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Quantitative Methods III - Practice 8
Dynamic Multipliers and Cumulative Dynamic Multipliers

Exercise The following table shows the estimates of the dynamic multipliers of the effect of the change in the price of oil per barrel (x_t) on US inflation (y_t), calculated on the basis of the respective time series, available quarterly from the first quarter 1970 to the last quarter of 2022 ($T = 208$).

	Dynamic multipliers	Cumulative dynamic multipliers
Constant	0.33	
	(0.05)	
Lag 0	0.04	
	(0.01)	
Lag 1	0.06	0.1
	(0.01)	(0.02)
Lag 2		0.12
	(0.02)	(0.02)
Lag 3		0.07
	(0.01)	(0.03)
Lag 4	-0.03	0.04
	(0.01)	(0.04)
Lag 5	-0.01	
	(0.01)	(0.04)
Remarks	208	208

1. What is the estimated impact effect?
2. Indicate the value of the constant in the column of cumulative dynamic multipliers.
3. Fill in the remaining missing values of the dynamic multipliers and cumulative dynamic multipliers columns.
4. Evaluate the significance of the cumulative dynamic multipliers.

5. Indicate which of the following statements are true:

   - ☐ the cumulative dynamic multiplier over 2 quarters is positive and significant.
   - ☐ the 4-quarter dynamic multiplier is positive and significant.
   - ☐ the cumulative dynamic multiplier over 4 quarters is positive and significant.
   - ☐ it is plausible that the effect of an increase in the price of crude oil in the current quarter on inflation will be reabsorbed within the next year.

Solution

1. The impact effect can be estimated within a DL model. For example, in a model with r delays:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} ... + \beta_{r+1} x_{t-r} + u_t$$

the impact effect is captured by the coefficient β_1.

In this case the impact effect is given by β_1, the coefficient at Lag 0, that is β_1=0.04.

The value of the t-ratio (the test statistic) will be:

$$t = \frac{0.04}{0.01} = 4$$

This value of the test statistic implies that the effect of the impact is statistically different from 0 at all conventional levels of significance.

2. It is possible to convert a DL model with r lags as:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} ... + \beta_{r+1} x_{t-r} + u_t$$

in a different DL model with r lags as:

$$y_t = \delta_0 + \delta_1 \Delta x_t + \delta_2 \Delta x_{t-1} + \delta_3 \Delta x_{t-2} ... + \delta_r \Delta x_{t-r+1} + \delta_{r+1} x_{t-r} + u_t$$

in which the new coefficients δ_i, called cumulative dynamic multipliers, are:

$\delta_0 = \beta_0$
$\delta_1 = \beta_1$
$\delta_2 = \beta_1 + \beta_2 = \delta_1 + \beta_2$
$\delta_3 = \beta_1 + \beta_2 + \beta_3 = \delta_2 + \beta_3$
...
$\delta_{r+1} = \beta_1 + \beta_2 + ... + \beta_{r+1}$

So, the constant and the impact effect of cumulative dynamic multipliers, δ_0 and δ_1, are the same of dynamic multipliers, β_0 and β_1.

3. The first missing value is that of the dynamic multiplier at Lag 2, β_3. Taking advantage of the estimate of the dynamic multiplier cumulative at Lag 2, δ_2, shown in the corresponding column and remembering what is reported above,

i.e. that $\delta_3 = \beta_1 + \beta_2 + \beta_3 = \delta_2 + \beta_3$, it is possible to derive β_3 by difference, as:

$$\beta_3 = \delta_3 - \delta_2 = 0.12 - 0.10 = 0.02$$

Similarly, the coefficient of Lag 3, β_4, can be obtained as the difference between δ_4 and δ_3:

$$\beta_4 = \delta_4 - \delta_3 = 0.07 - 0.12 = -0.05$$

Finally, the last missing cumulative dynamic multiplier, corresponding to Lag 5, δ_6 can be obtained by adding the previous cumulative dynamic multiplier, δ_5, to the dynamic multiplier of Lag 5, β_6:

$$\delta_6 = \delta_5 + \beta_6 = 0.04 + (-0.1) = 0.03$$

4. The significance of the cumulative dynamic multipliers can be tested through a classic hypothesis test on simple linear regression coefficients, comparing the t-ratio with the critical value.

   Since the number is large enough ($T > 30$), the central limit theorem allows us to use the Normal distribution.

   We have already evaluated the significance of the impact effect, which is equivalent to the first cumulative dynamic multiplier (Lag 0).

   For Lag 1, the t-ratio will be:

$$t = \frac{0.1}{0.02} = 5$$

   This value is greater than the critical values at all conventional levels of significance.

   Therefore, it is possible to add the symbol $***$ to the coefficient δ_2. For the other coefficients we proceed in the same way.

   The complete table of estimates and their relative significance is shown below.

	Dynamic multipliers	Cumulative dynamic multipliers
Constant	0.33	0.33∗ ∗ ∗
	(0.05)	(0.05)
Lag 0	0.04	0.04∗ ∗ ∗
	(0.01)	(0.01)
Lag 1	0.06	0.1∗ ∗ ∗
	(0.01)	(0.02)
Lag 2	0.02	0.12∗ ∗ ∗
	(0.02)	(0.02)
Lag 3	-0.05	0.07∗ ∗ ∗
	(0.01)	(0.03)
Lag 4	-0.03	0.04
	(0.01)	(0.04)
Lag 5	-0.01	0.03
	(0.01)	(0.04)
Observations	208	208

5. On the basis of the estimates obtained, indicate which of the following statements are true:

   ■ the cumulative dynamic multiplier over 2 quarters is positive and significant.

   □ the 4-quarter dynamic multiplier is positive and significant.

   □ the cumulative dynamic multiplier over 4 quarters is positive and significant.

   ■ it is plausible that the effect of an increase in the price of crude oil in the current quarter on inflation will be reabsorbed within the next year.