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Author(s): Francis T. Lui
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An Equilibrium Queuing Model of Bribery

Francis T. Lui

State University of New York at Buffalo

It is sometimes argued that bribery is inefficient because bureaucrats may cause delays for attracting more bribes. This hypothesis is examined in the context of a queue where customers having different values of time are ranked by their bribe payments to the queue's server. The Nash equilibrium strategies of the customers are derived. It is shown that the server is unlikely to slow down the allocation process when bribery is allowed. The model does not have stringent informational requirements, and the equilibrium outcome minimizes the average value of time costs of the queue. It also suggests a useful auctioning procedure.

I. Introduction

If prizes are awarded simultaneously at a specified time to the first customers who queue for them, the arrival times of the customers to the queue can serve the function of prices in the allocation process (Holt and Sherman 1982, 1983). When prizes are awarded in a continuous stream, as is common in practice, the arrival times cannot serve this function very well. Instead, bribes for buying better positions in the queue sometimes give useful signals similar to those of a pricing mechanism.[1]

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[1] Rigid prices often give rise to queues. The waiting time of queues may be regarded as part of the real resource costs of price rigidity. However, as argued by Alchian (1970), stable prices may sometimes be superior to flexible market-clearing prices because the former reduces the search costs of customers who look for lower prices. Bribery in this paper may be viewed as a means to reduce the resource costs further.

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That bribery may have beneficial effects is not a new idea (e.g., Leff 1970). It is often argued that bribes serve as "lubricants" in an otherwise sluggish economy and improve its efficiency. However, aside from the undesirable distributional consequences, an important opposing view on efficiency also exists. Myrdal (1968, chap. 20), quoting the Santhanam report on prevention of corruption by the Indian government, argues that the corrupt officials may deliberately cause administrative delays so as to attract more bribes. If this is indeed the case, the efficiency argument will be much less appealing. A serious study on bribery should not leave this question unanswered.

Myrdal's hypothesis can be examined in the context of a queue where customers come to one end of it to wait for prizes distributed by a server at the other end. However, the question whether the server can increase bribe revenue by slowing down the service does not have a trivial answer. Several issues are involved. For instance, what will happen to the number of incoming customers who choose not to join the queue because the expected waiting time is too long? For those who stay, do they always want to pay larger bribes when the wait is too long? A more fundamental difficulty, however, is that a customer's action affects others. Externalities must be incorporated in the behavioral model of the queue.

Several queuing models related to bribery are available in the literature. The Kleinrock model (1967), which will be discussed extensively later in this paper, assumes that a customer paying a bribe will be placed in front of those who have paid smaller bribes in the queue, but behind those who have paid larger bribes. This model has the desirable feature that it can generate socially optimal results. However, in this model the amounts of bribes to be paid by different customers are decided by the server (who acts as if he is also a social planner) rather than the customers themselves. To obtain optimal results, a great informational burden is imposed on the server, who is required to know the values of time of all customers. Some later models have less severe informational requirements. Naor (1969) discusses a queue where a uniform toll is imposed on those who want to join it. Rose-Ackerman (1978) proposes a system in which a customer entering a queue served with higher priority will have to pay a larger bribe than one joining a lower-priority queue. However, if customers differ in their opportunity costs of time, it can easily be shown that these two models give suboptimal solutions in the sense that the total value of time spent in waiting by the customers is not minimized.[2]

[2] Naor (1969) assumes that all customers have equal values of time. Suboptimal results are thus avoided in his model. Rose-Ackerman (1978) explicitly shows that her system is not optimal. It should be noted that when people differ in their values of time,

In this paper, I propose an equilibrium queuing model of bribery with decentralized decision making. This model has some desirable features. Under some specified conditions, it is capable of giving socially optimal solutions. At the same time, it does not have stringent informational requirements. To obtain optimal results, the model is based on the queuing discipline of Kleinrock. However, the amounts of bribe payments are not decided by the server, but by the customers themselves. As we shall see, this lightens the informational requirements significantly. Another important feature is that the desired socially optimal solution is consistent with individual optimization strategies. In other words, there exists a Nash equilibrium of this noncooperative game such that, under some specified conditions, the outcome is also socially optimal. Based on this equilibrium concept, we can assess the validity of Myrdal's hypothesis. It should also be pointed out that the model need not be confined to the study of bribery alone. If bribes are regarded as legitimate payments, the mechanism becomes a useful auctioning procedure when a queue is involved.

In the next section, I outline a modified version of the Kleinrock model and derive an explicit expression for the time that a customer expects to spend in the queue. A criterion of social optimality is also stated. In Section III, I derive the bribing function for the customers and show that the implied strategies form a Nash equilibrium. In Section IV, I determine the server's optimal speed when the latter is a choice variable. I also show that Myrdal's hypothesis is not necessarily true. In Section V, I consider an extension of the model by allowing the server to charge, in addition to other bribe payments, a uniform entry fee on all joining customers. Given this assumption, Myrdal's hypothesis is shown to be even less plausible. Section VI is a summary of the results. Finally, the Appendix discusses a mathematical generalization of the model.

II. The Queuing Model

The following assumptions are made for the queuing model in this paper:

1. Customers arrive at the end of the queue according to a Poisson process at a mean rate of m customers per unit of time.

2. At the other end of the queue, there is a server who distributes a gift to each customer. The gift can also be purchased in the market at

the Rose-Ackerman system dominates the Naor system in terms of efficiency because it can differentiate the customers to some extent. However, the former also has stronger informational requirements.

a monetary value of P. The service time required to give out a gift obeys an exponential distribution with a mean service time of $1/u$.

3. Let v represent the value of time of a customer. In general, different customers may have different values of time, so that v is a random variable. The cumulative distribution function of v is represented by $A(v)$. It is assumed that $A(v)$ is known to the customers, and the derivative of $A(v)$ is continuous throughout its domain.

4. When a customer comes to the end of a queue, he can follow either of two strategies: (i) he can decide not to join the queue at all or (ii) he can pay a bribe x to the server before he sees the queue length. He will be placed in front of those whose bribes $x' < x$ and behind those whose bribes $x'' \geq x$. He does not know the actual number of people in front of him if he pays x. However, he can estimate the expected time he has to spend in the queue (waiting time plus the time spent when he is being served). The expected time spent in the queue by a customer who pays x is represented by $W(x)$. He cannot revise his bribe.

5. A customer being served will be ejected from service, but not from the queue, if a newly entering customer offers a bribe larger than his.[3]

We also let x^* represent the maximum bribe received by the server. Let the truncated distribution function of x be $B(x)$ such that $B(x^*)$ is the proportion of customers who choose to stay in the queue. In general, $B(x^*) \leq 1$. It is assumed that $B(x)$ is continuous. The variable x^* is endogenous in the model and will be determined in Section III.

PROPOSITION 1 (variant of Kleinrock [1967]). Given the assumptions of the model, the expected time that a customer paying bribe x spends in the queue is given by

$$W(x) = \frac{r}{m[1 - rB(x^*) + rB(x)]^2}, \tag{1}$$

where r is defined by $r = m/u$.

Proof. A customer paying a bribe x has to wait for three things before leaving the system:

i) His own expected service time is $1/u$ because of the assumption of the exponential distribution of service time.

ii) The customer must wait until service has been given to all those still in the queue and who arrived before him and whose bribes are at least as big as his. Owing to Little's (1961) result, which states that the expected number of units in a system is equal to the product of their

[3] This assumption is not essential, but it greatly reduces the algebra involved. The model can be modified for the case when a customer being served cannot be ejected from service.

arrival rate and the expected time they spend in the system, the expected number of customers whose bribes lie in the region $(y, y + dy)$ is[4] $m(y)W(y)dy$, where $m(y) = m[dB(y)/dy]$. The total number of those customers whose bribes are at least as big as x is therefore

$$\int_x^{x^*} m\left[\frac{dB(y)}{dy}\right]W(y)dy.$$

Since each of these customers causes him to wait $1/u$ units of time, his expected waiting time for them is

$$\int_x^{x^*} \left(\frac{m}{u}\right)\left[\frac{dB(y)}{dy}\right]W(y)dy.$$

iii) The customer must wait until service is given to those who come after him while he is still in the system and whose bribes exceed his. The expected number of such people coming per unit of time is

$$m\int_x^{x^*} dB(y).$$

Hence, during the time $W(x)$ he expects to spend in the queue, the expected number of arrivals of these customers is

$$W(x)m\int_x^{x^*} dB(y).$$

Again, on the average, each of these customers causes him to wait $1/u$ units of time. It follows that his expected waiting time for them is

$$W(x)\int_x^{x^*} \left(\frac{m}{u}\right)dB(y).$$

Adding up, we get

$$W(x) = \left(\frac{1}{u}\right) + \int_x^{x^*} \left(\frac{m}{u}\right)W(y)dB(y) + W(x)\int_x^{x^*} \left(\frac{m}{u}\right)dB(y)$$

or

$$W(x) = \frac{(1/u) + r\int_x^{x^*} W(y)dB(y)}{1 - rB(x^*) + rB(x)}. \tag{2}$$

[4] Little's result follows mainly from the observation that if a queuing system has existed for a long time, the expected number of "births" of any particular kind of customer to the system is equal to the expected number of "deaths" of this kind of customer from the system. It is also assumed that the interval $(y, y + dy)$ is so infinitesimally small that any bribe payments within this interval cannot be distinguished from each other. A new customer who pays a bribe within this interval will always be placed after those who have come earlier and have paid bribes within the same interval.

Replacing $W(x)$ and $W(y)$ in equation (2) with the expression given in equation (1), we can establish the proposition if the following equality is true:

$$\frac{(1/u)}{[1 - rB(x^*) + rB(x)]^2} = \frac{\left(\frac{1}{u}\right) + r\int_x^{x^*} \frac{(1/u)dB(y)}{[1 - rB(x^*) + rB(y)]^2}}{1 - rB(x^*) + rB(x)}.$$

By simplifying the expression on the right-hand side, we see that this equality is true. Thus, equation (1) is indeed the solution to equation (2). This completes the proof.

We now substantiate in part our earlier claim that this model is capable of yielding socially optimal outcomes. We want to examine how the bribe of a customer should be related to his value of time so that the queue has optimal properties. In other words, we want to know the necessary restrictions on the bribing function $x(v)$ such that the ranking of the customers in the queue is "correct."

To determine an optimality criterion for the queue, we first notice that real time costs are spent by customers having different values of time v. A natural direction is to consider whether the queue is capable of minimizing the average value of time costs spent by customers in the queue, which is defined as

$$\frac{\int_0^{v^*} vW[x(v)]dA(v)}{\int_0^{v^*} dA(v)},$$

where v^* is the maximum value of time among those customers who choose to stay in the queue.

DEFINITION. A queue is socially quasi-optimal for a given mean service time if the customers are ranked in such a way that for a given number of customers in the queue, the average value of time costs spent by the customers is minimized.

PROPOSITION 2. For any given $A(v)$, the bribing function $x(v)$ results in a socially quasi-optimal queue if $x(v)$ is a strictly increasing function of v.

Proof. See Kleinrock (1967).

The intuition behind this proposition is simple. To minimize the average value of time costs of the queue, all that is needed is to rank customers according to their values of time so that people with higher values of time are placed in front of those with lower values and therefore are served first. Since the queuing rule is to rank customers according to x, it necessarily also ranks them according to v for any $x(v)$ that is a strictly increasing function of v.

We have used the term "quasi-optimal" rather than "optimal" because the queue is optimal only when the number of customers is given. Nothing has been said about the optimal number of people to join the queue. For example, if nobody joins the queue, the time cost is zero. But this need not be an optimal outcome. In Sections IV and V, it will be shown that the server may have the incentive to choose a speed of service such that all incoming customers join the queue. The model guarantees that the ranking of all these customers is correct.

III. The Bribing Function and the Nash Equilibrium

I now turn to the derivation of a bribing function that is both socially quasi-optimal and privately optimal. I proceed in two steps. First, I artificially construct a differentiable bribing function that satisfies the socially quasi-optimal outcome requirement. Second, I show that if all other customers follow this bribing function, there is no incentive for anyone to depart from it.

To guarantee social quasi optimality, I impose the restriction that $x'(v) > 0$ on the bribing function I construct. Since the ranking of x is the same as the ranking of v,

$$B[x(v)] = A(v). \tag{3}$$

It follows immediately that

$$B'(x)x'(v) = A'(v). \tag{4}$$

I also note that, given this restriction, the definitions of x^* and v^* imply that $x^* = x(v^*)$. Therefore,

$$B(x^*) = A(v^*). \tag{5}$$

Each customer with a given value of time $\bar{v}$ solves the following maximization problem:

$$\max_x G = P - [x + \bar{v}W(x)]. \tag{6}$$

Recall that P is the monetary value of the gift. The term in brackets is the expected total cost of joining the queue, and G is the expected net gain. Because of (1), equation (6) can also be stated as

$$\max_x G = P - x - \frac{\bar{v}r}{m[1 - rB(x^*) + rB(x)]^2}. \tag{7}$$

The first-order necessary condition is

$$\frac{dG}{dx} = -1 + \frac{2r^2vB'(x)}{m[1 - rB(x^*) + rB(x)]^3} = 0. \tag{8}$$

Equation (8) defines a relation between x and v. To get a more explicit solution, substitution of equations (3), (4), and (5) into (8) obtains

$$x'(v) = \frac{2r^2 vA'(v)}{m[1 - rA(v^*) + rA(v)]^3}. \tag{9}$$

To solve this differential equation, integrate it with respect to v:

$$x = \int \frac{2r^2 vA'(v)dv}{m[1 - rA(v^*) + rA(v)]^3} + K, \tag{10}$$

where K is a constant to be determined. It should be pointed out that in (7) the notation $\bar{v}$ is used to emphasize that it is a parameter in the maximization problem. In (8), (9), and (10) v is used rather than $\bar{v}$ to indicate that once the maximization problem is solved, the bribe x is dependent on the variable v.

It is also necessary to show that (10) is the solution of a maximization problem:

$$\frac{d^2G}{dx^2} = \left(\frac{2r^2v}{m}\right)\frac{[1 - rB(x^*) + rB(x)]B''(x) - 3r[B'(x)]^2}{[1 - rB(x^*) + rB(x)]^4}.$$

By using (8) to get expressions for $B'(x)$ and $B''(x)$, we can simplify this to

$$\frac{d^2G}{dx^2} = \frac{-1}{x'(v)v} < 0$$

for $v > 0$. If $v = 0$, from (7), the maximum of G obviously occurs at the lowest permissible value of x. By assumption, negative values of v are not allowed.

Since the main purpose of this paper is to assess the validity of Myrdal's hypothesis, it is desirable to see whether counterexamples to the hypothesis exist. To do this, I shall derive a more explicit bribing function than (10) by making the additional assumption that $A(v)$ is a uniform distribution function from $v = 0$ to $v = v_1$:

$$A(v) = Av \quad \text{for } v \in [0, v_1]. \tag{11}$$

The results for the general distribution function $A(v)$ will be discussed in the Appendix.

Equation (10) now becomes

$$x = \int \frac{2r^2 vAdv}{m(1 - rAv^* + rAv)^3} + K.$$

Solving this gives

$$x = \frac{-vr}{m(1 - rAv^* + rAv)^2} - \frac{1}{mA(1 - rAv^* + rAv)} + K. \tag{12}$$

A necessary condition for Nash equilibrium, as argued below, is that the customer with the lowest value of time does not pay any bribe. Because $x'(v) > 0$, other people with higher values of time always pay higher bribes than he does. If he pays a positive bribe, he can always improve his gain by paying less without affecting the time he expects to spend in the queue. The case for a minimum positive bribe imposed by the server will be fully discussed in Section V. In (12), $v = 0$ implies $x = 0$. This condition can be used to solve for K. The bribing function now becomes

$$x = \frac{1}{mA(1 - rAv^*)} - \frac{vr}{m(1 - rAv^* + rAv)^2} - \frac{1}{mA(1 - rAv^* + rAv)}. \tag{13}$$

It remains to determine v^*. Recall that x^* is the largest bribe paid by a customer in the queue and v^* is his corresponding value of time, which is also the largest among those who join the queue. For this customer, his expected net gain must be nonnegative. Otherwise, he will not join the queue. Moreover, as long as $v^* < v_1$, that is, some people do not join the queue, his gain cannot be positive. Otherwise, people with a value of time just above his will also join the queue.[5] Hence, for $v^* < v_1$, $G(x^*) = P - x^* - v^*W(x^*) = 0$. From (1),

$$\begin{aligned} x^* &= P - \frac{v^*r}{m[1 - rB(x^*) + rB(x^*)]^2} \\ &= P - \frac{v^*r}{m}. \end{aligned}$$

By substituting $v = v^*$ into (13), we also obtain

$$x^* = \frac{1}{mA(1 - rAv^*)} - \frac{v^*r}{m} - \frac{1}{mA}.$$

The solution of the last two equations is

$$v^* = \frac{mPA}{rA(1 + mPA)}. \tag{14}$$

For convenience, define $z = mPA$. Equation (14) now becomes

$$v^* = \frac{z}{rA(1 + z)}. \tag{15}$$

[5] The argument follows from the fact that the gain of joining the queue is a decreasing function of v, which can be proved easily.

By utilizing the fact that $Av_1 = 1$, we can readily show an alternative expression for v^* to be

$$v^* = \frac{zv_1}{r(1 + z)}.$$

The condition that $v^* < v_1$ is therefore equivalent to

$$r > \frac{z}{1 + z}. \tag{16}$$

In other words, (15) holds if (16) is true.

Suppose that condition (16) holds, that is, $v^* < v_1$, or that only some customers join the queue. Then by substituting (15) into (13), we get

$$x = \frac{1}{mA}\left(1 + z - \frac{rAv}{\{[1/(1 + z)] + rAv\}^2} - \frac{1}{[1/(1 + z)] + rAv}\right). \tag{17}$$

We can also differentiate x with respect to v:

$$\frac{dx}{dv} = \frac{2r^2Av}{m\{[1/(1 + z)] + rAv\}^3} > 0.$$

This is of course the quasi optimality condition we have imposed earlier.

Suppose $r \leq z/(1 + z)$. Obviously,

$$v^* = v_1. \tag{18}$$

This means that all customers decide to join the queue. Equation (13) then becomes

$$x = \frac{1}{mA(1 - r)} - \frac{vr}{m(1 - r + rAv)^2} - \frac{1}{mA(1 - r + rAv)}. \tag{19}$$

We have made use of the fact that $Av_1 = 1$. Again, we can check whether $x'(v) > 0$:

$$\frac{dx}{dv} = \frac{2r^2Av}{m(1 - r + rAv)^3},$$

which must be positive because the condition $r \leq z/(1 + z) < 1$ applies to the case here.[6]

Equations (17) and (19) express the bribe x in terms of the parameters m, r, A, P, and the variable v. If the customers know their own values of time, they can compute the optimal bribes they should pay.

PROPOSITION 3. (i) Suppose $r > z/(1 + z)$. If customers with $v \leq v^*$

[6] Obviously, the two expressions for dx/dv here are special cases of eq. (9). It can be proved that if the queue does not get infinitely long, the term $rA(v^*)$ in (9) must be less than one. The $x'(v)$ given in this equation must therefore be positive.

follow bribing strategies given by (17) and customers with $v > v^*$ do not join the queue, where v^* is determined by (15), then these strategies form a Nash equilibrium that is socially quasi-optimal. (ii) Suppose $r \leq z/(1 + z)$. If all the customers follow bribing strategies given by (19), then these strategies form a Nash equilibrium that is socially quasi-optimal.

Proof. (i) Suppose $r > z/(1 + z)$. From proposition 2, if everybody with $v \leq v^*$ follows (17), the solution must be socially quasi-optimal because it has already been shown that $x'(v) > 0$.

Suppose all customers with $v \leq v^*$ follow (17) and those with $v > v^*$ do not join the queue. From how we constructed the bribing function, (17) is clearly the solution of the maximization problem (7). Hence, there is no incentive for those with $v \leq v^*$ to change the bribe. Moreover, for those with $v > v^*$, their net gain in joining the queue is negative if other customers follow (17). Thus, these customers have no incentive to join the queue. Since $x(0) = 0$, the customers with $v = 0$ cannot further improve their gain by paying less. They will not depart from the strategy (17). The strategies outlined above therefore form a Nash equilibrium.

ii) The proof is almost identical and is omitted here. (Note that given $v^* = v_1$, the net gains for all the customers are nonnegative.)

The following example illustrates much of what has been discussed in this section. Let $r = 1$, $m = 1$, $P = 1$, and $A = 1$. It follows easily that $x^* = 0.5$, $v^* = 0.5$, and $B(x^*) = 0.5$. Figure 1 plots the net gain G against the bribe x for different values of v, assuming that all other customers follow their own equilibrium strategies. The optimal bribe paid by a customer with v is the x that gives the maximum point on the corresponding curve if the gain is positive. It can be seen that the maximum gain of a customer is a decreasing function of his value of time v. For $v > 0.5$, the customers cannot have positive gain and do not join the queue. For $x \geq 0.5$, the customer is already at the front of the queue. He cannot improve his position by paying more bribes. The curves therefore become decreasing straight lines.

IV. Optimal Speed of Service

The bribing function (17) depends on the parameters r, m, P, and A, while (19) depends on r, m, and A. Let us now consider the effects of changes in the speed of service on the average revenue received by the server per period of time.

PROPOSITION 4. In the model outlined above, if $r < z/(1 + z)$, increasing the mean service time per customer ($1/u$) will cause the average bribe revenue received by the server per period to go up. If $r \geq z/(1 + z)$, increasing $1/u$ will cause the average bribe revenue per period to go down.

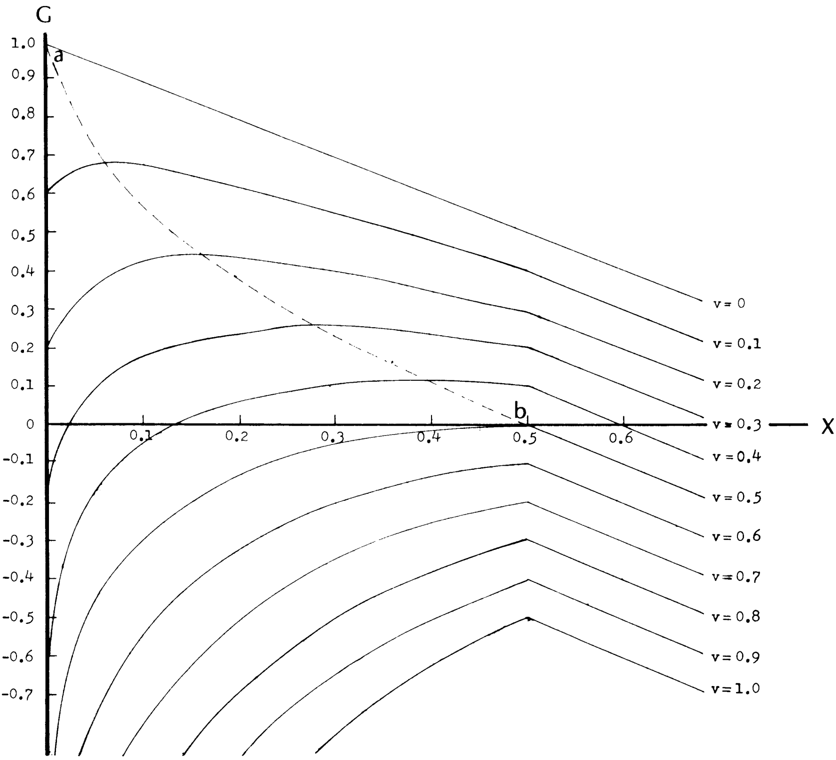

FIG. 1.—Relation between net gain and bribe. The line *ab* joins the maximum points of the curves.

Proof. The average bribe paid to the server by an incoming customer is given by

$$\bar{x} = \int_0^{v^*} x(v) A dv,$$

where $x(v)$ is the bribing function given by (17) if $r > z/(1 + z)$ and given by (19) if $r \leq z/(1 + z)$. Note that $x(v)$ also depends on the parameters v, m, A, and P, which have been suppressed in the notation here.

Since on the average there are m customers coming to the queue per period, the average bribe revenue per period is $m\bar{x}$. Suppose $r < z/(1 + z)$. Then $v^* = v_1$. All customers join the queue. We want to show that $d(m\bar{x})/d(1/u) > 0$. Since $r = m/u$, for fixed m, it suffices to show that $d\bar{x}/dr > 0$. From (19),

$$\begin{aligned} \bar{x} &= \int_0^{v_1} \left[\frac{1}{mA(1 - r)} - \frac{vr}{m(1 - r + rAv)^2} - \frac{1}{mA(1 - r + rAv)} \right] A dv \\ &= \frac{v_1}{m(1 - r)} + \frac{v_1}{m} + \frac{2 \ln(1 - r)}{rmA}. \end{aligned} \tag{20}$$

Differentiation of (20) yields

$$\frac{d\bar{x}}{dr} = \frac{v_1}{mr^2}\left[\frac{3r^2 - 2r}{(1 - r)^2} - 2\ln(1 - r)\right]. \tag{21}$$

Let J denote the term inside the brackets. Clearly, $r = 0$ implies $J = 0$. Moreover, $dJ/dr = 2r^2/(1 - r)^3 > 0$, since $r < z/(1 + z) < 1$. Hence, for $r \geq 0$ the smallest value of J is zero, which occurs at $r = 0$. Thus, for $0 < r < z/(1 + z)$,

$$\frac{d\bar{x}}{dr} > 0. \tag{22}$$

By differentiating (21), we can also show that $d^2\bar{x}/dr^2 > 0$.

Next assume that $r > z/(1 + z)$. Customers with $v > v^*$ will not join the queue and do not pay any bribe. We want to show that $d\bar{x}/dr < 0$. From equation (17),

$$\bar{x} = \int_0^{v^*}\left(\frac{1}{mA}\right)\left(1 + z - \frac{rAv}{\{[1/(1 + z)] + rAv\}^2} - \frac{1}{[1/(1 + z)] + rAv}\right)A\,dv.$$

By using the v^* given in equation (15), we can simplify this expression to

$$\bar{x} = \frac{1}{rmA}\left[z + \frac{z}{1 + z} - 2\ln(1 + z)\right]. \tag{23}$$

Differentiation of (23) yields

$$\frac{d\bar{x}}{dr} = \frac{1}{mAr^2}\left[2\ln(1 + z) - \left(z + \frac{z}{1 + z}\right)\right].$$

By a trick similar to that in obtaining (22), the term inside the brackets can be shown to be strictly negative. Hence, if $r > z/(1 + z)$,

$$\frac{d\bar{x}}{dr} < 0. \tag{24}$$

It also follows easily that $d^2\bar{x}/dr^2 > 0$.

We now consider the situation when $r = z/(1 + z)$. In this case, increasing r will cause it to become larger than $z/(1 + z)$. Result (24) will apply. Thus, increasing the mean service time reduces the average bribe revenue when $r \geq z/(1 + z)$.[7] This completes the proof.

Proposition 4 shows that if the objective of the server is to maximize bribe revenue and if he is free to change the speed of service, he will

[7] At $r = z/(1 + z)$, if r decreases, eq. (27) is applicable.

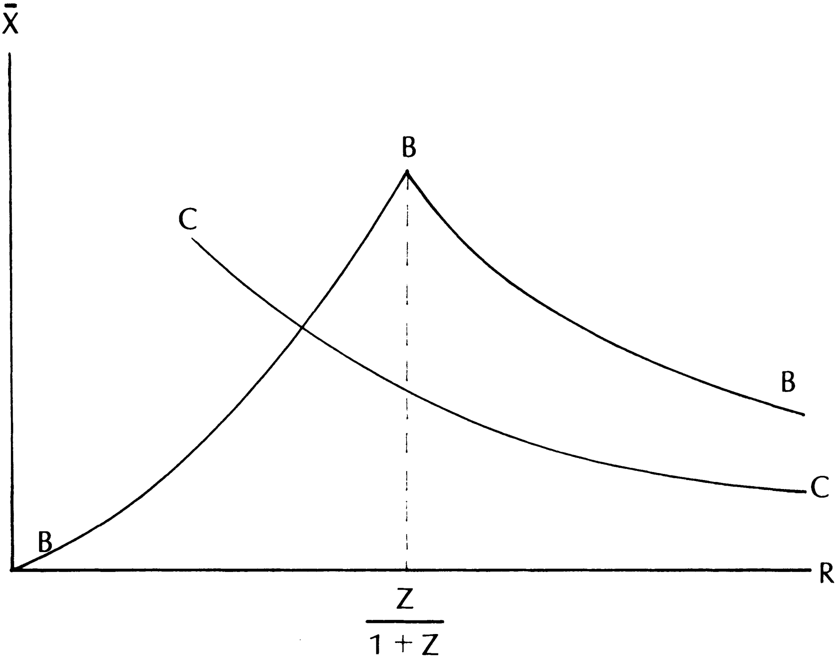

FIG. 2.—Relation between server's bribe revenue, cost, and r. BBB represents the bribe revenue function $x(r)$. CC represents total cost as a function of r. Maximum bribe revenue net of cost occurs at $r = z/(1 + z)$.

set $r = z/(1 + z)$. Let us call this value r^*. He will not choose a speed that is too slow because too few customers will want to join the queue or pay any bribes. He also will not choose a speed that is too fast because when waiting cost is very low, many people will have less incentive to pay bribes. The r^* can be considered an element of the set of Nash equilibrium strategies of the system. Suppose that before any bribing occurs the initial r (which can be any positive number) is larger than r^*; that is, only some customers join the queue. After bribery is permitted, the server has the incentive to speed up the system rather than to slow it down. The contrary of Myrdal's hypothesis is therefore possible.

Suppose there is a cost to the server to perform his service. His objective will then be to maximize the bribe revenue net of cost. Proposition 4 is useful in showing that it is unlikely for the server to slow down the system when bribery is allowed. Assuming increasing marginal cost for the speed, with no bribery, the optimal strategy for the server is to do nothing at all. In other words, the service rate $u = 0$, or r is infinitely large. If bribery is allowed, the speed can only be faster or remain the same. An example is illustrated in figure 2 where the server's optimal speed with bribery is given by $r^* = z/(1 + z)$, but that without bribery is given by an infinitely large r. Again, the contrary of Myrdal's hypothesis is true.

The chief results will still hold even when the uniform distribution of the v assumption is relaxed. In the Appendix, I show that for general distribution functions of v, counterexamples of Myrdal's hypothesis could also appear.

V. Optimal Speed When an Entry Fee Is Required

In Section III, a condition for Nash equilibrium is that a customer with $v = 0$ does not pay any bribe. Now suppose the server imposes a uniform entry fee F on all customers who decide to join the queue. Without paying this fee, a customer will never be served. In addition to the fee, as before, a customer can decide to pay more bribe x to buy a better position in the queue. The entry fee can also be interpreted as the lower bound on bribe payments by customers joining the queue. I assume that F is a choice variable of the server, and I shall show that given this new assumption in the model, Myrdal's hypothesis can never be true.

From the standpoint of a customer, the following two events are equivalent. (i) In addition to any bribe a customer wants to pay, the server charges an entry fee F on all joining customers. (ii) The value of the gift P is reduced by F. The equivalence of these two events permits us to use the same bribing functions developed earlier for our present analysis.

From the standpoint of the server, an increase in F will discourage people from joining the queue, a result that follows trivially from the analysis below. On the other hand, he can receive some revenues in the form of fees. We shall first determine the server's optimal F as a function of r. Then, assuming that the server always chooses the optimal F, we also derive the revenue function of the server and show that it is a strictly decreasing function of r.

Let us define $P^* = P - F$ and $z^* = mP^*A = z - mFA$. Any increase in F, as noted above, can be regarded as a decrease in the new value of the gift, P^*. Notice that F cannot be larger than P. Otherwise, nobody will join the queue. It follows that z^* and P^* must always be positive. We also let $\bar{v}^*$ represent the maximum value of time among customers who choose to stay in the queue after F has been imposed.

Suppose $\bar{v}^* < v_1$; that is, after imposing F, only some incoming customers join the queue. We can substitute z^* for z in equation (15) to obtain

$$\bar{v}^* = \frac{z - mFA}{rA(1 + z - mFA)}. \tag{25}$$

Since $v_1 = 1/A$, the condition $\bar{v}^* < v_1$ is equivalent to

$$r > \frac{z - mFA}{1 + z - mFA}. \tag{26}$$

Assume condition (26) holds. We can determine the server's expected bribe revenue from an average customer (who may or may not join the queue) by substituting z^* for z in equation (23). In addition to this bribe revenue, the server also receives the entry fee F if the customer joins the queue. Since the proportion of customers joining the queue is $A\bar{v}^*$, the expected fee from a customer coming to the end of the queue is $FA\bar{v}^*$. Hence, the revenue expected from a customer is given by

$$\bar{x}_F = \frac{1}{rmA}\left[z^* + \frac{z^*}{1 + z^*} - 2\ln(1 + z^*)\right] + FA\bar{v}^*. \tag{27}$$

After substituting $z^* = z - mFA$ and $\bar{v}^* = (z - mFA)/rA(1 + z - mFA)$ into (27), we can derive the first-order and second-order conditions:

$$\frac{d\bar{x}_F}{dF} = \frac{mA(P - 2F)}{r(1 + z - mAF)^2} = 0, \tag{28}$$

$$\frac{d^2\bar{x}_F}{dF^2} = \frac{-2mA(1 + mAF)}{r(1 + z - mAF)^3} < 0, \tag{29}$$

since $z^* = z - mAF > 0$. Thus, if condition (26) holds, the server's revenue attains a maximum if he chooses $F = P/2$. Assuming that the server always chooses the optimal F, we can substitute $F = P/2$ into (27). On simplification, this gives us the optimal revenue function of the server when only some customers join the queue:

$$\bar{x}_F = \frac{1}{rmA}\left[z - 2\ln\left(1 + \frac{z}{2}\right)\right]. \tag{30}$$

It follows immediately that

$$\frac{d\bar{x}_F}{dr} = \frac{-1}{r^2mA}\left[z - 2\ln\left(1 + \frac{z}{2}\right)\right].$$

By a trick similar to that in obtaining (22), the term in the brackets can be shown to be positive. Hence,

$$\frac{d\bar{x}_F}{dr} < 0. \tag{31}$$

Since F is always chosen optimally, we can also substitute $F = P/2$ into (26). In other words, as long as $r > z/(2 + z)$, the server can increase his revenue by decreasing r, that is, by increasing the speed of service. Again the contrary of Myrdal's hypothesis is true here.

We next consider the case when $r \leq z/(2 + z)$. We shall first determine the optimal F chosen by the server. Assume that in the beginning the F chosen is in fact $P/2$. Obviously, because (26) is not satisfied, all customers join the queue and $\bar{v}^* = v_1$. From the bribing function given by (20) and the fact that every customer pays the entry fee to the server, the latter's revenue expected from an average customer is

$$\bar{x}_F = \frac{v_1}{m(1 - r)} + \frac{v_1}{m} + \frac{2v_1 \ln(1 - r)}{mr} + F. \tag{32}$$

Obviously, $d\bar{x}_F/dF = 1 > 0$. In other words, the server can keep on increasing revenue by raising F. But this is true only if $r \leq (z - mFA)/(1 + z - mFA)$. When F is increased, the right-hand side of this inequality goes down. Eventually, when F is large enough, strict equality will occur. Let F^* be the entry fee such that this strict equality holds. It follows easily that

$$F^* = \frac{(1 - r)z - r}{mA(1 - r)}. \tag{33}$$

Suppose the server increases the fee further so that $F > F^*$. Then condition (26) again holds, and only some customers join the queue. The revenue function will then be (27) instead of (32). From (28), it can be seen that as long as only some of the customers join the queue, $d\bar{x}_F/dF < 0$ if $F > P/2$. Since the F under consideration is indeed larger than $P/2$, the server cannot further increase the revenue by raising F beyond F^*. Thus, as long as $r < z/(2 + z)$, the optimal F is F^*, which is just enough to make every customer join the queue. To obtain the optimal revenue function, we can substitute F^* given by (33) into equation (32). After simplification, this becomes

$$\bar{x}_F = \frac{1}{mA}\left[2 + z + \frac{2 \ln(1 - r)}{r}\right]. \tag{34}$$

Differentiation yields

$$\frac{d\bar{x}_F}{dr} = \frac{-2}{mAr^2}\left[\frac{r}{1 - r} + \ln(1 - r)\right].$$

By the same trick as the one used in getting (22), we can readily show that the term inside the brackets is strictly positive. Hence,[8]

$$\frac{d\bar{x}_F}{dr} < 0. \tag{35}$$

[8] The second derivative is positive. Thus, the function is concave downward.

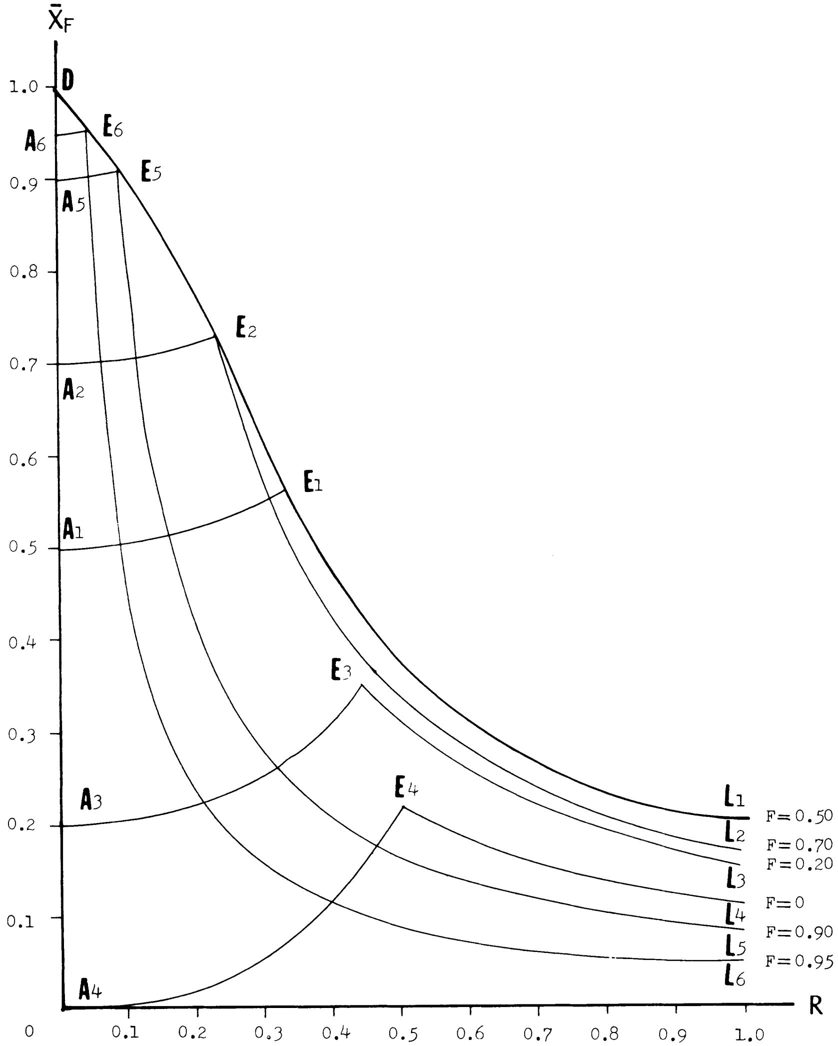

FIG. 3.—Relation between server's revenue and r for different values of the entry fee. DE_1L_1 represents the server's revenue function when F is chosen optimally.

Because of (31) and (35), the server can always increase the revenue $\bar{x}_F$ by increasing the speed of service, whether r is larger than, equal to, or smaller than $z/(2 + z)$. When the possibility of an entry fee is allowed, the positively sloping part of the bribe revenue function in proposition 4 does not appear. Whether or not it is costly to increase the speed of service, there is no possibility that Myrdal's hypothesis is true.

The analysis above can also be illustrated by an example depicted in figure 3. Let $m = 1, P = 1$, and $A = 1$. The $A_kE_kL_k$ curves, $k = 1, \ldots,$

6, are revenue functions of the server for different values of F. The curve DE_1L_1 is the revenue function when F is chosen optimally. This curve has a negative slope throughout, which is the main result of this section.

VI. Concluding Remarks

I have examined Myrdal's hypothesis in the context of an equilibrium queuing model. In this model, customers can decide to pay bribes for buying better positions in the queue. The bribing strategies of the customers who have different values of time have been derived. It has been shown that these strategies form a Nash equilibrium that minimizes the average value of the time costs of the queue. Based on this equilibrium, the server who wants to maximize either bribe revenue or bribe revenue net of cost of service will also choose an optimal speed of service. It has been shown that the server could choose to speed up the service when bribery is allowed. The contrary of Myrdal's hypothesis is therefore possible.[9]

The model has been extended to allow for the possibility of charging a uniform entry fee on the customers in addition to other bribes they want to pay. Since the fee discourages customers from joining the queue, the server wants to speed up further to get back as many bribers as possible. Given this new possibility, Myrdal's hypothesis cannot be true in the model.

If bribes are regarded as legitimate payments, the model suggests a useful auctioning procedure when a queue is involved. The informational requirements of this procedure are not stringent. The ranking of the customers is also "correct." However, if the server does not own the bribe payments, some of the results in this paper are not applicable to the auctioning procedure. Specifically, the server may not have the incentive to speed up the service.

Appendix

I examine how proposition 4 has to be modified if the distribution function of v is a general function $A(v)$ instead of the uniform distribution function Av

[9] This result does not strictly depend on the Nash equilibrium approach. Suppose the server sells the queue positions according to a pricing scheme that he announces. Then a customer has to pay attention only to the pricing scheme when he makes the decisions, not to the strategies of other people. If the announced pricing scheme is in fact the $W(x)$ given by eq. (1), the $B(x)$ and $B(x^*)$ in it are consistent with the Nash equilibrium bribing strategies derived in Sec. III, and the server follows the same queuing rule of Sec. II to determine the priority of service, then it can be shown (Lui 1985) that the supplies for the queue positions will be equal to their respective demands. The main result in this paper will also remain valid.

assumed in the paper. It is required that $A(v)$ be a differentiable function with the lowest v at zero and that $A'(v)$ be continuous. I make use of Leibniz's rule for this purpose.

For $v^* < v_1$,

$$\bar{x} = \int_0^{v^*(r)} x(v, r)A'(v)dv. \tag{A1}$$

In $x(v, r)$, the parameters other than r have been suppressed in the expression. Apply Leibniz's rule to (A1):

$$\frac{d\bar{x}}{dr} = \int_0^{v^*(r)} \frac{dx}{dr} A'(v)dv + x[v^*(r), r]A'[v^*(r)]\frac{dv^*(r)}{dr}. \tag{A2}$$

To determine the sign of $d\bar{x}/dr$, it is necessary to know the signs of dv^*/dr and dx/dr. First consider dv^*/dr. Equation (10) and the Nash equilibrium condition imply that

$$\begin{aligned} x^* &= \int_0^{v^*} \frac{2r^2 v dA(v)}{m[1 - rA(v^*) + rA(v)]^3} \\ &= \frac{-rv^*}{m} + \int_0^{v^*} \frac{rdv}{m[1 - rA(v^*) + rA(v)]^2}. \end{aligned}$$

However, we know from the text that $x^* = P - (rv^*/m)$. Hence,

$$P = \int_0^{v^*} \frac{rdv}{m[1 - rA(v^*) + rA(v)]^2}. \tag{A3}$$

Using Leibniz's rule again,

$$\begin{aligned} 0 &= \frac{dP}{dr} \\ &= \int_0^{v^*} \frac{d}{dr}\left\{\frac{rdv}{m[1 - rA(v^*) + rA(v)]^2}\right\} + \frac{r(dv^*/dr)}{m[1 - rA(v^*) + rA(v^*)]^2} \\ &= \int_0^{v^*} \frac{dv}{m[1 - rA(v^*) + rA(v)]^2} \\ &\quad + \int_0^{v^*} \frac{2r[A(v^*) - A(v) + r(dv^*/dr)A'(v^*)]dv}{m[1 - rA(v^*) + rA(v)]^3} \\ &\quad + \left(\frac{r}{m}\right)\left(\frac{dv^*}{dr}\right). \end{aligned} \tag{A4}$$

Suppose $dv^*/dr \geq 0$. Then since $A(v^*) - A(v)$ in the second term of (A4) is always positive because $v < v^*$ and $A'(v^*) > 0$, the second term must be strictly positive. The first and third terms are also positive. The right-hand side of (A4) must therefore be strictly positive. This is a contradiction. Hence,

$$\frac{dv^*}{dr} < 0. \tag{A5}$$

Consider dx/dr:

$$\frac{dx}{dr} = \frac{d}{dr}\int_0^v \frac{2r^2 v dA(v)}{m[1 - rA(v^*) + rA(v)]^3}$$

$$= \int_0^v \frac{6r^2 v[A(v^*) - A(v) + r(dv^*/dr)A'(v^*)]dA(v)}{m[1 - rA(v^*) + rA(v)]^4} + \int_0^v \frac{4rvdA(v)}{m[1 - rA(v^*) + rA(v)]^3}. \quad (A6)$$

The second term is clearly positive. In the first term, since dv^*/dr is negative, the sign of the numerator is indeterminate.

From (A5) and (A6), the first term of (A2) is indeterminate, while the second term is negative. Thus,

$$\frac{d\bar{x}}{dr} \gtrless 0 \quad \text{for } v^* < v_1. \quad (A7)$$

Now consider $v^* = v_1$. Equation (A1) then becomes

$$\bar{x} = \int_0^{v_1} x(v, r)A'(v)dv, \quad (A8)$$

$$\frac{d\bar{x}}{dr} = \int_0^{v_1} \frac{dx}{dr}A'(v)dv, \quad (A9)$$

since $dv_1/dr = 0$;

$$\frac{dx}{dr} = \int_0^v \frac{6r^2 v[1 - A(v) + r(dv_1/dr)A'(v_1)]dA(v)}{m(1 - r + rA(v)]^4} + \int_0^v \frac{4rvdA(v)}{m[1 - r + rA(v)]^3}. \quad (A10)$$

Since $dv_1/dr = 0$, the first term on the right-hand side must be positive. The second term is also clearly positive. Thus, $dx/dr > 0$. From (A9),

$$\frac{d\bar{x}}{dr} > 0 \quad \text{for } v^* = v_1. \quad (A11)$$

The first part of proposition 4 is therefore always true. For the case $v^* < v_1$, slowing down causes fewer people to join the queue because of (A5). Whether those who stay will pay more bribe is indeterminate. We cannot know unambiguously the sign of $d\bar{x}/dr$. The second part of proposition 4 should be modified accordingly. But even in this case, it is quite possible to encounter situations contradictory to Myrdal's hypothesis.

It should be pointed out that the algebraic results in the text can also be obtained by using Leibniz's rule.

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