*Quantitative Methods III - Practice 7*
*Time Series: trends and breaks*

Prof. Lorenzo Cavallo: lorenzo.cavallo.480084@uniroma2.eu
Prof. Marianna Brunetti: marianna.brunetti@uniroma2.it

**Stock & Watson - Chapter 15**

**Exercise 15.1** Consider the AR(1) model:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t.$$

Suppose the process is stationary.

a. Show that
$$E(Y_t) = E(Y_{t-1})$$

b. Show that
$$E(Y_t) = \frac{\beta_0}{1-\beta_1}$$

**Exercise 15.2** The Index of Industrial Production ($IP_t$) is a monthly time series that measures the quantity of industrial commodities produced in a given month. This problem uses data on this index for the United States.
All regressions are estimated over the sample period from 1986:M1 to 2017:M12 (that is, January 1986 through December 2017).

Let:
$$Y_t = 1200 \times ln(\frac{IP_t}{IP_{t-1}})$$
.

a. A forecaster states that $Y_t$ shows the monthly percentage change in IP, measured in percentage points per annum. Is this correct? Why?

b. Suppose she estimates the following AR(4) model for $Y_t$:

$$\hat{Y}_t = 0.749 + 0.071Y_{t-1} + 0.170Y_{t-2} + 0.216Y_{t-3} + 0.167Y_{t-4}$$

$$(0.488) \quad (0.088) \qquad (0.098) \qquad (0.078) \qquad (0.064)$$

Use this $AR(4)$ to forecast the value of $Y_t$ in January 2018, using the following values of IP for July 2017 through December 2017:

| Data | 2017:M7 | 2017:M8 | 2017:M9 | 2017:M10 | 2017:M1 | 2017:M12 |
|---|---|---|---|---|---|---|
| IP | 105.1 | 104.56 | 104.84 | 106.58 | 106.86 | 107.30 |

c. Worried about potential seasonal fluctuations in production, she adds $Y_{t-12}$ to the autoregression. The estimated coefficient on $Y_{t-12}$ is –0.061, with a standard error of 0.043. Is this coefficient statistically significant?

d. Worried about a potential break, she computes a QLR test (with 15% trimming) on the constant and AR coefficients in the AR(4) model. The resulting QLR statistic is 1.80. Is there evidence of a break? Explain.

**Exercise 15.3** Using the same data as in Exercise 15.2, a researcher tests for a stochastic trend in $ln(IP_t)$, using the following regression:

$$\begin{aligned}\Delta \widehat{ln(IP_t)} = \quad & 0,026 \quad +0,000097t - 0,0070ln(IP_{t-1}) + 0,068\Delta ln(IP_{t-1}) \\ & (0,013) \qquad (0,000067) \quad (0,0037) \qquad\qquad (0,050) \\ & +0,169 \quad \Delta ln(IP_{t-2}) + 0,219\Delta ln(IP_{t-3}) + 0,173\Delta ln(IP_{t-4}) \\ & (0,049) \qquad\qquad\qquad (0,050) \qquad\qquad\qquad (0,051)\end{aligned}$$

where the standard errors shown in parentheses are computed using the homoskedasticity-only formula and the regressor $t$ is a linear time trend.

a. Use the ADF statistic to test for a stochastic trend (unit root) in $ln(P_t)$.

b. Do these results support the specification used in Exercise 15.2? Explain

**Exercise 15.4** The forecaster in Exercise 15.2 augments her AR(4) model for IP growth to include four lagged values of $\Delta R_t$, where $R_t$ is the interest rate on three-month U.S. Treasury bills (measured in percentage points at an annual rate).

a. The F-statistic on the four lags of $\Delta R_t$ is 3.91. Do interest rates help predict IP growth? Explain.

b. The researcher also regresses $\Delta R_t$ on a constant, four lags of $\Delta R_t$, and four lags of IP growth. The resulting F-statistic on the four lags of IP growth is 1.48. Does IP growth help to predict interest rates? Explain.

**Solutions**

15.1. (a) Since the probability distribution of $Y_t$ is the same as the probability distribution of $Y_{t-1}$ (this is the definition of stationarity), the means (and all other moments) are the same.

(b) $E(Y_t) = \beta_0 + \beta_1 (Y_{t-1}) + E(u_t)$, but $E(u_t) = 0$ and $E(Y_t) = E(Y_{t-1})$.
Thus $E(Y_t) = \beta_0 + \beta_1 E(Y_t)$, and solving for $E(Y_t)$ yields the result.

15.2. (a) The statement is correct. The monthly percentage change in IP is $\frac{IP_t - IP_{t-1}}{IP_{t-1}} \times 100$ which can be approximated by $[\ln(IP_t) - \ln(IP_{t-1})] \times 100 = 100 \times \ln(\frac{IP_t}{IP_{t-1}})$ when the change is small. Converting this into an annual (12 month) change yields $1200 \times \ln(\frac{IP_t}{IP_{t-1}})$.

(b) The values of $Y$ from the table are:

| Date | 2017:M7 | 2017:M8 | 2017:M9 | 2017:M10 | 2017:M11 | 2017:M12 |
|---|---|---|---|---|---|---|
| IP | 105.01 | 104.56 | 104.82 | 106.58 | 106.86 | 107.30 |
| Y | | -5.14 | 2.98 | 19.98 | 3.15 | 4.93 |

The forecasted value of $Y_t$ for January 2008 is:

$$\hat{Y}_{2018:M1|2017:M12} = 0.749 + [0.071 \times 4.93] + [0.170 \times 3.15] + [0.216 \times 19.98] + \\ + [0.167 \times 2.98] = 6.45$$

(c) The $t$-statistic on $Y_{t-12}$ is $t = \frac{-0.061}{0.043} = -1.42$ with an absolute value less than 1.96, so the coefficient is not statistically significant at the 5% level.

(d) For the QLR test, there are 5 coefficients (including the constant) that are being allowed to break. Compared to the critical values for $q = 5$ in Table 15.5, the QLR statistic 1.80 is smaller than the 10% critical value (3.26). Thus, the hypothesis that these coefficients are stable is not rejected at the 10% significance level.

15.3. (a) To test for a stochastic trend (unit root) in ln($IP$), the ADF statistic is the $t$-statistic testing the hypothesis that the coefficient on $\ln(IP_{t-1})$ is zero versus the alternative hypothesis that the coefficient on $\ln(IP_{t-1})$ is less than zero.

The test is a left-tailed test (the rejection region is in the left tail of the distribution).

If the null hypothesis is rejected ln($IP$) is stationary (no unit root), if the null hypothesis is not rejected ln($IP$) contains a stochastic trend (unit root).

The calculated $t$-statistic is $t = \frac{-0.0070}{0.0037} = -1.89.$

From Table 15.4, the 10% critical value with a time trend is −3.12.

Because −1.89 > −3.12, the test does not reject the null hypothesis that ln($IP$) has a unit autoregressive root at the 10% significance level, so the ln($IP$) contains a stochastic trend.

**TABLE 15.4 Large-Sample Critical Values of the Augmented Dickey–Fuller Statistic**

| Deterministic Regressors | 10% | 5% | 1% |
|---|---|---|---|
| Intercept only | −2.57 | −2.86 | −3.43 |
| Intercept and time trend | −3.12 | −3.41 | −3.96 |

(b) The ADF test supports the specification used in Exercise 15.2. The use of first differences in Exercise 15.2 eliminates random walk trend in ln($IP$).

15.4. (a) The critical value for the $F$-test is 3.32 at a 1% significance level. Since the $F$-statistic 3.91 is larger than the critical value, we can reject the null hypothesis that interest rates have no predictive content for IP growth at the 1% level.

(b) The $F$-statistic of 1.48 is smaller than the 10% critical value, so we cannot reject the null hypothesis that $IP$ growth does not help forecast future interest rates.