Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

$$\frac{f(x_1, x_2, \ldots, x_n|\lambda_0)}{f(x_1, x_2, \ldots, x_n|\lambda_1)} < k$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

$$\frac{f(x_1, x_2, \ldots, x_n|\lambda_0)}{f(x_1, x_2, \ldots, x_n|\lambda_1)} < k$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

- $H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$
- Reject when

$$
\begin{aligned}
\frac{f(x_1, x_2, \ldots, x_n|\lambda_0)}{f(x_1, x_2, \ldots, x_n|\lambda_1)} &< k \\
\frac{\prod_i \lambda_0 exp(-\lambda_0 x_i)}{\prod_i \lambda_1 exp(-\lambda_1 x_i)} &< k \\
\frac{\lambda_0^n exp(-\lambda_0 \sum x_i)}{\lambda_1^n exp(-\lambda_1 \sum x_i)} &< k
\end{aligned}
$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

- $H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$
- Reject when

$$
\begin{aligned}
\left(\frac{\lambda_0}{\lambda_1}\right)^n exp\left(-(\lambda_0 - \lambda_1)\sum_i x_i\right) &< k \\
exp\left(-(\lambda_0 - \lambda_1)\sum_i x_i\right) &< k_2 \\
-(\lambda_0 - \lambda_1)\sum_i x_i &< k_3 \\
\sum_i x_i &> k
\end{aligned}
$$

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

Neyman Pearson test - Exponential

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

$$R = \{(x_1, x_2, \dots, x_n) : \sum_i x_i > k\}$$

Neyman Pearson test - Exponential

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

$$R = \{(x_1, x_2, \dots, x_n) : \sum_i x_i > k\}$$

Which value for k? USE α

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

Reject when

$$R = \{(x_1, x_2, \ldots, x_n) : \sum_i x_i > k\}$$

Which value for k? USE α

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

$$R = \{(x_1, x_2, \ldots, x_n) : \sum_i x_i > k\}$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

$$R = \{(x_1, x_2, \ldots, x_n) : \sum_i x_i > k\}$$

$$P\left((x_1, x_2, \ldots, x_n) \in R | H_0\ true\right) = \alpha$$

$$P\left(\sum_i x_i > k \in R | \lambda = \lambda_0\right) = \alpha$$

$$\sum_i X_i \sim Gamma(\lambda_0, n)$$

$$R = \{\underline{x} : \sum_i x_i > InvGamma(1 - \alpha, \lambda_0, n)\}$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

$$R = \{(x_1, x_2, \ldots, x_n) : \sum_i x_i > k\}$$

$$P\left((x_1, x_2, \ldots, x_n) \in R | H_0 \; true\right) = \alpha$$

$$P\left(\sum_i x_i > k \in R | \lambda = \lambda_0\right) = \alpha$$

$$\sum_i X_i \sim Gamma(\lambda_0, n)$$

$$R = \{\underline{x} : \sum_i x_i > InvGamma(1 - \alpha, \lambda_0, n)\}$$

Neyman Pearson test - Exponential

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Exponential*(λ) such that $E(X) = \frac{1}{\lambda}$ known

$H_0 : \lambda = \lambda_0$ vs $H_1 : \lambda = \lambda_1 \qquad \lambda_0 > \lambda_1$

$$R = \{(x_1, x_2, \ldots, x_n) : \sum_i x_i > k\}$$

$$P\left((x_1, x_2, \ldots, x_n) \in R | H_0 \; true\right) = \alpha$$

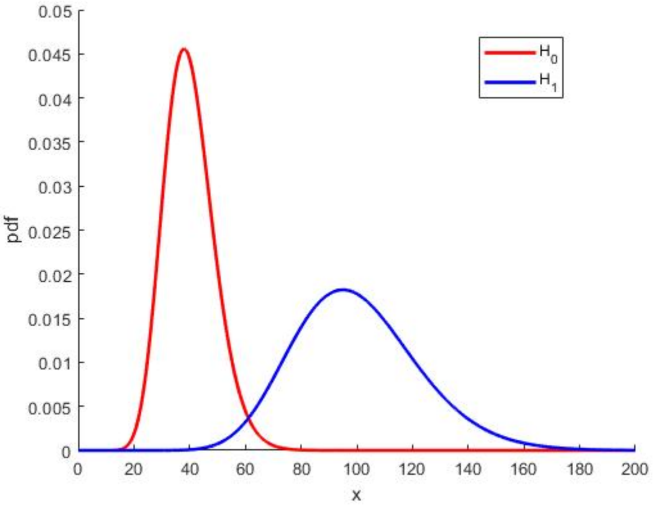
$$P\left(\sum_i x_i > k \in R | \lambda = \lambda_0\right) = \alpha$$

$$\sum_i X_i \sim Gamma(\lambda_0, n)$$

$$R = \{\underline{x} : \sum_i x_i > InvGamma(1 - \alpha, \lambda_0, n)\}$$

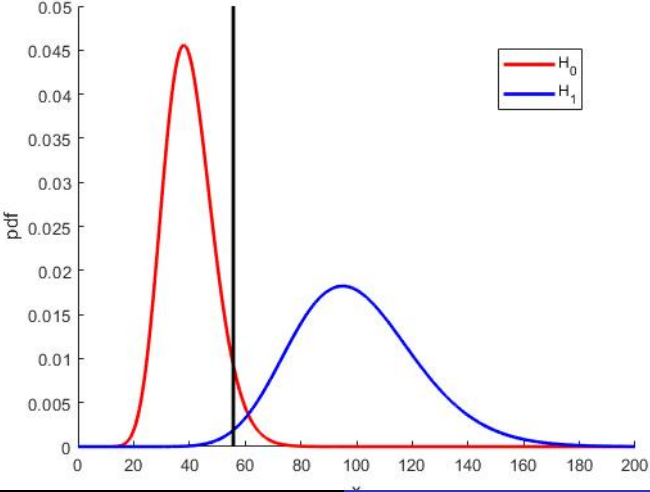
Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis



Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis and Rejection Region



Neyman Pearson - Exponential, $\lambda_0 = 0.5\, \lambda_1 = 0.2\, n = 20$, α

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis

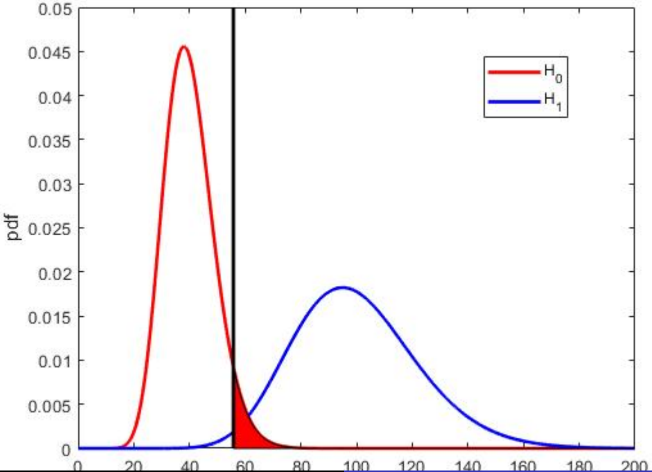
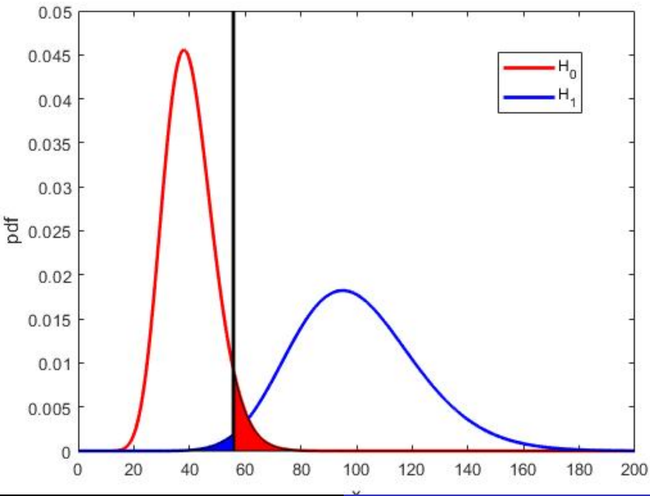


Figure: Distribution of $\sum X_i$ under null and alternative hypothesis, α and β



Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\frac{f(x_1, x_2, \ldots, x_n|\mu_0)}{f(x_1, x_2, \ldots, x_n|\mu_1)} < k$$

$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i-\mu_0)^2}{2\sigma^2}\right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i-\mu_1)^2}{2\sigma^2}\right)} < k$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\frac{f(x_1, x_2, \ldots, x_n | \mu_0)}{f(x_1, x_2, \ldots, x_n | \mu_1)} < k$$

$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i - \mu_0)^2}{2\sigma^2}\right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i - \mu_1)^2}{2\sigma^2}\right)} < k$$

Neyman Pearson test - Gaussian

$$
\begin{aligned}
\frac{f(x_1, x_2, \ldots, x_n|\mu_0)}{f(x_1, x_2, \ldots, x_n|\mu_1)} &< k \\
\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i-\mu_0)^2}{2\sigma^2}\right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) exp\left(-\frac{(x_i-\mu_1)^2}{2\sigma^2}\right)} &< k \\
\frac{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n exp\left(-\sum_i \frac{(x_i-\mu_0)^2}{2\sigma^2}\right)}{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n exp\left(-\sum_i \frac{(x_i-\mu_1)^2}{2\sigma^2}\right)} &< k
\end{aligned}
$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$
\begin{aligned}
\left(-\sum_i \frac{(x_i-\mu_0)^2}{2\sigma^2}\right) - \left(-\sum_i \frac{(x_i-\mu_1)^2}{2\sigma^2}\right) &< k_1 \\
\left(-\sum_i (x_i-\mu_0)^2\right) - \left(-\sum_i (x_i-\mu_1)^2\right) &< k_2 \\
-\mu_0^2 + \mu_1^2 + 2\sum_i x_i(\mu_0-\mu_1) &< k_3 \\
\sum_i x_i &< k
\end{aligned}
$$

Neyman Pearson test - Gaussian

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\left(-\sum_i \frac{(x_i - \mu_0)^2}{2\sigma^2}\right) - \left(-\sum_i \frac{(x_i - \mu_1)^2}{2\sigma^2}\right) < k_1$$
$$\left(-\sum_i (x_i - \mu_0)^2\right) - \left(-\sum_i (x_i - \mu_1)^2\right) < k_2$$
$$-\mu_0^2 + \mu_1^2 + 2\sum_i x_i(\mu_0 - \mu_1) < k_3$$
$$\sum_i x_i < k$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

Which value for k? USE α

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

Which value for k? USE α

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

$$R = \left\{ \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq -z_{1-\alpha} \right\}$$

Neyman Pearson test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1 \qquad \mu_0 > \mu_1$

Reject when

$$\sum_i x_i \quad < \quad k$$

$$R = \left\{ \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq -z_{1-\alpha} \right\}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\lambda(\mathbf{x}) = \frac{sup_{\mu_0} L(\mu|\mathbf{x})}{sup_{\mu} L(\mu|\mathbf{x})} \leq k$$

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\lambda(\mathbf{x}) = \frac{sup_{\mu_0} L(\mu|\mathbf{x})}{sup_{\mu} L(\mu|\mathbf{x})} \leq k$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}_0^2}}\right) exp\left(-\frac{(x_i-\mu_0)^2}{2\hat{\sigma}_0^2}\right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}^2}}\right) exp\left(-\frac{(x_i-\hat{\mu})^2}{2\hat{\sigma}^2}\right)} < k$$

$$\hat{\sigma}_0 = \frac{\sum_i (x_i - \mu_0)^2}{n}$$

$$\hat{\sigma} = \frac{\sum_i (x_i - \bar{X})^2}{n}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}_0^2}}\right) exp\left(-\frac{(x_i-\mu_0)^2}{2\hat{\sigma}_0^2}\right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}^2}}\right) exp\left(-\frac{(x_i-\hat{\mu})^2}{2\hat{\sigma}^2}\right)} \quad < \quad k$$

$$\hat{\sigma}_0 = \frac{\sum_i (x_i - \mu_0)^2}{n}$$

$$\hat{\sigma} = \frac{\sum_i (x_i - \bar{X})^2}{n}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\frac{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}_0^2}}\right)^n exp\left(-\frac{n}{2}\right)}{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}^2}}\right)^n exp\left(-\frac{n}{2}\right)} < k$$

$$\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2} < k$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\frac{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}_0^2}}\right)^n exp\left(-\frac{n}{2}\right)}{\left(\frac{1}{\sqrt{2\pi\hat{\sigma}^2}}\right)^n exp\left(-\frac{n}{2}\right)} < k$$

$$\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2} < k$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$\begin{aligned}
\frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \mu_0)^2} &< k \\
\frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \bar{X})^2 + n(\bar{x} - \mu_0)^2} &< k \\
\frac{1}{1 + n\frac{(\bar{x}-\mu_0)^2}{\sum_i (x_i - \bar{x})^2}} &< k
\end{aligned}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$
\begin{aligned}
\frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \mu_0)^2} &< k \\
\frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \bar{X})^2 + n(\bar{x} - \mu_0)^2} &< k \\
\frac{1}{1 + n\frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2}} &< k
\end{aligned}
$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$n \frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2} \geq k$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$n \frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2} \quad \geq \quad k$$

Which value for k?

Use α

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$n \frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2} \quad \geq \quad k$$

Which value for k?

Use $\quad \alpha$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$R = \left\{ \left| \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \right| \geq t_{1-\alpha/2}^{n-1} \right\}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as *Gaussian*(μ, σ^2), σ^2 unknown,

$H_0 : \mu = \mu_0$ vs $H_1 : \mu \neq \mu_0$

Reject when

$$R = \left\{ \left| \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \right| \geq t^{n-1}_{1-\alpha/2} \right\}$$

Likelihood ratio test: Example

Suppose that the distribution of lifetimes of TV tubes can be adequately modelled by an exponential distribution with mean λ so

$$f(x|\lambda) = \frac{1}{\lambda} exp\left(-\frac{x}{\lambda}\right)$$

for $x \geq 0$. Under usual production conditions, the mean lifetime is 50 hours but if a fault occurs in the process, the mean lifetime drops to 40 hours. A random sample of 20 tube lifetimes is to taken in order to test the hypotheses

$$H_0 : \lambda = 50 \qquad H_1 : \lambda = 30.$$

And the following statistics is observed $\sum_i x_i = 680$
Use the Neyman-Pearson lemma to find the most powerful test with significance level $\alpha = 0.05$.

Likelihood ratio test: Example

$$R = \left\{ x : \sum_i x_i \leq K \right\}$$

$$R = \left\{ x : \sum_i x_i \leq 662.73 \right\}$$

$$R = \{ x : \bar{x} \leq 33.14 \}$$

ACCEPT NULL HYPOTHESIS