# Statistics, Fall 2025 - Practice Session 4
# Interval Estimation

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## Problem 1

Let $\{X_i\}_{i=1}^n$ be a random sample such that $X_i \sim N(\mu, \sigma^2)$, where $\sigma^2$ is an unknown parameter. Calculate the shortest confidence interval for $\sigma$ given some significance level $\alpha$.

## Problem 2

Suppose that a random sample $\{X_i\}_{i=1}^n$ follows a density

$$f(x; b) = bx^{b-1}\mathbb{1}_{[0,1]}(x)$$

Find the symmetric $(1-\alpha)$ confidence interval for $b$.

## Problem 3

Suppose a random sample $\{X_i\}_{i=1}^n$ follows the Geometric distribution with parameter $p$. For $n \to \infty$, find the $(1-\alpha)$ confidence interval of $p$.

## Notes

- The CDF $F(X)$ of any random variable $X$ is distributed as $F(X) \sim U(0,1)$
- If $X_i \sim Exp(c)$, then $\frac{2}{c}\sum_{i=1}^{n} X_i \sim \chi^2(2n)$
- The probability mass function of the Geometric distribution with parameter $1-p$ is

$$(1-p)p^x \mathbb{1}_{\mathbb{N}_0}(x)$$

- If $\{X_i\}_{i=1}^{n}$ and $X_i \sim N(\mu, \sigma^2)$, then

$$(n-1)\frac{S^2}{\sigma^2} \sim \chi(n-1)$$

  where $S^2$ is the sample variance.