ESERCITAZIONE di MATEMATICA GENERALE - CLEF

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Es. 1. Calcolare il rango delle seguenti matrici utilizzando il Teorema di Kronecker.

(1.a) $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$. [2]

(1.b) $\begin{pmatrix} \pi & -3 \\ 2\pi & -6 \end{pmatrix}$. [1]

(1.c) $\begin{pmatrix} 6 & 6 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$. [2]

(1.d) $\begin{pmatrix} 2 & -1 \\ -4 & -2 \\ 1/3 & -2/3 \end{pmatrix}$. [1]

(1.e) $\begin{pmatrix} -1 & 3 & 4 \\ 0 & 3 & 3 \\ 2 & -6 & -8 \end{pmatrix}$. [2]

(1.f) $\begin{pmatrix} -1 & -1 & 7 & 2 \\ -1 & 3 & 0 & 0 \\ 2 & 2 & -14 & 4 \end{pmatrix}$. [3]

(1.g) $\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$. [2]

(1.h) $\begin{pmatrix} 1 & -3 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 1 \end{pmatrix}$. [2]

(1.i) $\begin{pmatrix} 5 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 3 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. [4]

Es. 2. Calcolare la matrice inversa, quando esiste, delle seguenti matrici.

(2.a) $\begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$. $\left[\begin{pmatrix} -1 & \frac{2}{3} \\ 1 & -\frac{1}{3} \end{pmatrix}\right]$

(2.b) $\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}$. [$\nexists$]

(2.c) $\begin{pmatrix} 3 & 2 \\ 5 & -1 \end{pmatrix}$ $\left[\begin{pmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{pmatrix}\right]$

(2.d) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$ $\left[\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 1 \end{pmatrix}\right]$

(2.e) $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 0 & 0 \\ 2 & 0 & 1 \end{pmatrix}$. $\left[\begin{pmatrix} 0 & -1 & 0 \\ -1 & -3 & 0 \\ 0 & 2 & 1 \end{pmatrix}\right]$

(2.f) $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & 0 & 3 \end{pmatrix}$. $\left[\begin{pmatrix} 0 & \frac{3}{7} & \frac{2}{7} \\ \frac{1}{2} & -\frac{1}{14} & -\frac{3}{14} \\ 0 & -\frac{2}{7} & \frac{1}{7} \end{pmatrix}\right]$

(2.g) $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & -1 \end{pmatrix}$. $\left[\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 1 & -1 & -1 \end{pmatrix}\right]$

Sistemi lineari

Es. 3. Risolvere i seguenti sistemi lineari.

1. $$\begin{cases} 2x - 5y = 0 \\ x - 4y + 2 = 0 \end{cases}$$

2. $$\begin{cases} x - 3y = 3 \\ -2x + 6y = -1 \end{cases}$$

3. $$\begin{cases} x + y = 4 \\ 2x - 3y = 7 \end{cases}$$

4. $$\begin{cases} x + y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

5. $$\begin{cases} x + y - z = 1 \\ 2x + 2y + z = 0 \\ x + y + 2z = -1 \end{cases}$$

6. $$\begin{cases} -2x + y + z = 1 \\ x - 2y + z = -2 \\ x + y - 2z = 4 \end{cases}$$

7. $$\begin{cases} 2x - y + z = 0 \\ 3x + 2y - 5z = 1 \\ x + 3y - 2z = 4 \end{cases}$$

8. $$\begin{cases} x + 2y = 1 \\ -x + 2y + 2z = -2 \\ x - y - z = 3 \end{cases}$$

9. $$\begin{cases} x + 3y - z = 2 \\ 2x + y - z = 1 \\ 2x - y = -1 \end{cases}$$

10. $$\begin{cases} 2x - y - z - 4w = 9 \\ 4x - 3z - w = 0 \\ 8x - 2y - 5z - 9w = 18 \\ 2y - 3z + w = 3 \end{cases}$$

Es. 4. Discutere le soluzioni dei seguenti sistemi lineari al variare del parametro $k \in \mathbb{R}$.

1. $$\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (k^2 - 14)z = k + 2 \end{cases}$$

2. $$\begin{cases} kx - y + z = 2 \\ x - ky + z = 3 - k^2 \\ x - y + kz = k + 1 \end{cases}$$

3. $$\begin{cases} x + y + z = k \\ x - ky - z = 1 \\ 2x + y + kz = k + 1 \end{cases}$$

4. $$\begin{cases} x + y + kz = 2k - 1 \\ x + ky + z = k \\ kx + y + z = 1 \end{cases}$$

5. $$\begin{cases} 2x + kz = 1 \\ 3x + ky - 2z = 2 \\ kx + 2z = 1 \end{cases}$$

6. $$\begin{cases} x + y - z = 1 \\ 2x + 3y + kz = 3 \\ x + ky + 3z = k \end{cases}$$

7.
$$\begin{cases} kx + y + z = 1 \\ x + ky + z = 1 \\ x + y + kz = k \end{cases}$$

8.
$$\begin{cases} x - y + z = 5 \\ 2x + y + 2z = k \\ -3x - 3y + kz = 1 \end{cases}$$

Autovalori e autovettori

Es. 5. Trovare gli autovalori e gli autovettori delle seguenti matrici.

1.
$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

2.
$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 5 \end{pmatrix}$$

3.
$$\begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$

4.
$$\begin{pmatrix} 1 & -2 & -1 \\ -2 & 0 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

5.
$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 3 & 4 & -1 \end{pmatrix}$$

6.
$$\begin{pmatrix} 0 & -2 & -2 \\ 2 & 4 & 2 \\ -2 & -2 & 0 \end{pmatrix}$$

7.
$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & 3 & 2 \end{pmatrix}$$

8.
$$\begin{pmatrix} 1 & 2 & -4 \\ 2 & -2 & -2 \\ -4 & -2 & 2 \end{pmatrix}$$

9.
$$\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 6 & 6 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & -2 & -2 \end{pmatrix}$$