

Statistics Fall 2024 - TA Session 3

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Problem 1

Prove that \bar{X} , the mean of a random sample of size n from a distribution that is $\mathcal{N}(\theta, \sigma^2)$ ($-\infty < \theta < +\infty$) is, for every known $\sigma^2 > 0$, an efficient estimator of θ .

Problem 2

Let X_1, X_2, \dots, X_n be a random sample on X that has a **Gamma** ($\alpha = 4, \beta = \theta$) distribution with $0 < \theta < \infty$.

- (a) Find the MLE of θ .
- (b) Find the Fisher information $I(\theta)$.
- (c) Show that the MLE of θ , which was derived in point (a), is an efficient estimator of θ .
- (d) Obtain the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$.

Problem 3

If X_1, X_2, \dots, X_n is a random sample from a distribution with pdf:

$$f(x; \theta) = \begin{cases} \frac{3\theta^3}{(x+\theta)^4} & 0 < x < \infty \\ 0 & elsewhere \end{cases}$$

Show that $Y = 2\bar{X}$ is an unbiased estimator of θ and determine its efficiency.