MATHEMATICS
Friday April 1 2016
Fourth Exercise Class

1) Determine the parametric equation and the Cartesian equation of the line on a plane

a) passing through the points $A(1,2)$, $B(-1,3)$.

b) passing through the point $C(2,3)$ and parallel to the vector $\overrightarrow{OP} = (-1,2)$

c) of Cartesian equation $y = 2x + 5$

2) Determine the parametric equation and the Cartesian equation of the line on the space

a) passing through the points $A(1,0,2)$, $B(3,-1,0)$.

b) passing through the point $P(1,3,1)$ and parallel to the vector $\overrightarrow{OQ} = (2,0,0)$

c) of Cartesian equation $\begin{cases} y = 3x + 1 \\ y - x + z = 0 \end{cases}$.

3) Determine the parametric equation and the Cartesian equation of the plane π that passes through the points $A(1,3,1)$, $B(2,0,0)$ and $C(0,1,1)$. Does the point $P(0,2,0)$ belong to the plane?
Determine the equation of the line passing through A and orthogonal to π.

4) Determine the reciprocal position of the lines r and r' of parametric equations

$$r: \begin{cases} x = 2t \\ y = t + 1 \\ z = t + 3 \end{cases} \qquad r': \begin{cases} x = s \\ y = 2 \\ z = s + 2 \end{cases}$$

5) Determine parametric equations of the line r passing through the points $A(2,3,1)$ and $B(0,0,1)$ and of the line s passing through the points $A(0,0,0)$ and $B(4,6,0)$.
Establish if r and s lie on the same plane. If so find a cartesian equation of the plane containing r and s.

6) Consider the lines of Cartesian equations

$$r: \begin{cases} x + 2y = 0 \\ y - z = 0 \end{cases} \qquad s: \begin{cases} 2x = 0 \\ x + y + z = 0 \end{cases}$$

a) after the verification that the two lines intersect, determine the cartesian equation of the line passing through $P(1,1,1)$ and that intersects r and s.

b) determine the cartesian equation of the plane passing through $C(1,2,-3)$ and perpendicular to r.

c) determine cartesian equations of the line passing through the point $P(1,1,1)$ and perpendicular to the two lines r and s.

7) Consider the line r of equation

$$r: \begin{cases} x = 2 + t \\ y = -3 - 2t \\ z = 1 \end{cases}$$

and the family of planes $\pi_k: \ 2x + ky - z = 1$ where k is a real number. Determine for which k the plane π_k is parallel to r.

8) Establish if the vectors $\vec{v_1} = (1,5,7)$ and $\vec{v_2} = (1,3,4)$ in $\mathbb{R}^3$ are linearly dependent.

9) Study the linear dependence or independence of the following vectors in $\mathbb{R}^3$

$$\vec{v_1} = (1,-1,2), \ \vec{v_2} = (2,-1,-1), \ \vec{v_3} = (-4,2,2)$$

If they are linearly dependent, when possible, express:

$\vec{v_1}$ as a linear combination of $\vec{v_2}$ and $\vec{v_3}$

$\vec{v_2}$ as a linear combination of $\vec{v_1}$ and $\vec{v_3}$

$\vec{v_3}$ as a linear combination of $\vec{v_1}$ and $\vec{v_2}$

10) Determine for which values of k the following vectors in $\mathbb{R}^5$ are linearly independent

$$\vec{v_1} = (0,1,-1,0,1), \ \vec{v_2} = (1,0,1,0,k), \ \vec{v_3} = (-1,2,-3,0,0)$$

11) Calculate the rank of the following matrix A

$$A = \begin{pmatrix} 1 & k+2 & 0 \\ k^2-1 & 0 & 4-k \\ 1 & 2k-3 & 0 \end{pmatrix} \quad k \in \mathbb{R}$$