

Statistics Fall 2024 - TA Session 4

TA: Giacomo Caserta - giacomo.caserta@uniroma2.it
Office Hour: By appointment, office 3D-8, third floor building B

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Problem 1

Let x_1, \dots, x_n be a random sample from $\mathcal{N}(\mu, \sigma^2)$.

- (a) Find the Cramer-Rao lower bound for $\hat{\sigma}^2$
- (b) Compare the following estimators in terms of MSE and their variances with the lower bound computed before.

$$\tilde{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{X})^2$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{X})^2$$

Problem 2

Let x_1, \dots, x_n be a random sample from an exponential distribution with unknown parameter λ .

- (a) Find the likelihood ratio (LR) statistic for testing $H_0 : \lambda = \lambda_0$
- (b) Find the Wald statistic for testing $H_0 : \lambda = \lambda_0$ with $\hat{\lambda}_{ML} = \frac{1}{\bar{x}}$
- (c) Find the Score statistic for testing $H_0 : \lambda = \lambda_0$

Problem 3

Let x_1, \dots, x_{100} be a random sample from:

$$f(x|\theta) = x \cdot \frac{1}{\theta} \exp\left(-\frac{x^2}{2\theta}\right), \quad x > 0, \theta > 0$$

With:

$$\sum_{i=1}^{100} x_i^2 = 2600$$

- (a) Test $H_0 : \theta = 10$ according to the Likelihood Ratio Test at a 5% confidence level.
- (b) Test $H_0 : \theta = 10$ according to the Wald statistic at a 5% confidence level.
- (c) Test $H_0 : \theta = 10$ according to the Score statistic at the 5% confidence level.