$$f(x) = -f(-x)$$

$$\frac{1}{\sqrt{2}} \simeq 0.7071$$

$$\sqrt{\frac{3}{2}} \simeq 1.2247$$

$$\int f'(x)\, f(x)^{a}\, dx = \dots = \frac{f(x)^{a+1}}{a+1} + c$$

$f(x) = x\,e^{-x^2}$ $\qquad$ $f(x) = -f(-x)$

1) $D = \mathbb{R}$

2) $f(x) \geq 0 \iff x\,\underbrace{e^{-x^2}}_{>0} \geq 0 \iff x \geq 0$

$f(0) = 0$ $\qquad$ $x\,\underbrace{e^{-x^2}}_{>0} = 0 \iff x = 0$

$$\lim_{x \to +\infty} x\,e^{-x^2} = \lim_{x \to +\infty} \frac{x}{e^{x^2}} = \frac{\infty}{\infty} = 0$$

$$\lim_{x \to -\infty} x\,e^{-x^2} = \lim_{t \to +\infty} -t\,e^{-t^2} = -\lim_{t \to +\infty} t\,e^{-t^2} = 0$$

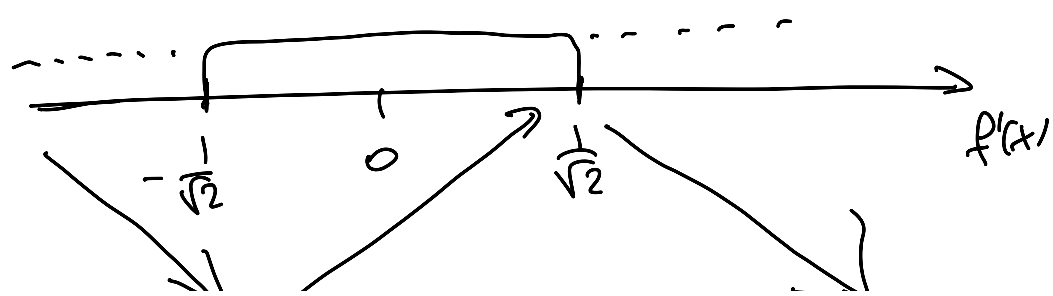
$t = -x \to +\infty$

$y=0$ È UN ASINTOTO ORIZZONTALE

3) $f'(x) = (x\,e^{-x^2})' = e^{-x^2} + x(-2x)\cdot e^{-x^2} =$

$$= \underbrace{e^{-x^2}}_{>0}\left[1 - 2x^2\right] = 0$$

$f'(x) = 0 \iff 1 - 2x^2 = 0 \iff x = +\frac{1}{\sqrt{2}},\ x = -\frac{1}{\sqrt{2}}$



$x = -1/\sqrt{2}$ minimo $\qquad x = +1/\sqrt{2}$ MAX

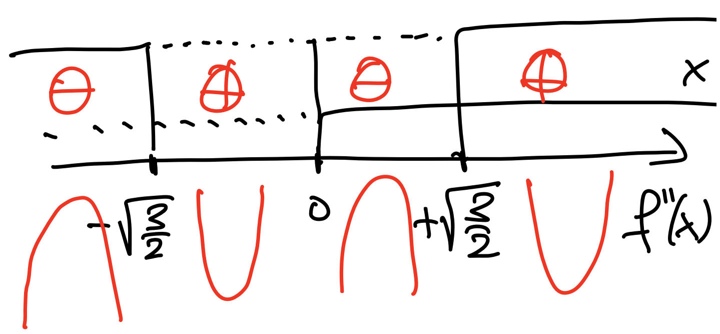
$$f''(x) = \left(e^{-x^2}\left[1-2x^2\right]\right)' = -2x e^{-x^2}\left[1-2x^2\right] + e^{-x^2}(-4x)$$

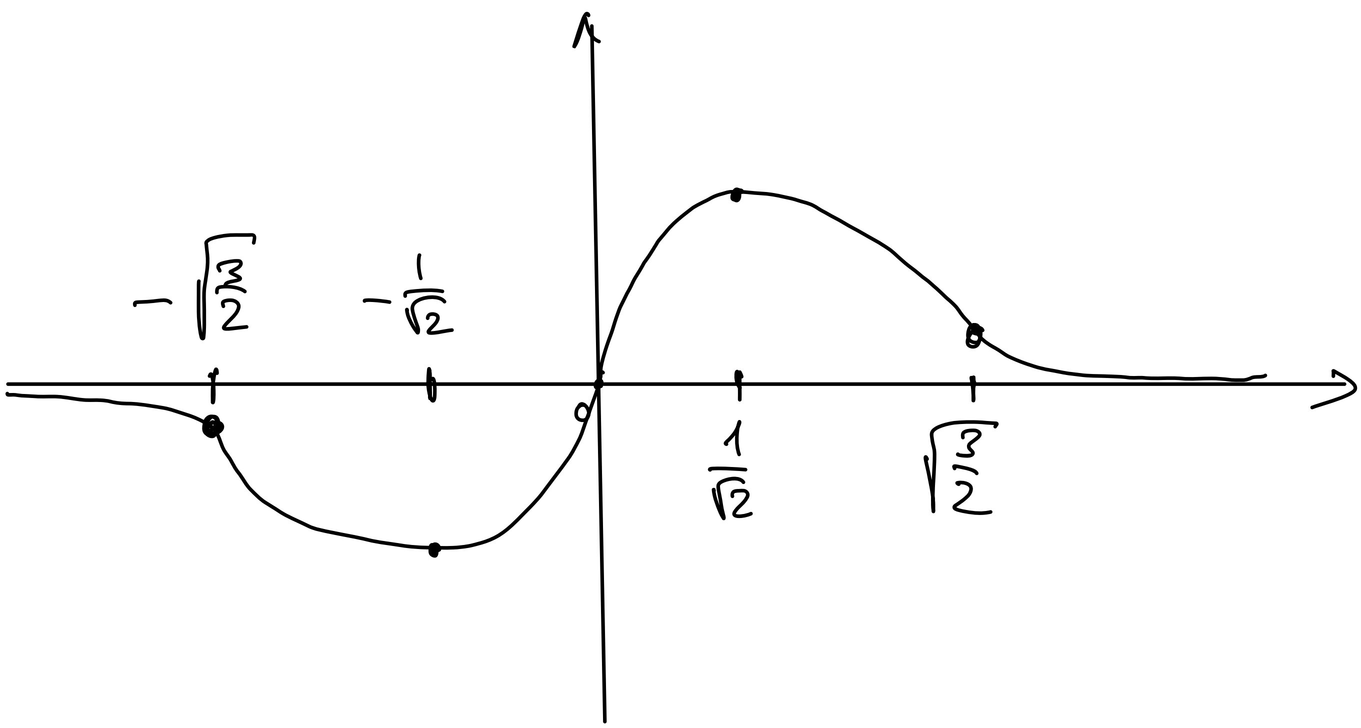
$$= x e^{-x^2}\left[-2(1-2x^2) - 4\right]$$

$$= x e^{-x^2}\left[-2+4x^2-4\right]$$

$$= x e^{-x^2}\left[4x^2-6\right] = 2x e^{-x^2}\left[2x^2-3\right]$$

$$f''(x) = 0 \iff \begin{cases} x = 0 \\ x = -\sqrt{3/2} \\ x = +\sqrt{3/2} \end{cases}$$





$$\int f'(x) \cdot f(x)^{\alpha}\, dx = \frac{f(x)^{\alpha+1}}{\alpha+1} + c$$

$$\left(\frac{f(x)^{\alpha+1}}{\alpha+1} + c \right)' = \frac{\cancel{\alpha+1}}{\cancel{\alpha+1}} f(x)^{\alpha} \cdot f'(x)$$

$$\int \frac{1}{x+3} \cdot \log(x+3)\, dx \qquad \left(\log(x+3) \right)' = \frac{1}{x+3}$$

$\alpha = 1 \quad f(x) = \log(x+3)$

$$\int \frac{1}{x+3} \cdot \log(x+3)\, dx = \frac{1}{2} \left(\log(x+3) \right)^2 + c$$

$$\begin{cases} x + ky - z = 1 \\ (2k+1)x + ky = 0 \\ -x + kz = -1 \end{cases}$$

$$A = \begin{pmatrix} 1 & k & -1 \\ 2k+1 & k & 0 \\ -1 & 0 & k \end{pmatrix}$$

$$\det(A) = 1 \cdot \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} - (2k+1) \cdot \begin{vmatrix} k & -1 \\ 0 & k \end{vmatrix} - 1 \cdot \begin{vmatrix} k & -1 \\ k & 0 \end{vmatrix}$$

$$= 1 \cdot k^2 - (2k+1)\, k^2 - 1 \cdot k$$

$$= k^2 - k^2(2k+1) - k =$$

$$= k\,[k - k(2k+1) - 1] =$$

$$= k\,[\cancel{k} - 2k^2 - \cancel{k} - 1] =$$

$$= -k\,[1 + 2k^2] \neq 0 \iff k \neq 0$$

SE $k \neq 0 \Rightarrow \det(A) \neq 0 \Rightarrow$ IL SISTEMA ESSENDO QUADRATO AMMETTE UN' UNICA SOLUZIONE

VEDIAMO COSA SUCCEDE PER $k = 0$

$$\tilde{A} = (A|b) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \end{array}\right)$$

$$r(A) = r\begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = 2$$

$$r(A|b) = r\begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 \end{pmatrix} = 3$$

PER $k=0$ SISTEMA INCOMPATIBILE

$$\det\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix} = (-1)(-1) = 1$$

$$\sum_{n=0}^{+\infty} \left(\frac{2k}{3}\right)^n \qquad \boxed{\left|\frac{2k}{3}\right| < 1} \iff |k| < \frac{3}{2}$$

$$\sum_{n=0}^{+\infty} \left(\frac{2k}{3}\right)^n = \frac{1}{1-\frac{2k}{3}} = \frac{3}{3-2k}$$

$$f(x) = \log(1+\sin(x))$$

$$f(\pi) \qquad f'(\pi) \qquad f''(\pi) \qquad x_0 = \pi$$

$$f(x) = f(\pi) + f'(\pi)(x-\pi) + \frac{1}{2!} f''(\pi)(x-\pi)^2 + o$$

$$f(\pi) = \log(1) = 0$$

$$f'(x) = \frac{1}{1+\sin(x)} \cdot \cos(x) \Rightarrow f'(\pi) = -1$$

$$f''(x) = (\cos(x))' \cdot \frac{1}{1+\sin(x)} + \cos(x) \cdot \left(\frac{1}{1+\sin(x)}\right)'$$

$$= -\frac{\sin(x)}{1+\sin(x)} + \cos(x)\left(-\frac{1}{(1+\sin(x))^2}\right)\cos(x)$$

$$= -\frac{\sin(x)}{1+\sin(x)} - \frac{\cos^2(x)}{(1+\sin(x))^2}$$

$$= -\frac{\sin(x)(1+\sin(x))+\cos^2(x)}{(1+\sin(x))^2} =$$

$$= -\frac{\sin(x)+\overbrace{\sin^2(x)+\cos^2(x)}^{1}}{(1+\sin(x))^2} = -\frac{\cancel{(1+\sin(x))}}{(1+\sin(x))^{\cancel{2}}}$$

$$= -\frac{1}{1+\sin(x)}$$

$$f''(\pi) = -1$$

$$f(\pi) = 0 \qquad f'(\pi) = -1 \qquad f''(\pi) = -1$$

$$f(x) = -(x-\pi) - \frac{1}{2}(x-\pi)^2 + o\left((x-\pi)^2\right)$$

$$\sum_{n=0}^{+\infty} q^n = \frac{1}{1-q}$$