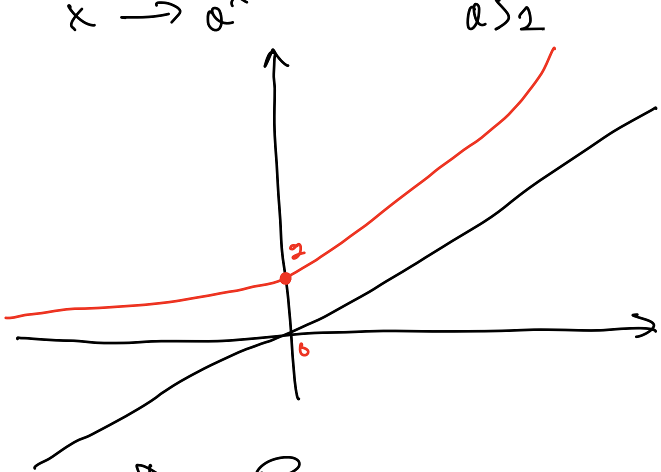
$0<a<1,\ a>1$

$x \to a^x$

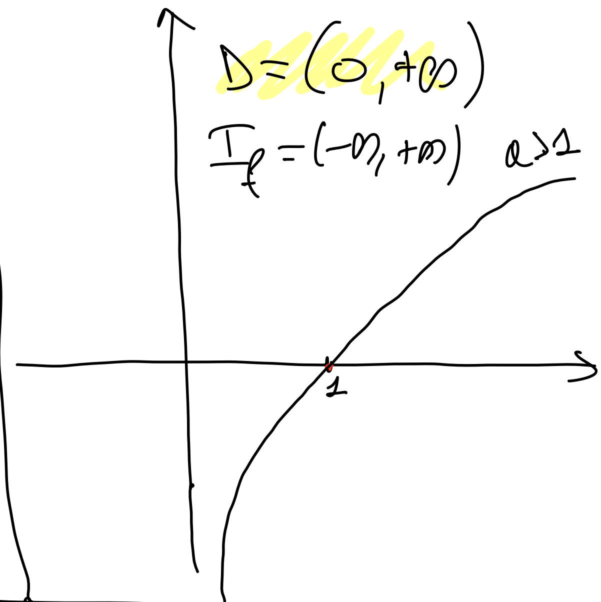


$D = \mathbb{R}$

$I_f = (0, +\infty)$

$0<a<1 \qquad a>1$

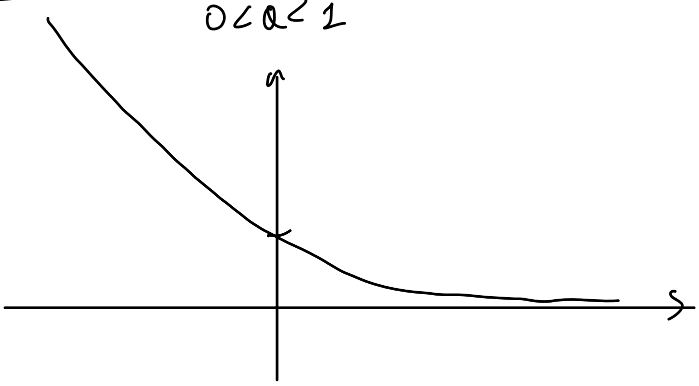
$x \to \log_a(x)$



$D = (0, +\infty)$

$I_f = (-\infty, +\infty)$

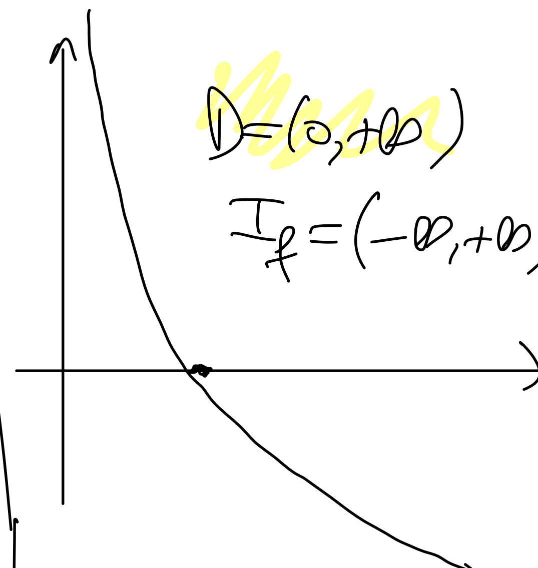
$0<a<1$



$D = \mathbb{R}$

$I_f = (0, +\infty)$

$0<a<1$



$D = (0, +\infty)$

$I_f = (-\infty, +\infty)$

$$a^{x+y} = a^x \cdot a^y$$

$$(a^x)^y = (a^y)^x = a^{xy}$$

$$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$$

$$\log_a(x^y) = y \log_a(x)$$

$a^0 = 1$ | $\log_a(1) = 0$

ESERCIZIO: SI TROVI IL DOMINIO DI

$$f(x) = \log_2(\underbrace{1 - x^2})$$

$$1 - x^2 > 0 \iff x \in (-1, 1)$$

$$D = (-1, 1)$$

$$f(x) = \log_2(\underbrace{\log_2(\log_2(x))}) \qquad \log_2(g(x))$$

$g(x) > 0$

$$\log_2(\log_2(x)) > 0 \iff \log_2(x) > \log_2(2) = 1$$

$a > b$

$$\Rightarrow 2^{\log_2(x)} > 2 \qquad 2^a > 2^b$$

$$x > 2$$

$$D = (2, +\infty)$$

$$f(x) = \log_2\left(\frac{x}{1+2^x}\right) \qquad D = ? = (0, +\infty)$$

$$\frac{x}{1+2^x} > 0 \iff x > 0$$

$$f(x) = \log_2\left(\frac{1}{1+x^2}\right) \qquad D = \mathbb{R}$$

PER QUALI x SI HA CHE $f(x) \geq 0$?

$$\frac{1}{1+x^2} \geq 1 \iff 1 \geq 1 + x^2$$
$$\iff x = 0$$

$$\log_2\left(\frac{1}{1+x^2}\right) \leq 0 \quad \text{E} \quad \log_2\left(\frac{1}{1+x^2}\right) = 0 \iff x = 0$$

$$\log_2\left(\frac{1}{1+x^2}\right) = \log_2\left(\left(1+x^2\right)^{-1}\right) = -\underbrace{\log_2\left(1+x^2\right)}$$

$$f(x) = \log_{\left(\frac{1}{3}\right)}(|x|) \qquad D = \mathbb{R}\setminus\{0\}$$

$$\sqrt{x^2} = |x|$$

PRINCIPIO DI INDUZIONE:

SIA $P(n)$ UNA PROPRIETÀ DEFINITA PER OGNI INTERO $n \in \mathbb{N}$

SE PER UN DATO n^* SI HA CHE $P(n^*)$ È VERA

E SE $\forall\, n \geq n^*$ POSSO DIMOSTRARE CHE

$$P(n) \Rightarrow P(n+1)$$

ALLORA $P(n)$ È VERA $\forall\, n \geq n^*$

ESEMPIO: SIA S_n LA SOMMA DEI PRIMI n NUMERI NATURALI

$$S_n = 1+2+3+\dots+n$$

$S_1 = 1$ $\quad n=1$

$S_2 = 1+2 = 3 = \frac{2 \cdot 3}{2}$ $\quad n=2$

$S_3 = 1+2+3 = 6 = \frac{3 \cdot 4}{2}$ $\quad n=3$

$S_4 = 1+2+3+4 = 10 = \frac{4 \cdot 5}{2}$ $\quad n=4$

$$P(n):\ S_n = \frac{n \cdot (n+1)}{2} = 1+2+3+\dots+n$$

$$P(n) \Rightarrow P(n+1)$$

ASSUMENDO CHE $P(n)$ SIA VERA DEVO DIMOSTRARE $P(n+1)$

$$S_{n+1} = 1+2+\dots+n+n+1$$

$$= \frac{n \cdot (n+1)}{2} + n+1 = \frac{(n+1)(n+2)}{2} \quad P(n+1)$$

ESEMPIO: SIA $x \in \mathbb{R}$ $m \in \mathbb{N}$, $m \geq 1$ $x \neq 0$

$$x^0 + x^1 + x^2 + \dots + x^m =$$

$$1 + x + x^2 + \dots + x^m \underset{?}{=} \frac{1-x^{m+1}}{1-x}$$

VOGLIO DIMOSTRARE QUESTA IDENTITÀ USANDO IL PRINCIPIO DI INDUZIONE

1) È VERA PER UN DATO m^*?

PROVO $m^* = 1$ VERA!

$$1+x = \frac{1-x^2}{1-x} = \frac{(1-x)(1+x)}{1-x} = 1+x$$

2) DEVO DIMOSTRARE CHE SE È VERA LA FORMULA PER UN GENERICO m ALLORA ESSA È VERA PER $m+1$

$$1+x+x^2+\dots+x^m+x^{m+1} = \frac{1-x^{m+1}}{1-x} + x^{m+1}$$

$$1+x+\dots+x^m = \frac{1-x^{m+1}}{1-x}$$

$$= \frac{1-\cancel{x^{m+1}}+\cancel{x^{m+1}}-x^{m+2}}{1-x}$$

VERA

$$= \frac{1-x^{(m+1)+1}}{1-x} \quad P(m+1)$$

• $(1+x)^n \ge 1+nx \qquad \forall x \ge -1 \qquad \forall n \ge 1$

DISUGUAGLIANZA BINOMIALE

$\hookrightarrow \boxed{|x|>0} \iff \underline{x \neq 0}$

$|x| \ge 0 \quad$ e $\quad |x| = 0 \iff x = 0$

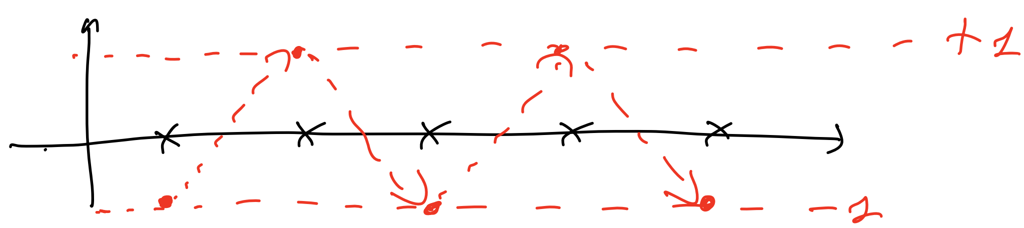
DEF:

- $\lim\limits_{n \to +\infty} S_n = L \iff \forall \varepsilon > 0 \ \exists n_\varepsilon \in \mathbb{N} : \forall n \ge n_\varepsilon \Rightarrow |S_n - L| < \varepsilon$
- $\lim\limits_{n \to +\infty} S_n = +\infty \iff \forall M > 0 \ \exists n_M \in \mathbb{N} : \forall n \ge n_M \Rightarrow S_n > M$
- $\lim\limits_{n \to +\infty} S_n = -\infty \iff \forall M > 0 \ \exists n_M \in \mathbb{N} : \forall n \ge n_M \Rightarrow S_n < -M$

TEO: SE $\lim\limits_{n \to +\infty} S_n = L \Rightarrow \begin{cases} \lim\limits_{n \to +\infty} S_{2n} = L & \Leftarrow \\ \lim\limits_{n \to +\infty} S_{2n+1} = L & \Leftarrow \end{cases}$

ESEMPIO: $S_n = (-1)^n$

$S_1 = -1 \quad S_2 = 1 \quad S_3 = -1 \quad S_4 = +1 \quad \dots$

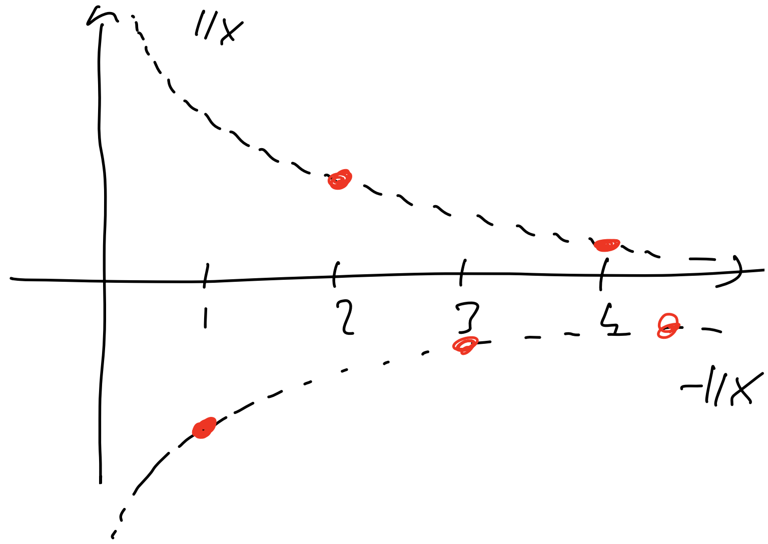


$$S_{2n} = (-1)^{2n} = +1 \longrightarrow +1$$

$$S_{2n+1} = (-1)^{2n+1} = -1 \longrightarrow -1$$

$$\boxed{\nexists \lim_{n \to +\infty} (-1)^n}$$

$$S_n = \frac{(-1)^n}{n}$$



$$\begin{cases} S_{2n} = \dfrac{1}{n} \longrightarrow 0 \\ S_{2n+1} = -\dfrac{1}{n} \longrightarrow 0 \end{cases}$$

DISUGUAGLIANZE TRIANGOLARI

SIANO a E b NUMERI REALI.

① $-|a| \leq a \leq |a|$

② $-|b| \leq b \leq |b|$

① + ② $\Rightarrow$ $-|a| - |b| \leq a + b \leq |a| + |b|$

$$-(|a| + |b|) \leq a+b \leq |a| + |b|$$

$$\boxed{|a+b| \leq |a| + |b|}$$

$$a = a - b + b$$

$$|a| = |a-b+b| \leq |a-b| + |b|$$

1) $|a| - |b| \leq |a-b|$

2) $|b| - |a| \leq |b-a| = |a-b|$

$$\boxed{\big||a| - |b|\big| \leq |a-b|}$$

TEO: SE $S_n \longrightarrow L \Rightarrow |S_n| \longrightarrow |L|$

VERIFICO LA DEFINIZIONE DI LIMITE

NEL CASO $|S_n| \longrightarrow |L|$

$$\forall \varepsilon > 0 \ \exists n_\varepsilon : \forall n \geq n_\varepsilon \Rightarrow \big||S_n| - |L|\big| < \varepsilon$$

$$\big||S_n| - |L|\big| \leq |S_n - L| < \varepsilon$$

CVD

QDE

SE $S_n \to L \Rightarrow |S_n| \to |L|$

$S_n = (-1)^n \quad |S_n| = |(-1)^n| = 1 \to 1$

TEO: SE $|S_n| \to 0 \Rightarrow S_n \to 0$

$\forall \varepsilon > 0 \ \exists n_\varepsilon : \forall n \geq n_\varepsilon \Rightarrow |S_n - 0| < \varepsilon$

$|S_n| < \varepsilon$

$\forall \varepsilon > 0 \ \exists n_\varepsilon : \forall n \geq n_\varepsilon \Rightarrow ||S_n|| < \varepsilon$

$$\underset{|S_n|}{\overset{\|}{||S_n||}}$$

$$S_n = \frac{(-1)^n}{n} \qquad |S_n| = \left|\frac{(-1)^n}{n}\right| = \frac{1}{n} \to 0$$

$\Rightarrow S_n \to 0$

ESERCIZIO: $S_n = \sqrt{n^2+n} - n$

$(a-b)(a+b)$

$$\sqrt{n^2+n} - n = \frac{\left(\sqrt{n^2+n} - n\right)\left(\sqrt{n^2+n} + n\right)}{\sqrt{n^2+n} + n}$$

$$= \frac{\cancel{n^2}+n-\cancel{n^2}}{\sqrt{n^2+n} + n} = \frac{n}{n+\sqrt{n^2+n}}$$

$$= \frac{\cancel{n}}{\cancel{n}+\cancel{n}\sqrt{1+\frac{1}{n}}} = \frac{1}{1+\sqrt{1+\frac{1}{n}}}$$

$$\longrightarrow \frac{1}{1+1} = \frac{1}{2}$$