ESERCITAZIONE
MATEMATICA GENERALE
CLEF

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Determinante Rango e Inversa di Matrice

Esercizio 1. Calcolare il determinante delle seguenti matrici.

1. $\begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}$. [2]
2. $\begin{pmatrix} 5 & -1 \\ 3 & -1 \end{pmatrix}$. [−2]
3. $\begin{pmatrix} 4 & -4 \\ -6 & 6 \end{pmatrix}$. [0]
4. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 3 \\ 1 & 0 & 1 \end{pmatrix}$. [0]
5. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 4 & 0 & 5 \end{pmatrix}$. [13]
6. $\begin{pmatrix} 1 & -1 & 1 \\ -3 & 1 & 3 \\ -2 & 0 & 4 \end{pmatrix}$. [0]
7. $\begin{pmatrix} 6 & 0 & -3 & 5 \\ 4 & 1 & 6 & 8 \\ -1 & 0 & 7 & 4 \\ 8 & 6 & 0 & 2 \end{pmatrix}$. [2]

Esercizio 2. Calcolare il rango delle seguenti matrici.

1. $\begin{pmatrix} 3 & 6 \\ 2 & 5 \end{pmatrix}$. [2]
2. $\begin{pmatrix} 0 & 1 \\ -1 & \pi \end{pmatrix}$. [2]
3. $\begin{pmatrix} 3 & 6 \\ -2 & 0 \\ 1 & 2 \end{pmatrix}$. [2]
4. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$. [1]
5. $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$. [2]
6. $\begin{pmatrix} 1 & 3 & 4 \\ 0 & 3 & 3 \\ 7 & -6 & 1 \end{pmatrix}$. [2]
7. $\begin{pmatrix} 1 & 0 & 1 & 2 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 2 & 4 \end{pmatrix}$. [3]
8. $\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. [3]
9. $\begin{pmatrix} 0 & -2 & 0 & 1 \\ -2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$. [2]
10. $\begin{pmatrix} 1 & -2 & 1 & -2 \\ -2 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 2 & -1 & 3 & 0 \end{pmatrix}$. [4]

Esercizio 3. Calcolare, se esiste, l'inversa delle seguenti matrici:

1. $\begin{pmatrix} 3 & 6 \\ 2 & 5 \end{pmatrix}$.
2. $\begin{pmatrix} 3 & 6 \\ 6 & 12 \end{pmatrix}$.
3. $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$
4. $\begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix}$
5. $\begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 1 \end{pmatrix}$.
6. $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 4 & 0 & 5 \end{pmatrix}$.
7. $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$.

Dipendenza e Indipendenza Lineare

Esercizio 4. Determinare per se esistono e per quali valori dei parametri reali indicati le seguenti uguaglianze risultino valide:

1. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} = a\begin{pmatrix} 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$
2. $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = a\begin{pmatrix} 2 \\ 2 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$
3. $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad a, b, c \in \mathbb{R}$
4. $\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad a, b \in \mathbb{R}$
5. $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad a, b, c \in \mathbb{R}$
6. $\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + b\begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} + c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \quad a, b, c \in \mathbb{R}$
7. $\begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} = a\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}; \quad a, b, c \in \mathbb{R}$
8. $\begin{pmatrix} 1 \\ 0 \\ -2 \\ 2 \end{pmatrix} = a\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + d\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad a, b, c, d \in \mathbb{R}$

Esercizio 5. Stabilire per ciascun dei seguenti sottoinsiemi di vettori il numero di vettori **linearmente indipendenti**, quindi la dimensione del sottospazio vettoriale generato. Si indichi quando i sottoinsiemi sono una base per l'opportuno spazio vettoriale.

$A = \{(1,0),(3,5),(0,7)\} \subseteq \mathbb{R}^2$;

$B = \{(1,0),(1,0),(-1,-2)\} \subseteq \mathbb{R}^2$;

$C = \{(12,-2),(-3,\frac{1}{2})\} \subseteq \mathbb{R}^2$;

$D = \{(0,-1),(-3,4)\} \subseteq \mathbb{R}^2$;

$E = \{(1,0,1),(0,3,5),(0,0,7)\} \subseteq \mathbb{R}^3$;

$F = \{(0,-1,0),(-2,2,0),(0,2,-2)\} \subseteq \mathbb{R}^3$;

$G = \{(0,-1,1),(2,1,0),(2,0,1)\} \subseteq \mathbb{R}^3$;

$H = \{(1,0,2,1),(1,2,0,0),(0,1,0,3),(0,0,2,-1)\} \subseteq \mathbb{R}^4$;

$I = \{(1,0,0,1),(0,1,1,0),(0,2,0,-2),(-2,0,2,0)\} \subseteq \mathbb{R}^4$;

$J = \{(1,0,1,0),(1,1,1,0),(0,2,2,0)\} \subseteq \mathbb{R}^4$;

Esercizio 6. Dire se i seguenti sottoinsiemi di vettori sono una base per lo spazio vettoriale indicato. In caso negativo si determinino i vettori linearmente indipendenti e si esprimano i rimanenti come combinazione lineare di quelli Linearmente Indipendenti.

$A = \{(1,2),(2,1)\} \subseteq \mathbb{R}^2$

$B = \{(0,2),(0,1)\} \subseteq \mathbb{R}^2$

$C = \{(1,1),(0,-1)\} \subseteq \mathbb{R}^2$

$D = \{(-3,-4),(2,\frac{8}{3})\} \subseteq \mathbb{R}^2$

$E = \{(1,0,-1),(2,2,2),(1,1,1)\} \subseteq \mathbb{R}^3$

$F = \{(0,\frac{1}{2},3),(0,\frac{1}{3},2),(0,1,-1)\} \subseteq \mathbb{R}^3$

$G = \{(7,7,7),(6,6,6),(-1,-1,-1)\} \subseteq \mathbb{R}^3$

$H = \{(1,0,0),(-1,-1,0),(0,0,4)\} \subseteq \mathbb{R}^3$

$I = \{(1,1,1,0),(2,2,0,2),(3,0,3,3),(-1,-1,-1,-1)\} \subseteq \mathbb{R}^4$

$J = \{(-1,1,0,0),(1,1,0,1),(1,1,1,0),(-1,0,0,0)\} \subseteq \mathbb{R}^4$

$K = \{(1,3,2,1),(0,1,-2,0),(2,6,4,2),(1,4,0,1)\} \subseteq \mathbb{R}^4$