Corso di Matematica Generale
Esercitazione 3 - Soluzioni

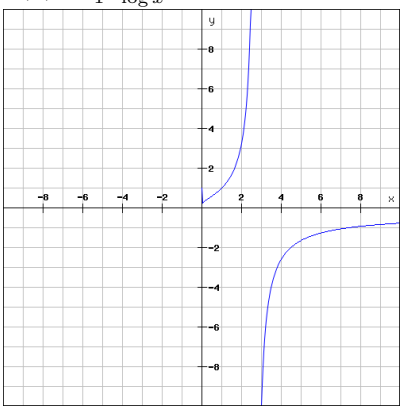
Dr. Stefano Guarino
guarino@mat.uniroma3.it

9 Ottobre, 2014

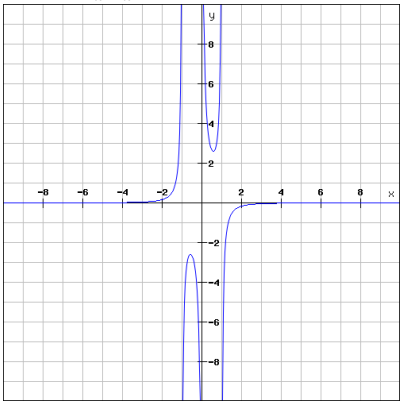
0. 1. $f(x) = |x^2 - 1| + x$
   $D = \mathbb{R}$, $Im = [-1, +\infty)$, $f(x) > 0$ se $x < \frac{-1-\sqrt{5}}{2}$ o $x > \frac{1-\sqrt{5}}{2}$.
   2. $f(x) = \ln(\sin x)$
   $D = \cup_{k\in\mathbb{Z}}(2k\pi, (2k+1)\pi)$, $Im = (-\infty, 0]$, $f(x) \leq 0\ \forall x \in D$.

1. (a) $f(x) = 2^x$
   $f((-\infty, 1]) = (0, 1], \inf = 0, \max = 1$, $f^{-1}((-\infty, 1])$ mal definito: $f^{-1}(y) = \log_2(y)$ è definito solo per $y > 0$.
   (b) $f(x) = \sqrt{x}$
   $f\left([0, \frac{9}{4}]\right) = [0, \frac{3}{2}], \min = 0, \max = \frac{3}{2}$, $f^{-1}((-1, 2]) = [0, 4], \min = 0, \max = 4$.
   (c) $f(x) = \frac{1}{x}$
   $f((-1, 0)) = (-1, -\infty), \inf = -\infty, \sup = -1$, $f((0, 2]) = [\frac{1}{2}, +\infty), \min = \frac{1}{2}, \sup = +\infty$,
   $f^{-1}([0, 1])$ è mal definito: $f^{-1}(y) = \frac{1}{y}$ è definito solo per $y \neq 0$.
   (d) $f(x) = \cos x$
   $f((-\infty, \pi)) = [-1, 1], \min = -1, \max = 1$, $f\left((-\frac{\pi}{6}, \frac{2\pi}{3}]\right) = (-\frac{1}{2}, 1], \inf = -\frac{1}{2}, \max = 1$,
   $f^{-1}\left([-1, \frac{1}{2})\right) = (\frac{\pi}{3}, \pi], \inf = \frac{\pi}{3}, \max = \pi$.
   (e) $f(x) = \log_2 x$
   $f((0, 32]) = (-\infty, 5], \inf = -\infty, \max = 5$, $f^{-1}(-\infty, 1] = (0, 1], \inf = 0, \max = 1$, $f^{-1}((-\infty, 1])$.

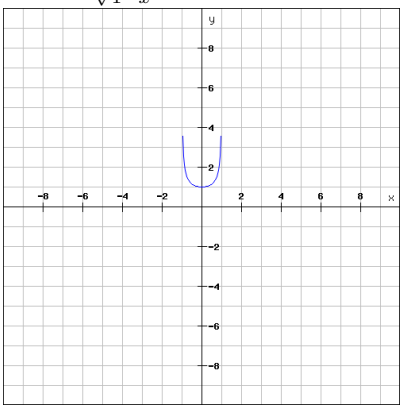
2. Nota: per questo esercizio, invece della soluzione forniremo il grafico della funzione, dal quale dovrebbe essere possibile ricavare facilmente dominio, intersezioni e segno.
   (a) $f(x) = \frac{1}{1-\log x}$



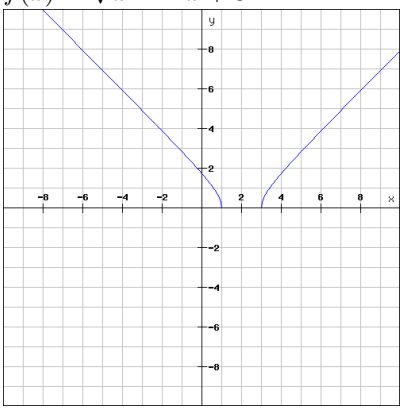
   (b) $f(x) = \frac{1}{x-x^3}$



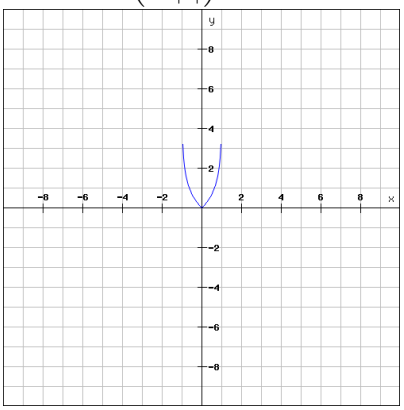
(c) $f(x) = \frac{1}{\sqrt{1-x^2}}$



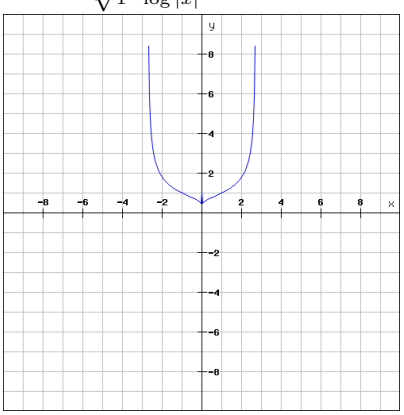
(d) $f(x) = \sqrt{x^2 - 4x + 3}$



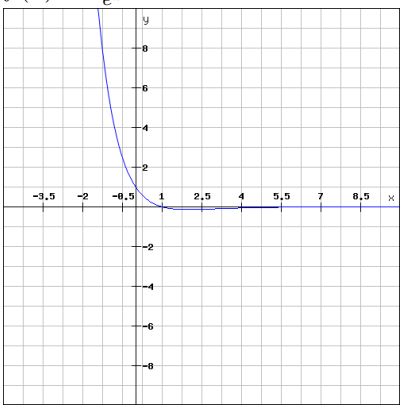
(e) $f(x) = \log\left(\frac{1}{1-|x|}\right)$



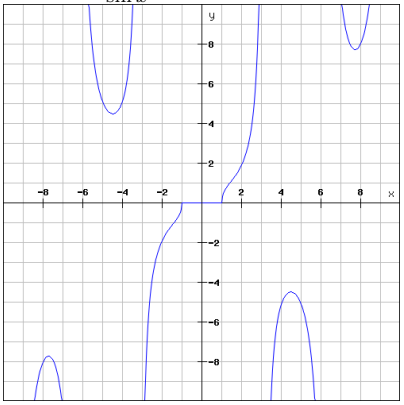
(f) $f(x) = \frac{1}{\sqrt{1-\log|x|}}$



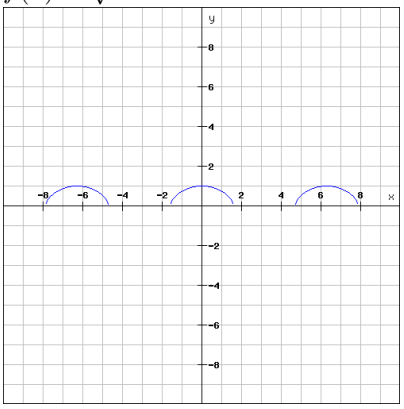
(g) $f(x) = \frac{1-x}{e^x}$



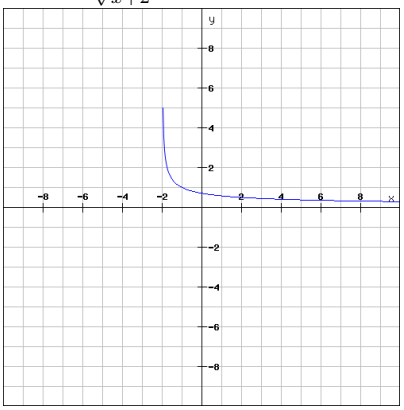
(h) $f(x) = \frac{\sqrt{x^2-1}}{\sin x}$



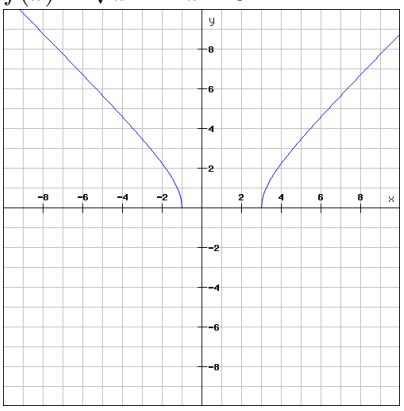
(i) $f(x) = \sqrt{\cos x}$



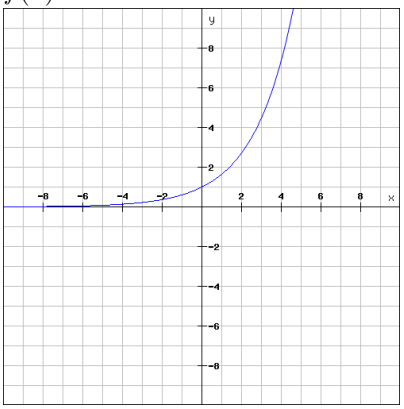
(j) $f(x) = \frac{1}{\sqrt{x+2}}$



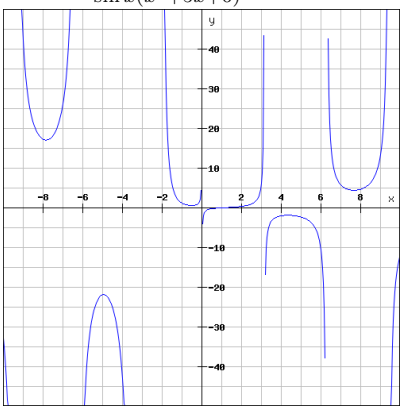
(k) $f(x) = \sqrt{x^2 - 2x - 3}$



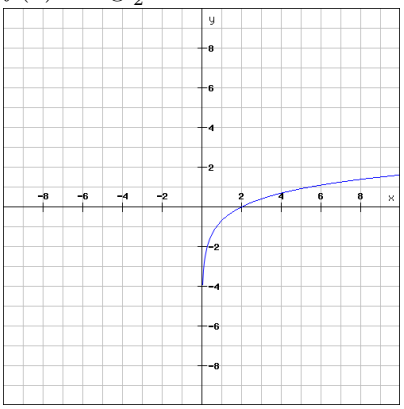
(l) $f(x) = e^{\frac{x}{2}}$



(m) $f(x) = \frac{x^3-1}{\sin x(x^2+5x+6)}$



(n) $f(x) = \log \frac{x}{2}$



(o) $f(x) = \frac{1-x^2}{x^3-4x}$