Point Estimation

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Outline

1. Method of Moments
2. MLE

Method of Moments

The Method of Moments

consists of equating sample moments and population moments moments

Population moments

$$\mu_r(\theta) = E_\theta(X^r) = \int_{\infty}^{\infty} x^r f(x;\theta)dx$$

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$$M_r = \frac{1}{n}\sum_i X_i^r$$

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If a population has q parameters, the MOM consists of solving the system of q equations

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Equate the k-the population moment to the k-th sample moment

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Solve the resulting system of equations for θ

$X \sim N(\mu, \sigma^2)$

$$
\begin{aligned}
\hat{\mu}_{MOM} &= \bar{X} \\
\hat{\sigma}^2_{MOM} &= \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}
\end{aligned}
$$

MOM and unbiasedness

Are MOM estimators unbiased estimators?

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$X \sim U(0, \theta)$

$\hat{\theta}_{MOM}$

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MOM and Sufficiency

Are MOM estimators function of sufficient statistics?

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MOM and Sufficiency

Are MOM estimators function of sufficient statistics?

Intuition

Suppose I tell you I have 100 cookies in my backpack. The cookies are of two types: chocolate chip cookies and fortune cookies. Moreover, I tell you that the number of fortune cookies is either 10 or 90. You draw a cookie out of my backpack at random and see that it is a fortune cookie.

Intuition

Based on this data, what is more likely: there are

1. 10 fortune cookies and 90 chocolate chip cookies, or
2. 90 fortune cookies and 10 chocolate chip cookies?

Based solely on one sample (fortune cookie), 2 is more likely.

Intuition

This is exactly the idea behind maximum likelihood estimation.

The method asks: what value of the a parameter is most consistent with the data?

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In other words, what value of a parameter makes the data most likely?

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Bernoulli

$X \sim Bernoulli(\theta)$

Observations: 3 successes and 2 failures

Which of the following values for θ is the most likely:

0.2 0.4 0.6 0.8

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Likelihood?

$$
\begin{aligned}
L(\theta|x) &= \prod_i \theta^{x_i}(1-\theta)^{1-x_i} \\
L(\theta|x) &= \theta^{\sum_i x_i}(1-\theta)^{n-\sum x_i}
\end{aligned}
$$

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Likelihood

$$
\begin{aligned}
L(0.2|x) &= 0.0051 \\
L(0.4|x) &= 0.023 \\
L(0.6|x) &= 0.0346 \\
L(0.8|x) &= 0.0205
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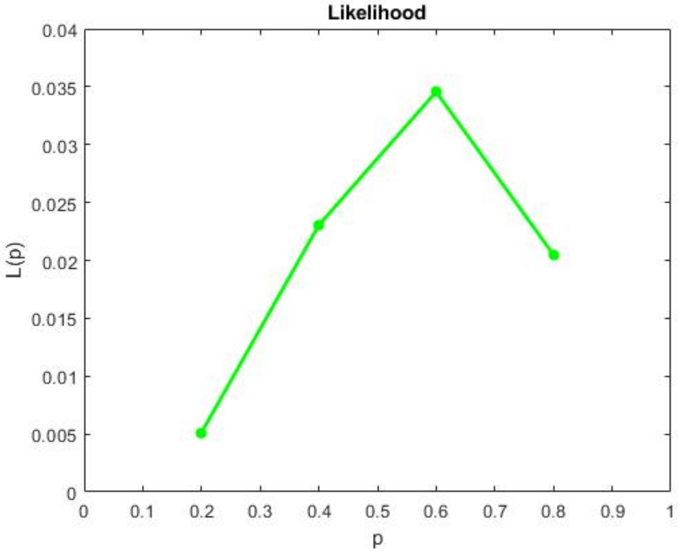
Likelihood
L(p)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p

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More likely value? $\hat{p} = 0.6$

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Observations: 3 successes and 2 failures

Likelihood

$$L(\theta|x) = \theta^{\sum_i x_i}(1-\theta)^{n-\sum x_i}$$

$$\hat{\theta}_{MLE} = \hat{\theta}(x) \qquad such \quad that \qquad L(\hat{\theta}) = sup_{\theta\in\Theta} L(\theta)$$

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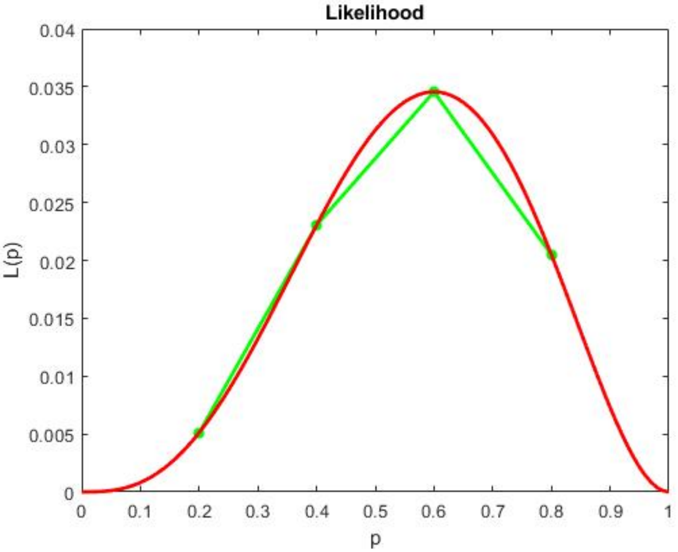
Likelihood
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
L(p)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p

Gaussian

$X \sim N(\mu, \sigma = 10)$

Observations:

160 170 166 173 172

Likelihood

$$L(\mu|x) = \prod_{i=1}^{n}\left(\frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)\right)$$

$$L(\mu|x) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{n/2} exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right)$$

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Likelihood

$$L(164|x) = 3.7 \times 10^{-8}$$
$$L(166|x) = 5.09 \times 10^{-8}$$
$$L(168|x) = 5.74 \times 10^{-8}$$
$$L(170|x) = 5.3 \times 10^{-8}$$
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$$160 \quad 170 \quad 166 \quad 173 \quad 172$$

Likelihood

$$log(L(164|x)) = -17.113 \qquad log(L(166|x)) = -16.793$$
$$log(L(168|x)) = -16.673 \qquad log(L(170|x)) = -16.753$$
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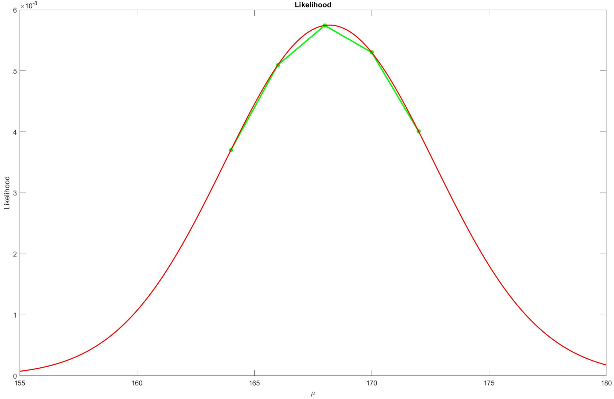
Gaussian-Likelihood



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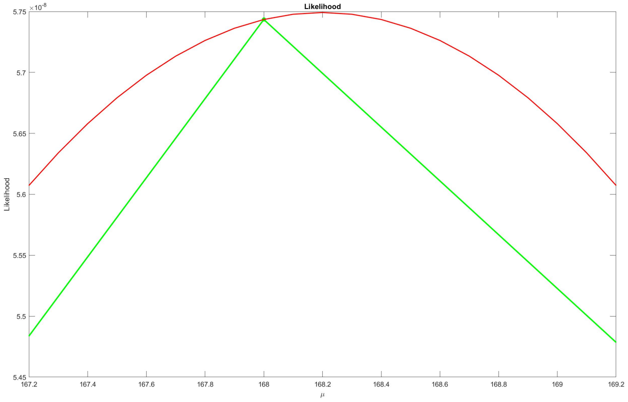
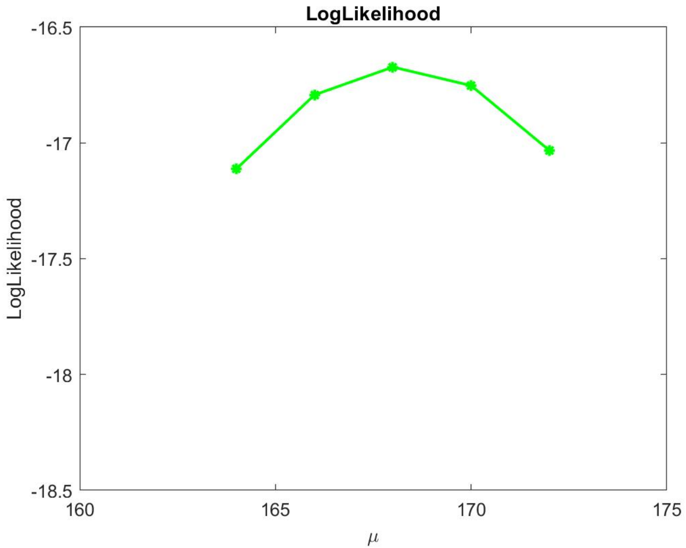


Gaussian-LogLikelihood



Gaussian-LogLikelihood