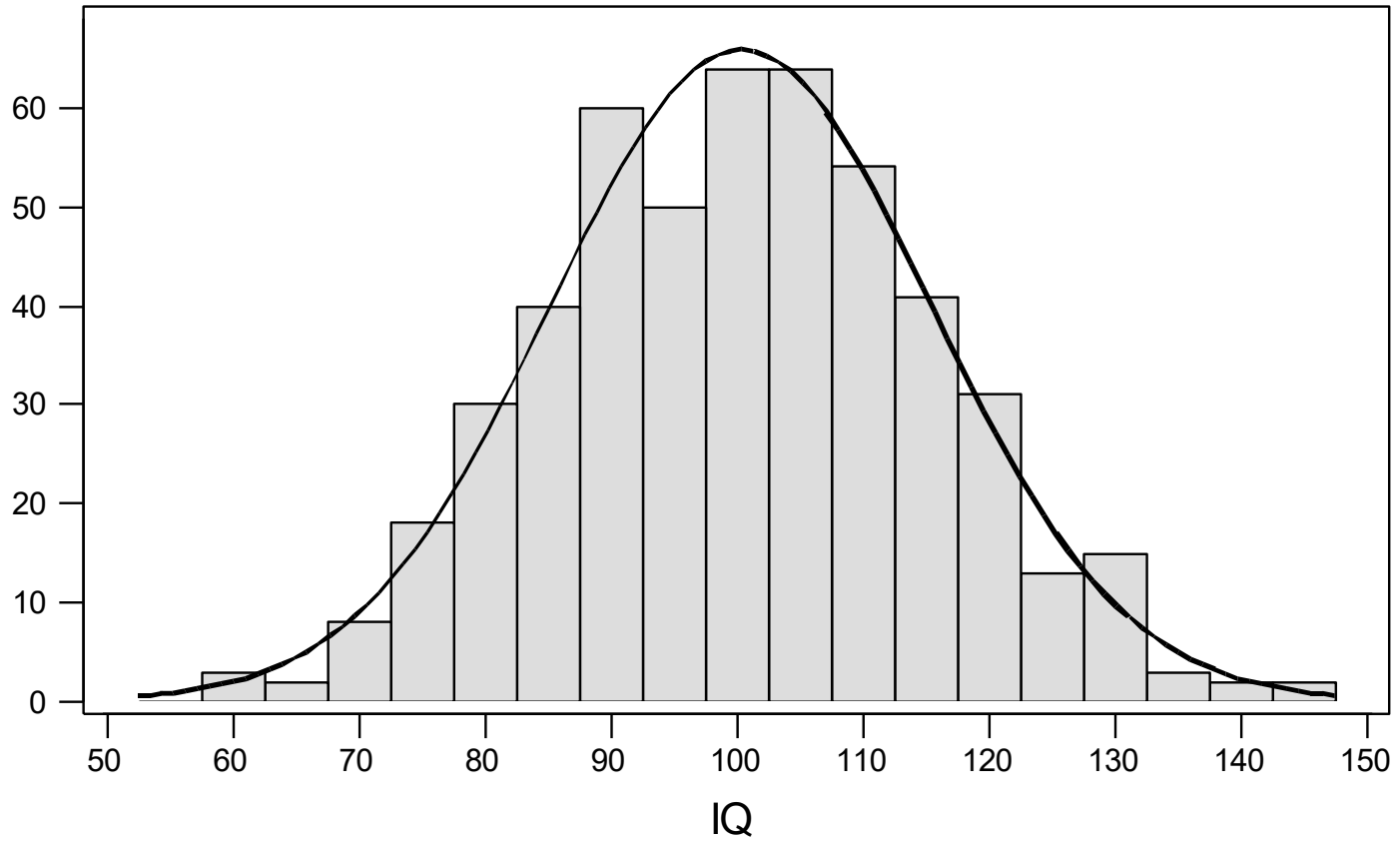


Z	.00	.01	0.2
	.0000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

$z = 0.31$

z	Seconda cifra decimale di z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981

Histogram of IQ data, with Normal Curve



EXAMPLE

The mean and sd of IQ are 100 and 15.

Therefore:

95% of people have IQs between
 $100 - 2 \times 15$ and $100 + 2 \times 15$.

i.e. between 70 and 130

Virtually all IQs lie between
 $100 - 3 \times 15$ and $100 + 3 \times 15$.

i.e. between 55 and 145

EXERCISE

Approximately, what proportion of people have IQs

greater than 100?

0.5

greater than 115?

0.16

less than 85?

0.16

greater than 130?

0.025

Esercizio

L'altezza di un uomo adulto è una variabile aleatoria gaussiana con valore atteso 170 cm e deviazione standard 12 cm

- Qual è la probabilità che un uomo adulto selezionato casualmente sia più alto di 158cm?
- Qual è la probabilità che un uomo adulto selezionato casualmente sia più alto di 194cm?
- trovare il valore a , tale che il 10% dei maschi adulti sia più alto di tale valore (o equivalentemente trovare il 90^{esimo} percentile della distribuzione)

Soluzione

a)
$$P(X > 155) = P\left(\frac{X - 170}{12} > \frac{155 - 170}{12}\right) = P(Z > -1) = P(Z \leq 1) = 0.8413$$

b)
$$\begin{aligned} P(X > 194) &= P\left(\frac{X - 170}{12} > \frac{194 - 170}{12}\right) = P(Z > 2) = 1 - P(Z \leq 2) = \\ &= 1 - 0.9772 = 0.0228 \end{aligned}$$

c)
$$\begin{aligned} 0.10 &= P(X \geq a) = P\left(\frac{X - 170}{12} \geq \frac{a - 170}{12}\right) \\ P(Z \geq z_{0.90}) &= 0.10 \\ z_{0.90} &= 1.28 \\ \frac{a - 170}{12} &= 1.28 \\ a &= 185.36 \end{aligned}$$

Esercizio

Il tempo di percorrenza da casa al lavoro del Signor XXX si distribuisce come una Gaussiana di media μ 20 minuti e deviazione standard di 10 minuti.

- Supponendo che il Signor XXX esce di casa solitamente alle 8.00 per arrivare al lavoro alle 8.30
- Qual è la probabilità che oggi il Signor XXX arriverà in ritardo al lavoro?
- Qual è la probabilità che il Signor XXX arriverà in ritardo al lavoro sia oggi sia domani?
- Qual è la probabilità che in una settimana con 5 giorni lavorativi il Signor XXX arriverà in ritardo esattamente due volte?
- Quanti giorni vi aspettate che arriverà in ritardo al lavoro il Signor XXX in un anno (considerando 240 giorni lavorativi)?

Soluzioni

- $P(X > 30) = p((X - 20)/10 > (30 - 20)/10) =$
 $= P(Z > 1) = 0.1587$
- $P(X > 25) = p((X - 20)/10 > (25 - 20)/10) =$
 $= P(Z > 0.5) = 0.3085$
- $\text{Binomiale}(n=5, k=0; p=(1-0.1587)) =$
 $= 0.8413^5 = 0.4215$

$$P(X = 2) = \binom{5}{2} 0.1587^2 (1 - 0.1587)^3 =$$

$$= 0.15$$

$$\binom{5}{2} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3!}{2 \times 3!} = 10$$

Example: Pep Zone

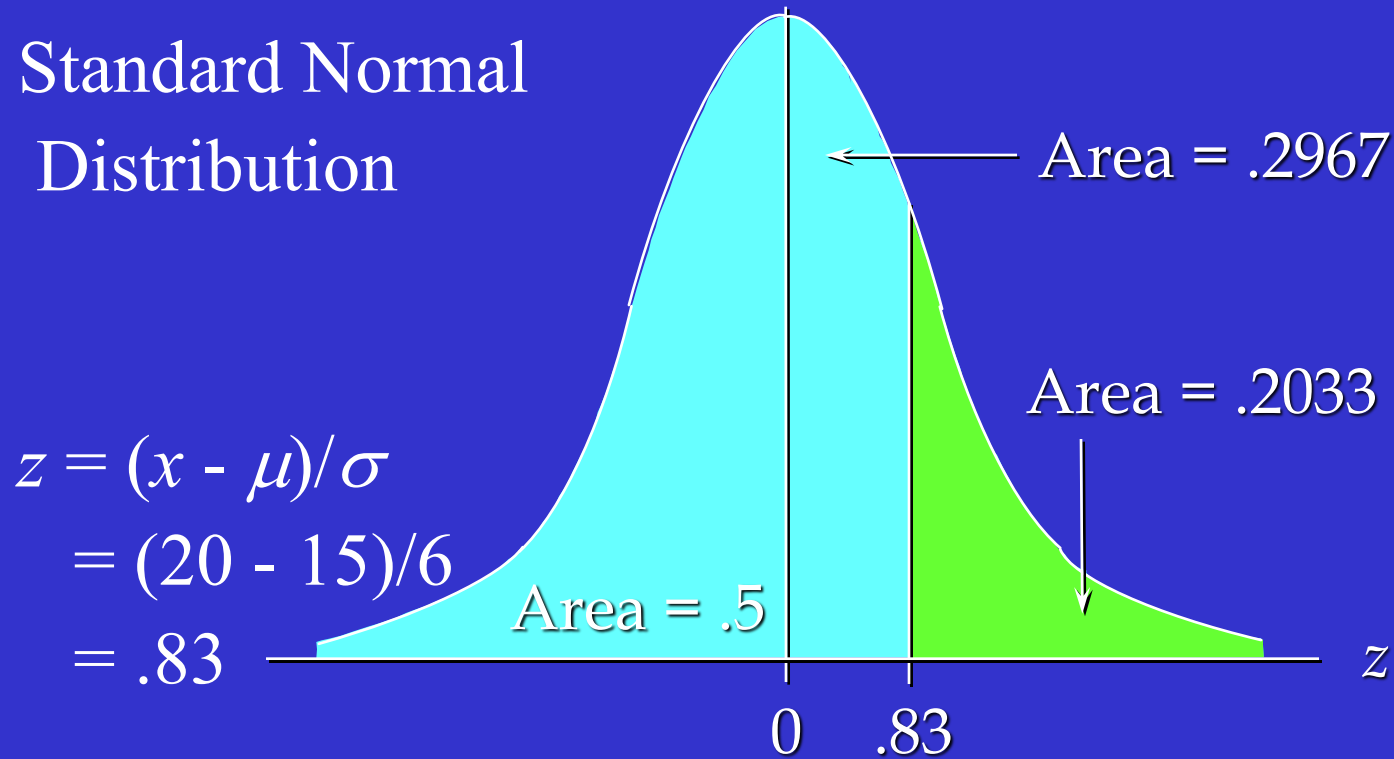
Pep Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed.

The store manager is concerned that sales are being lost due to stockouts while waiting for an order. It has been determined that leadtime demand is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stockout, $P(X > 20)$.

Example: Pep Zone

- Standard Normal Distribution



The Standard Normal table shows an area of .2967 for the region between the $z = 0$ line and the $z = .83$ line above. The shaded tail area is $.5 - .2967 = .2033$. The probability of a stockout is .2033.

EXAMPLE: Food Expenditure Study

It was reported in a local newspaper that the monthly food (grocery) expenditure for a family of four in this region averaged \$420 with a standard deviation of \$80. Furthermore, we suspect the the distribution of these expenditures is approximately normal.

19.22%

What percentage of households spend less than \$350?

What percentage of households spend between \$260 and \$460?

0.6687

Determine the 75th percentile of this distribution

474.4

Example

What is the probability a woman randomly chosen is shorter than 60 inches?

$$z = \frac{60 - 64}{2.6} = -1.54$$

$$P(Z < -1.54) = 0.0618$$

What is the probability a woman randomly chosen is higher than 60 inches?

$$1 - 0.0618$$